

**J.A. CROWTHER**

**IONS, ELECTRONS,  
AND  
IONIZING RADIATIONS**



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## CHAPTER I

### INTRODUCTION

1. **Electrons.** Historically the subject dealt with in the present text-book may be said to have taken its origin in experiments on the discharge of electricity through gases at low pressure contained in some form of discharge tube. When a high-tension discharge is sent through a gas at low pressure (say some fraction of a millimetre of mercury) the gas becomes luminous showing a series of glows and striae (see Fig. 27), which are often very beautiful, but which, at the time of their discovery, were very difficult to explain on the current theories of the time. If the pressure is sufficiently low, a series of streamers known as the cathode rays can be observed proceeding in straight lines from the cathode and crossing the other glows in the tube. The experiments of Sir William Crookes led him to the conclusion that these rays were material in nature, consisting of some new manifestation of matter to which he gave the term *radiant matter*. On the other hand there existed, especially in Germany, a large school of thought which was inclined to assign the effects to some sort of wave motion in the ether, probably of very short wavelength, and thus analogous to ultra-violet light.

The controversy raged hotly and somewhat inconclusively for nearly twenty years. It was not until 1897 that Sir J. J. Thomson, following up the work of Perrin, proved conclusively that the cathode rays consisted of negatively charged particles. These experiments may be said to mark a new epoch in physical science. The method evolved enabled him to measure not only the velocity of the particles but also the ratio of the mass of an individual particle to the charge upon it. It was found that the velocity of the particles was a function of the difference of potential across the terminals of the discharge tube. In general its value lay between  $10^9$  and  $10^{10}$  cm. per second, or roughly between one-thirtieth and one-third of that of light. The ratio of the mass of a cathode particle to the charge upon it was, however, within the limits of experimental error, perfectly constant and independent of all the circumstances of the experiment. It was found to have the same value no matter what the potential across the tube, no matter what the chemical nature of the substance used as the cathode, and



no matter what the nature and pressure of the residual gas within the tube. It was in fact a universal constant. These particles can be produced from and thus are contained in all chemical substances.

It was later found possible to determine separately both the mass and the charge on these particles or *electrons* as they are now called. It was found that the mass of an electron was very small compared with that of the lightest known atom, being only about 1/1834th of that of a hydrogen atom. The mass of an electron has been found to be  $9.0 \times 10^{-28}$  gm. and its charge  $4.80 \times 10^{-10}$  electrostatic units or  $1.60 \times 10^{-20}$  electromagnetic units of charge. The radius of the electron has been estimated at approximately  $1.9 \times 10^{-13}$  cm. The radius of an atom is about  $10^{-8}$  cm. The radius of an electron is thus about 1/50,000th of that of the atom. Such electrons must, as we have already seen, form a part of every known kind of matter.

No charge has yet been observed which is smaller than that on the electron. Every charge observed which is sufficiently small to be directly compared with that of the electron has been found to be an exact integral multiple of it. We are thus led to the conclusion that *electricity is atomic in structure*; the smallest possible unit of charge being that of the electron, which thus constitutes our fundamental unit of electricity. It is generally supposed that the "mass" of the electron is entirely an electrical effect, and is due solely to the charge which it carries. The electron therefore constitutes a real atom of electricity.

Recently a *positively* charged counterpart to the negative electron has been discovered. The term electron, however, had been used for so long as signifying a negative electron that it was convenient to continue to use it in this sense, and to find a new name for the positively charged particle. It has been agreed to call the latter a *positron*.

**2. Conduction through gases.** Even before the discoveries outlined in the previous section, it had become known that gases which ordinarily are among our best insulators of electricity can be rendered partially conducting by the action of certain agents upon them. The discovery of X-rays by Röntgen in 1895 was quickly followed by the observation that charged bodies rapidly lost their charge when an X-ray bulb was worked in their vicinity. This loss of charge was shown to take place through the surrounding air, which became temporarily conducting. It was shown that the conductivity of the gas was due to the creation in it of positively and negatively charged systems, which by analogy with the carriers in electrolytic conduction were termed *gaseous ions*. These ions must, on the electron theory,

consist of molecules of the gas which have lost or gained one or more electrons. Further investigation showed that similar properties could be conferred upon a gas by the action of cathode rays. Owing partly to the small number of the carriers, partly to the great simplicity in the structure of gases, a study of these carriers has thrown considerable light on some obscure parts of science.

The existence of these gaseous ions was controverted for some little time, but no other explanation has been found adequate to explain the phenomena. Their objective reality was placed beyond all reasonable doubt by the experiments of C. T. R. Wilson, who showed that these ions could serve as nuclei for the condensation of drops of water in a supersaturated space. The ions were thus rendered visible by the deposition of the water upon them, and could readily be observed, and even photographed. Photographs such as that of Fig. 17, which represents the track of a cathode particle through air, make it certain that the production of conductivity in a gas by these ionizing radiations is due to the formation in the gas of definite charged systems.

**3. Radio-active substances and their radiations.** The discovery of X-rays and the somewhat startling manner in which these rays were able to penetrate considerable thicknesses of materials which were opaque to ordinary light naturally led to an exhaustive search among natural substances to see if any of these emitted radiations having properties at all resembling those of X-rays. The research was pursued with vigour by Becquerel and afterwards by the Curies, and led to the discovery in 1896 that radiations capable of penetrating opaque substances and of causing conductivity in gases were emitted by the salts of uranium and thorium. The subsequent observation that naturally occurring ores of uranium were several times more active than the metal itself led to the isolation in 1898 of polonium, radium, and subsequently of other similar substances which were found to possess these properties in a very remarkable degree. Substances which are capable of emitting spontaneously these ionizing radiations are classed together under the title *radio-active substances*.

It has been shown that each radio-active substance is a definite chemical element, and that its activity is due to a spontaneous decomposition or disintegration as it is usually termed of the radio-active atom into one of smaller weight, the process taking place according to definite laws which have been determined. This naturally leads us to the view that the atoms of the various chemical elements are not essentially different in kind but represent complex systems built up out of one or two fundamental particles according to certain definite laws.

On further investigation it was found that the rays given out by these radio-active substances were of three kinds, which were denoted by the non-committal symbols  $\alpha$ ,  $\beta$ , and  $\gamma$ . All three types possess in common with X-rays the properties of converting a gas into a conductor, of affecting a photographic plate, and of causing luminescence in a fluorescent screen. In other respects however they differ very markedly in their properties.

The  $\alpha$ -rays were found to be positively charged and were of the mass of atoms. They have been proved to consist of helium atoms which have lost two electrons and thus possess a positive charge of  $2 \times (4.80 \times 10^{-10})$  or  $9.6 \times 10^{-10}$  e.s.u. They are very easily absorbed in matter, being completely stopped by  $\frac{1}{16}$  mm. of aluminium or a few centimetres of air at atmospheric pressure. They produce very intense ionization in the gas through which they pass. Their velocity depends on the substance from which they are ejected; their nature is always the same.

The  $\beta$ -rays on the other hand carry a negative charge. They can penetrate much greater thicknesses of matter than the  $\alpha$ -rays, producing measurable effects through a thickness of even a centimetre of aluminium. The ionization they produce in a gas is, however, small compared with that of the  $\alpha$ -rays. Their nature is found to be identical with that of the cathode rays. They consist of electrons, carrying the universal electronic charge, and differ from the cathode rays only in velocity. Whereas it is difficult to obtain cathode rays with a velocity much greater than one-half that of light, the  $\beta$ -rays from radium possess velocities which range up to within one or two per cent. of that of light itself.

The  $\gamma$ -rays are exceedingly penetrating, those from radium producing quite measurable effects through a thickness of 20 or 30 cm. of iron. Their ionizing power is small as the fraction of their energy which can be absorbed by any reasonable thickness of gas is minute. They are now known to be pulses in the ether of very short wavelength (less than  $10^{-9}$  cm.). X-rays are also ether pulses; those most commonly employed having wavelengths between  $10^{-9}$  cm. and  $10^{-8}$  cm. Recent developments in X-ray generators have, however, made it possible not merely to overlap with the  $\gamma$ -ray spectrum but even to extend beyond it to still shorter wavelengths. The only distinction between X-rays and  $\gamma$ -rays is their mode of origin. Together they represent the extreme short-wave end of the electromagnetic wave-scale.

It has been found that in some of its interactions with matter it is necessary to suppose that the energy of radiation is concentrated

in numerous definite small bundles, or "quanta," and that, in these cases, the action of the radiation can best be compared with that of a hail of bullets. When it is desired to emphasize this corpuscular character of the radiation each of these corpuscles is referred to as a *photon*. Since the energy of a photon is inversely proportional to the wavelength, the effect is most prominent in the case of X- and  $\gamma$ -radiation, but it can also be observed with ordinary light.

## CHAPTER II

### PASSAGE OF A CURRENT THROUGH AN IONIZED GAS

**4. Conductivity of gases in the normal state.** A gas in its normal state is one of the best insulators of electricity known. It was indeed for a long time a moot point whether gases conducted electricity at all, and whether the gradual loss of charge which always occurs from charged bodies might not be due to slight defects in the insulating

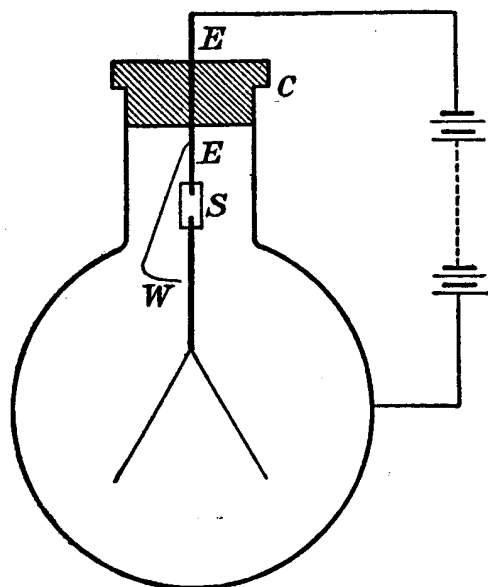


FIG. 1. Apparatus for demonstrating the conductivity of air

qualities of the solid materials used to support the body. The matter was settled by C. T. R. Wilson using the ingenious apparatus shown in Fig. 1. The gas is enclosed in a large spherical vessel which is silvered inside to make it conducting. The charged system takes the form of a brass rod carrying a pair of thin gold leaves. In this way the charged body is made to serve as its own electroscope. Since the electrical capacity of a system such as this is very small, a very small loss of charge will be sufficient to produce a large fall of potential, and hence a considerable movement of the gold leaves, which are

closely observed with a long focus microscope having a micrometer scale in the eye-piece. The apparatus is thus exceedingly sensitive.

The end of the rod carrying the gold leaves is fused into a block of sulphur *S*, sulphur when carefully prepared being an exceedingly good insulator. The sulphur block is supported by another brass rod *E* passing through an ebonite stopper *C* in the neck of the vessel.

The rod *E* is charged to some suitable potential (about 400 volts) and is maintained at this potential throughout the experiment, most conveniently by connecting it to one end of a cabinet of small accumulator cells, the other terminal of which is connected to the silver coating



of the vessel. Contact between  $E$  and the gold leaves can be made, when required, by means of the fine iron wire  $W$  which can be attracted by a magnet so as to touch the gold leaf system, and then allowed to fly back leaving the latter perfectly insulated. The gold leaves are then at the potential of  $E$ .

Since the latter remains at the same potential throughout, any leak across the sulphur plug will only tend to keep the deflection of the leaves at their original value. Any loss of charge must take place therefore across the gas. Even under these circumstances it was found that the leaves gradually became discharged, the loss of charge per second being about  $10^{-8} V$  e.s.u. of charge where  $V$  is the volume of the vessel.

We see therefore that even in its normal state a gas allows the passage of a small current of electricity. Under certain circumstances, however, the gas acquires a conductivity many times greater than the normal, and these currents have acquired considerable importance in the development of modern electrical theory.

If, for example, a quantity of radium is brought near the electroscope of Fig. 1, the leaves collapse with considerable rapidity, and similar effects are observed if an X-ray tube is worked in the vicinity. Gases drawn from a Bunsen flame also possess temporarily the power to discharge a conductor, and this capacity is enormously increased if the flame is fed with a volatile salt such as sodium or potassium chloride.

The power to discharge a conductor persists in the gas for some little time after the active agent has been removed, and is transferred from place to place with the air. Thus if air from a Bunsen flame (Fig. 2) is drawn

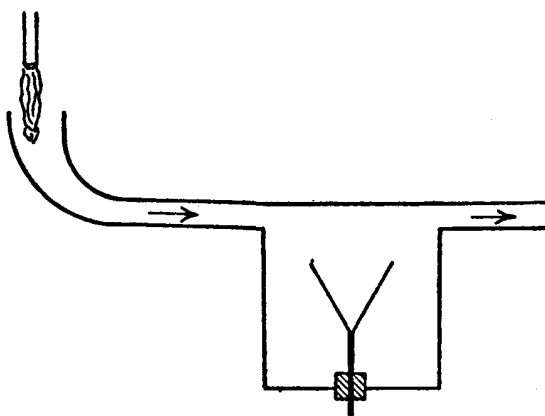


FIG. 2. Transference of conductivity by a current of air

in a continuous current into an electroscope, the latter will be discharged. If, however, the gas is filtered through tightly packed glass-wool or passed through water the effect disappears, and it can also be removed by subjecting the gas to a strong electric field. It disappears spontaneously if the gas is allowed to stand for a few minutes.

**5. Ionization theory of gaseous conduction.** The phenomena of the passage of electricity through gases can best be explained by assuming that charged systems are set up in the gas by the action of the various

agencies, and that these charged systems or gaseous ions, as they are called, convey the current across the gas by moving in opposite directions through the gas under the action of the electrostatic field set up by the charged body, in much the same way that a current is conveyed through the solution of an electrolyte by the motion of the positively and negatively charged electrolytic ions.

Thus if the body has a positive charge and is surrounded by a gas containing gaseous ions the negative ions will be attracted to the body and, giving up their charges to it, will gradually bring about its discharge. The difference between electrolytic and gaseous conduction which experimentally is very marked is due to two principal causes. In the first place the electrolytic ions are formed by the mere process of solution and persist so long as the salt remains dissolved, while the gaseous ions are only formed by the action of an external agency which we may call the *ionizing agent*, and gradually disappear when the latter is removed. In the second place the number of charged systems present in a gas under the most intense ionizing agent is only a very small fraction of those present even in very dilute electrolytic solutions.

**6. The electrostatic method of measuring current.** Before proceeding to describe the phenomena attending the passage of an electric current through gases it will be convenient to explain briefly the methods available for measuring such currents. The discharge through a normal gas is, as we have seen, about  $10^{-8}$  electrostatic units (e.s.u.) per unit volume. If the volume of gas is one litre this corresponds to a current of  $10^{-5}$  e.s.u., since current is the rate of flow of electricity. The current through the gas is thus equal to  $10^{-5}/3 \times 10^{10}$  in absolute electromagnetic units (e.m.u.), that is  $\frac{1}{3} \times 10^{-15}$  e.m.u. or about  $3.3 \times 10^{-15}$  amperes. This is about one ten thousandth of the smallest current which could be detected on our most sensitive galvanometers. Thus even if the conductivity of the gas is increased a thousandfold by the ionizing agent the current will still be beyond the range of a galvanometer.

Let  $Q$  be the charge at any instant on an electrical conductor of capacity  $C$  immersed in an ionized gas. The potential  $V$  of the conductor is given by the equation

$$Q = C \cdot V.$$

The current  $i$  through the gas at any instant is given by the rate of loss of charge of the conductor. Thus

$$i = dQ/dt = C \cdot dV/dt \quad . \quad . \quad . \quad . \quad . \quad (1)$$

since the capacity  $C$  is constant.

Thus if the rate of loss of potential can be measured, as for example by measuring the rate of collapse of the leaves of an electroscope, the current through the gas can be calculated when the capacity of the charged system is known.

In this form of measurement the potential across the gas must be adjusted to suit the instrument used. Thus if the gold leaf electroscope requires a potential difference of, say, 100 volts to produce a sensible deflection of the leaves we cannot work with potential differences of less than this amount. It is often, therefore, more convenient to measure the rate at which an insulated conductor receives electric charge from the gas under the action of a field supplied by another electrode immersed in the gas. The current will clearly be measured by  $i = C \cdot dV/dt$  as before, where  $C$  is the combined capacity of the receiving electrode and the measuring instrument, and  $dV/dt$  the rate at which the potential of the latter increases.

**7. Measuring instruments.** For measuring the variation in potential some type of electroscope or electrometer is employed. The *Dolezalek electrometer* (see Fig. 3) is an improved form of quadrant electrometer, in which special attention is paid to obtaining not only a high degree of sensitiveness but also a small capacity. The quadrants are small and are carefully insulated on amber supports. The needle is made as light as possible and supported by a fine strip of phosphor-bronze. The instrument is used heterostatically, the needle being kept at a constant potential of about 120 volts by means of a high-tension battery. One pair of quadrants of the electrometer is connected to the electrode at which the current is to be measured, the other pair being permanently earthed. If the collecting electrode is initially earthed, the rate of increase of deflection of the needle is proportional to the rate of rise of potential of the insulated quadrants, that is, to  $dV/dt$ .

If the deflection is measured by a reflected beam of light in the usual way, a deflection of 1000 mm. per volt can be obtained on a scale placed 1 metre from the instrument. The capacity of the electrometer is generally of the order of 50 e.s.u. An improved form of this instrument, introduced by Compton, with a still smaller capacity, gives a sensitivity under similar conditions of 10,000 mm. per volt.

The simple type of gold-leaf electroscope, described in § 4, in which the gold-leaf system is actually surrounded by the ionized gas, is still employed in certain types of measurement, particularly in comparing the ionization produced by the  $\gamma$ -radiation from different samples of

radio-active material. Its small voltage sensitivity is compensated for by its very small electrostatic capacity and its high insulation.

The *Lindemann electrometer* combines some of the merits of both instruments. It consists of a vertical gilded quartz fibre *nn* (Fig. 4), which is supported by stretched horizontal quartz fibres, so that it lies symmetrically between four parallel brass plates, *ABCD*. The plates are charged oppositely, in pairs, as shown in the diagram, to a potential

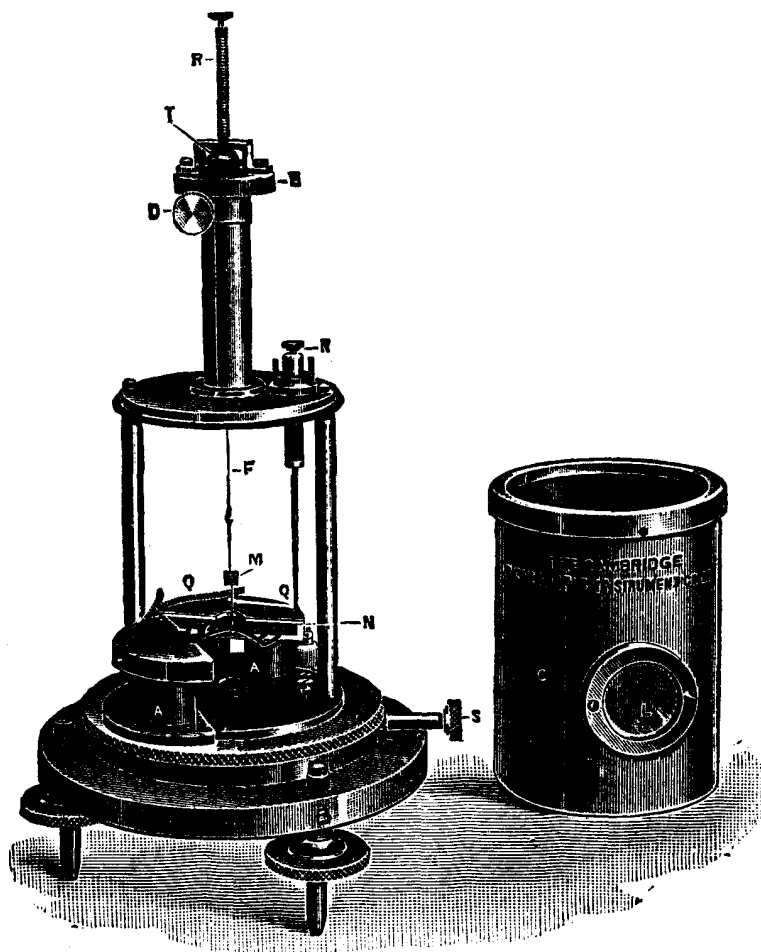


FIG. 3. The Dolezalek electrometer

of approximately  $\pm 30$  volts. The small potential to be measured is applied to the quartz needle. As the potentials of the plates are gradually raised the needle approaches a condition of instability, and just below this point the instrument is very sensitive. The quartz needle makes an ideal object for high-power magnification, and owing to the mode of suspension the instrument is insensitive both to vibrations and tilt. A sensitivity of the order of 100 divisions per volt can be reached, with a capacity of only a few e.s.u.

A comparison of the current sensitivity of two instruments affords a useful exercise in the principles of the method of measurement. Let us assume that a Dolezalek electrometer gives a deflection of 1000 divisions per volt and a Lindemann 100, and that in each case a rate of deflection of one division in 10 seconds can be measured. In the case of the Dolezalek this corresponds to a value of  $dV/dt$  of  $10^{-4}$  volts per second. The capacity of the instrument is, say, 50 e.s.u., or since one farad or practical unit of capacity is equal to  $9 \times 10^{11}$  e.s.u. the capacity of our electrometer is  $50/(9 \times 10^{11})$  or approximately  $6 \times 10^{-11}$  farads. The smallest current which can be measured with accuracy is therefore  $6 \times 10^{-11} \times 10^{-4}$ , that is  $6 \times 10^{-15}$  amperes.

For the Lindemann the rate of change of potential is on the same assumptions 1/1000 volt per sec., while the capacity is, say, 2 e.s.u., or about  $2.2 \times 10^{-12}$  farads. The smallest measureable current is thus  $2.2 \times 10^{-12} \times \frac{1}{1000}$  or  $2.2 \times 10^{-15}$  amperes. Thus, the instrument with the higher voltage sensitivity may have the smaller current sensitivity owing to its greater electrostatic capacity.

**8. Ionization current measurement.** The actual form of the vessel used for containing, and applying a field to, the gas varies very much with the nature of the effect to be observed. A suitable form for investigating the variation of current with potential, and for many other experiments in which the electric field needs to be definite, is shown in Fig. 5.

The gas under investigation is contained in a metal box which is connected to earth, thus serving to shield the electrodes *A* and *B* from any stray electrostatic field. For a similar reason the wire leading from the insulated electrode *A* to the electrometer should also be surrounded by an earthed metal tube (not shown in the diagram). The electrodes are placed parallel to each other a few centimetres apart; *A* is connected to one pair of quadrants of a Dolezalek, or to the needle of a Lindemann electrometer, and *B* is raised to any required potential by means of a high-tension battery. The system *A* can be placed at zero potential by means of a key *K*, which can be earth-connected. When the key *K* is raised the system *A* is completely insulated. If *K* is connected to a potentiometer, instead of directly to

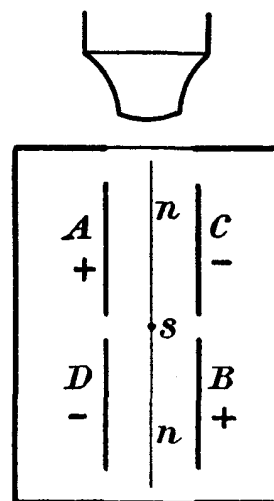


FIG. 4. The Lindemann electrometer



earth, the system *A* with its attached quadrants can be raised to a suitable known potential, and thus the electrometer can be calibrated.

The ionizing agent may be enclosed in the box, but it is generally more convenient to have it outside, in which case a window *W* of some substance transparent to the ionizing rays, and generally consisting of a very thin sheet of aluminium foil, must be made in the side of the box. The electrodes are insulated from the box by insulating stoppers *S, S*. The insulation of *A* is very important, and should be of amber, sulphur, or quartz. The box and its electrodes form what is known as an *ionization chamber*.

Suppose *B* is now raised to some constant potential, say 200 volts positive, while *A* is earthed by the key *K*, and the gas exposed to the

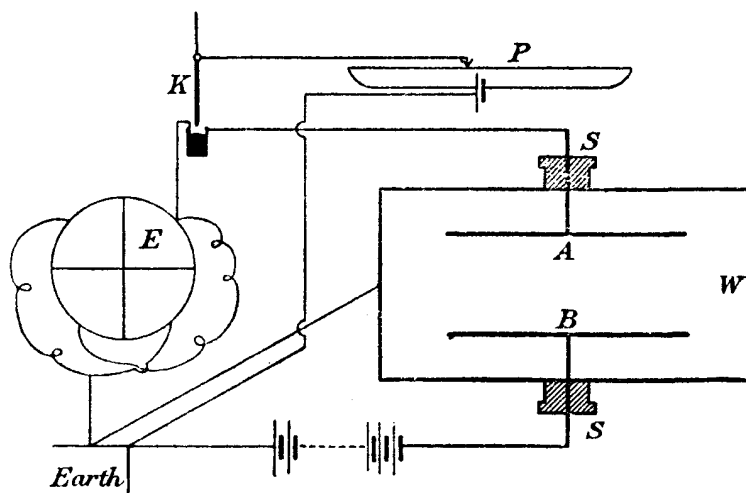


FIG. 5. Apparatus for measuring ionization currents

action of the ionizing agent. The ions in the gas begin to move under the action of the field between the plates, the positive towards *A*, the negative towards *B*. As long as the wire *K* remains in the mercury cup, the potential of *A* remains zero. As soon as it is withdrawn *A* begins to charge up positively, and the needle of the electrometer begins to move. Let *t* be the time taken for the needle to reach the deflection corresponding to a potential *V*. Then if *C* is the capacity of *A* and its connected quadrants, etc., the average current through the gas is given by

$$i = C \cdot \frac{V}{t}.$$

The actual potential corresponding to a given deflection can easily be found by lowering the key *K* again into the mercury cup, and sliding the end of the wire connected to *K* along the potentiometer wire

until the deflection reaches the given value. The corresponding potential can then be read off on the potentiometer.

For the measurement of very small ionization currents, less than, say,  $10^{-13}$  amperes, the methods outlined in the preceding sections are essential. For larger currents a convenient alternative is provided by the *electrometer valve*. The electrometer valve differs from an ordinary triode valve mainly in the insulation of the grid, the lead from which is brought out through a long tube of special high-resistance glass. In use, the grid  $G$  is connected directly to the insulated plate of the ionization chamber (Fig. 6) and any leakage would reduce the charge collected from the gas in the chamber. The filament  $F$  and anode  $A$

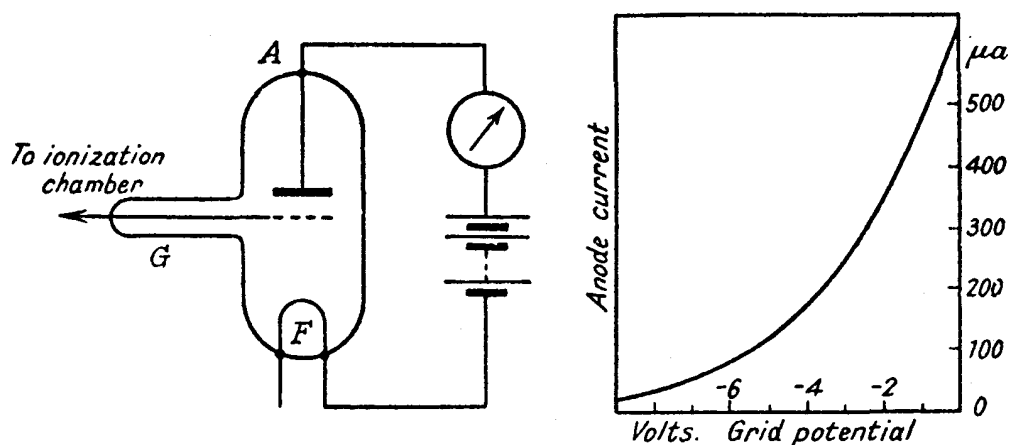


FIG. 6. The electrometer valve

of the valve are connected through a micro-ammeter and a 6-volt battery.

Owing to the space-charge effect, the current in the anode circuit is controlled by the voltage of the grid, an increase in the negative potential of the grid producing a decrease in the anode current of about 100 micro-amperes per volt. After calibration, the readings of the micro-ammeter thus provide a measure of the potential of the grid and the attached electrode, and the ionization current can be deduced from the capacity and rate of change of potential, in the usual way. Since the calibration curve of the valve is a function of the filament current and the plate voltage great care has to be taken to maintain the constancy of the working conditions, and the actual circuits employed are generally more complicated than is indicated in Fig. 6.

**9. Variation of the current with the potential across the gas.** Suppose now the potential of  $B$  (Fig. 5) is gradually increased from some small

initial value. For simplicity we will suppose that the maximum voltage acquired by  $A$  in charging up is not sufficient to affect materially the potential difference between the plates. We can then study the magnitude of the current through the ionized gas for different potential differences between the plates. In the case of a metallic conductor or of an electrolyte with non-polarizable electrodes, the relation is expressed by Ohm's law, that is, the current is simply proportional to the potential difference. The relation for a gas is, however, more complex, and is represented by a curve such as that of Fig. 7, which is plotted from an actual set of readings.

For very small voltages, say, less than 1 volt, the current is approximately proportional to the applied potential difference; that is at this

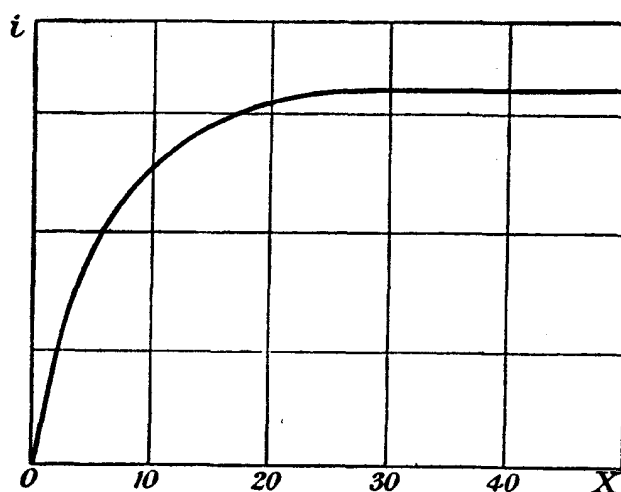


FIG. 7. Current-potential graph for a uniformly ionized gas

stage the gas obeys Ohm's law, though not with the same degree of accuracy as a metallic conductor. As, however, the potential of  $B$  is raised, a stage is reached when the current falls appreciably below the value which would be given by Ohm's law.

If the potential is increased still further a point is reached at which further increase in the potential of  $B$  produces no further increase in the current through the gas. After this point the current is independent of the voltage. This maximum current is called the *saturation current* through the gas, and the potential necessary to produce this current is known as the *saturation voltage*. The actual value of the saturation voltage depends upon the distance apart of the electrodes, the pressure, and the intensity of the ionization in the gas. Except in cases of very intense ionization, a field of from 20 to 30 volts per cm. between the electrodes is sufficient to produce the saturation current.

**10. Variation of the saturation current with the distance between the electrodes, and the pressure of the gas.** If one of the electrodes is made movable the variation of the saturation current with the distance apart of the electrodes can be studied. In the case of a metallic conductor the current for a given potential difference is inversely proportional to the length of the conductor, and a similar result holds for electrolytes. For gases however, if we arrange that the effect of the ionizing agent is the same in all parts of the gas, the saturation current through the gas is directly proportional to the distance apart of the electrodes. It can also be shown, by using electrodes of different sizes, to be directly proportional to the area of the electrodes. If the pressure of the gas is varied it is found that the saturation current is directly proportional to the pressure of the gas. In other words the saturation current through a gas is directly proportional to the mass of gas between the electrodes.

The effects when the current is not saturated are in general very complex, and need not concern us here.

**11. Theory of conduction through gases.** Let us assume that the ionizing agent is producing ions uniformly throughout the gas and at a constant rate, so that  $q$  pairs of ions are formed per c.c. per second in the gas. These ions are charged positively and negatively, and, since the gas as a whole is uncharged, the number of positive and negative systems must be equal, supposing that each carries a charge of the same magnitude  $e$ .

In general ions may be lost by the gas in three ways. In the first place since the ions are oppositely charged they attract each other according to the ordinary laws of electrostatics. On collision their charges may neutralize each other, and the ions return to their ordinary uncharged state. The chance of a given positively charged ion meeting a negative ion in a given time is obviously proportional to the number,  $n_2$ , of negative ions present per c.c., while the number of positive ions finding partners in a given time is proportional to the number,  $n_1$ , of positive ions actually present per c.c. of the gas. The rate at which combination goes on is thus proportional to  $n_1 n_2$ . It can, therefore, be written  $\alpha n_1 n_2$ , where  $\alpha$  is a constant under given conditions and is known as the *coefficient of recombination*. If, as is usually the case, the numbers of positive and negative ions are equal, this reduces to  $\alpha n^2$ .

Ions are also withdrawn by the action of the electric field, and by diffusion to the electrodes or to the walls of the chamber. In general the effect of diffusion is small and may be neglected. In special

circumstances it may become appreciable, and in these cases it can be measured.

Let us assume for a moment that there is no electric field across the gas and that the diffusion is negligible. Then the rate of increase in the number of ions per unit volume of the gas is equal to the number formed per second by the action of the ionizing agent less the number lost by recombination. Hence

$$\frac{dn}{dt} = q - an^2.$$

The ionization reaches a steady state when  $dn/dt$  is equal to zero or

$$q = \alpha n^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the ionizing agent is then withdrawn  $q$  becomes zero, and the ionization in the gas decreases according to the equation

$$\frac{dn}{dt} = -\alpha n^2,$$

$$\text{or} \quad \frac{1}{n_0} - \frac{1}{n} = -\alpha t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where  $n_0$  is the value of the ionization at the moment when the agent was removed, and  $t$  the time which has elapsed.

**12. Motion of the ions under a uniform electric field.** If  $X$  is the strength of the electric field across the gas the force acting on any ion is equal to  $Xe$ . If the ion were perfectly free to move this would produce a constant acceleration of  $Xe/m$  where  $m$  is the mass of the ion. The ion is, however, moving through other uncharged gas molecules; its path therefore is made up of a series of short runs, each terminating in a collision in which all the accumulated energy of the ion is lost. Under these circumstances it is easy to show that the ion will move not with a uniform acceleration but with a constant average velocity which is proportional to the field applied.

For let  $\lambda$  be the mean free path of the ion in the gas, and  $\tau$  the time taken by the ion to describe its mean free path under the action of the field. We may suppose that the whole of the velocity added by the field is removed at each collision and that the ion thus starts its course again with zero velocity. Between two collisions the ion moves freely under the action of the field. If  $X$  is the intensity of the field and  $e$  the charge on the ion, the mechanical force is  $Xe$  and the velocity  $v'$  of the ion on reaching the end of its path is therefore given by  $Xe\tau = mv'$ , or

$$v' = \frac{X_{e\tau}}{m}.$$





loss of ions is due solely to recombination and  $n$  is a constant, its value being given by (2).

If the electrodes are arranged so that the field is uniform, as for example in the case of two parallel plates, the field is equal to  $V/d$ , where  $V$  is the potential difference between the electrodes and  $d$  their distance apart. In this case Equation (5) becomes

$$i = Ane(k_1 + k_2)X = Ane(k_1 + k_2)\frac{V}{d} \quad . \quad . \quad . \quad . \quad (6)$$

where  $A$  is the area of either electrode. This equation is of the same form as that for conduction through an electrolyte, if  $n$  is a constant. Under these circumstances the current through the gas obeys Ohm's law, and this case corresponds to the first straight portion of our experimental curve.

(2) *The field is so large that the ions are conveyed to the electrodes without appreciable recombination.*

If the field is large the velocities become so great that the number recombining in the small time taken by an ion to reach an electrode becomes negligible. In this case it is obvious that all the ions formed in the gas by the ionizing agent will be conveyed to one or other of the electrodes. If  $B$  is the volume of the gas between the electrodes the number of ions formed per second is  $Bq$  and the total charge conveyed to either electrode in 1 second is therefore  $Bqe$ . This is the maximum current through the gas, and, provided that the voltage is sufficient to produce saturation, is obviously independent of the difference of potential between the plates. This case therefore corresponds to the saturation current through the gas. It will be seen that the saturation current should be directly proportional to the volume of the gas, a result which we have already found experimentally.

Since the time taken for the ions to reach the electrodes is proportional to the distance which they have to travel, that is, to the distance apart of the electrodes, while the field for a given potential difference is inversely proportional to the distance between the electrodes, it follows that the potential necessary to produce the saturation current through the gas is directly proportional to the square of the distance apart of the electrodes. It has also been shown to be proportional to the square root of the intensity of ionization.

Since the saturation current is found to be proportional to the pressure, it follows that  $q$ , the rate of ionization for a given strength of the ionizing radiation, is directly proportional to the pressure; that

is to say, the number of ions formed is proportional to the number of molecules present.

(3) *The general case* has been considered by Sir J. J. Thomson, who found that on certain assumptions the current through the gas could be represented by an expression of the form

$$V = Ai + Bi^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where  $A$  and  $B$  are complicated constants and  $V$  is the P.D. across the gas. This expression is found to express the intermediate portion of the experimental curve (Fig. 7) reasonably well. This part of the curve is of no practical importance except in the case of the currents through flames where the ionization is so intense that the maximum fields which we can apply are insufficient to produce saturation.

## THE PROPERTIES OF THE GASEOUS IONS

**14. Measurement of ionic mobilities.** The mobility of an ion is defined as its velocity in cm. per sec. under the action of an electric field of 1 volt per cm. Ionic mobilities may be measured in various ways, the earliest and most direct of which was due to Rutherford.<sup>(1)</sup>

Two large metal plates were placed parallel to each other and 16 cm. apart on insulating blocks. One of these, *B*, was connected to a

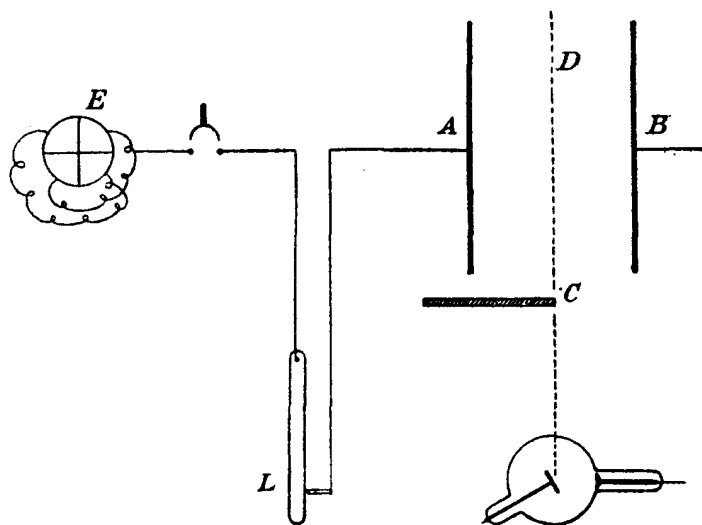


FIG. 8. Rutherford's method of measuring ionic mobilities

battery so that any suitable field could be applied across the gas, the other, *A*, was connected through an insulated lever to an electrometer *E*.

A heavy pendulum, not shown in the diagram (Fig. 8), was arranged so that at one point of its swing it made the circuit of the primary of an induction coil the secondary of which was connected to the X-ray tube used for ionizing the gas. Rays were therefore generated and the gas became ionized. After a given interval (which could be calculated from the time of swing of the pendulum and the distance apart of the switches) the pendulum swung against the lever *L* and so disconnected the plate *A* from the electrometer.

Owing to the screen *C* only the portion of the gas to the right of the

plane  $CD$  is ionized. Hence to reach  $A$  the ions must travel through a distance equal at least to that between  $A$  and the plane  $CD$ . Hence unless the time taken by the ions to travel this distance is less than the time which elapses between the switching on of the rays and the breaking of the electrometer circuit at  $L$ , the electrometer will show no charge. If the time is gradually increased a point is reached when the electrometer begins to show a deflection, that is when the ions from  $CD$  are just able to reach the plate  $A$  before the connection at  $L$  is severed. This critical time is evidently the time taken for the ions to travel under the action of the field from  $CD$  to  $A$ . In Rutherford's experiment the critical time was 0.36 sec., when the potential of the plate  $B$  was 220 volts and the distance to be travelled was 8 cm. The velocity of the ion was therefore  $8/0.36$  cm. per sec., with a potential gradient of  $220/16$  volts per cm. The mobility of the ions was therefore  $8/0.36 \div 220/16$ , or about 1.6 cm./sec. per volt/cm.

The method just described, though admirably direct, is not susceptible of great accuracy. A more promising procedure, also first used by Rutherford, is to employ an alternating field. Let  $AB, CD$ , be a pair of parallel plates, the plate  $AB$  being connected to an electrometer, and  $CD$  to a source of alternating potential. We will suppose that the ionization is confined to a very thin layer of the gas at the surface  $CD$ , and that the ions are all of the same sign. This can be effected in practice by making the plate  $CD$  of some metal such as zinc and illuminating it with ultra-violet light (Chap. ix).

Since the potential applied to the plate  $CD$  is alternating, the ions will during one half of the complete cycle be acted on by an electric field urging them towards the plate  $AB$ . If the distance they can travel during this half cycle is greater than the distance  $d$  between the plates some of them will reach the electrode  $AB$  and the electrometer will show a charge which is added to at each alternate half cycle. If, however, the ions fail to reach  $AB$  before the field reverses they will be dragged back to  $CD$  by the reverse field which exists during the following half cycle, and thus the electrometer will receive no charge.

If the applied potential is represented by  $a \sin pt$ , the field is  $a \sin pt/d$  and the velocity of the ions  $= dx/dt = ka \sin pt/d$ , where  $k$  is the mobility and  $x$  is the distance of the ion from the plane  $CD$ . Since  $x=0$  when  $t=0$ , the solution of this equation is

$$x = \frac{ka}{pd}(1 - \cos pt).$$

Thus the greatest distance that an ion can travel from  $CD$  is  $2ka/pd$ .

36  
42  
276

2/36

2/36

$V = \frac{Q}{r}$

$V = \frac{Q}{r}$

$V = \frac{Q}{r}$



If the distance between the plates is gradually reduced until the electrometer begins to collect a charge we thus have for this critical distance  $d_0$

$$d_0 = \frac{2ka}{pd_0} \text{ or } k = \frac{pd_0^2}{2a}.$$

In the simple form outlined above the method is not very sensitive, since the determination of the point at which an electrometer begins to collect a charge is always a somewhat uncertain procedure. Various modifications have been made by later experimenters to increase what we may call the resolving power of the method. A particularly elegant and accurate method has been devised by Tyndall<sup>(2)</sup> and his collaborators, and is illustrated very diagrammatically in Fig. 9a.  $A$ ,  $B$ ,  $C$ ,

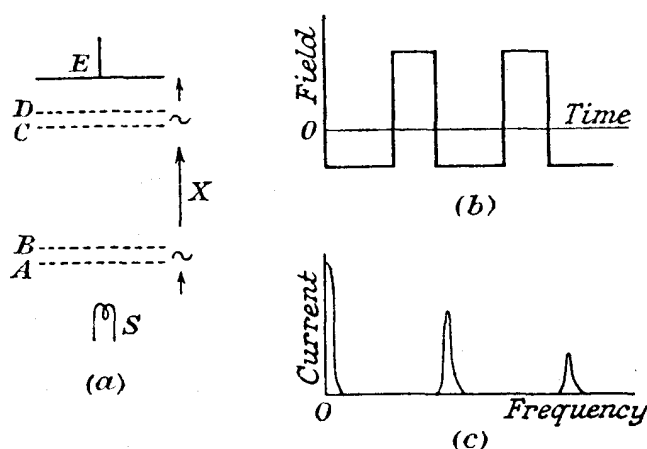


FIG. 9. Tyndall's method of measuring mobilities

and  $D$  are four similar plane parallel grids of fine wire gauze;  $E$  is a metal plate connected to an electrometer, and  $S$  is the source of ions under experiment. A steady uniform electric field is maintained between  $B$  and  $C$ , in such a direction as to drive the ions under experiment from  $B$  to  $C$ , the distance between  $B$  and  $C$  being accurately known. An auxiliary field directs the ions from the source to  $A$ , while another drives any ions which emerge from  $D$  to the plate  $E$ . The apparatus is enclosed in a vacuum-tight glass container (not shown in the diagram) so that the whole can be evacuated and "de-gassed" to remove all gaseous impurities which might contaminate the gas in which the mobilities are to be measured, and which is either generated inside the vessel itself or admitted to it after the cleaning up has been completed. We shall see later how essential this is if the results are to have any real significance.

An alternating difference of potential is established between  $A$  and  $B$ , so that ions directed through  $A$  from the source are alternately

attracted and repelled by  $B$ . A similar alternating field, from the same alternating source, is established between  $C$  and  $D$ ; the two alternating fields being, therefore, in the same phase.

Let us suppose, for simplicity, that we are dealing with ions of a single kind, which travel in the fixed fields in the direction of the arrows. Ions which reach  $A$  during the half-cycle when  $A$  is positive to  $B$  will be urged towards  $B$ , and if the distance  $AB$  is less than the critical distance  $d_0$ , some of them will reach  $B$ , and passing through the meshes will enter the main field,  $X$ , and be carried away towards  $C$ . Those failing to reach  $B$  will be swept back to  $A$  during the negative half-cycle. Thus once every cycle a pulse of ions will leave  $B$  and proceed on its journey to  $C$ . By making  $AB$  only slightly less than  $d_0$  this pulse can obviously be confined to a very thin sheet. The density of the ions in this sheet can be increased by using a "square-topped" potential wave-form (Fig. 9b), instead of the more usual sinusoidal wave-form.

Consider now the sheet of ions as it reaches the electrode  $C$ . If  $C$  is just entering upon its positive half cycle, the ions will be directed towards  $D$ , and, if  $CD$  is less than the critical distance  $d_0$ , will all pass through it and be collected by the plate  $E$ . Arriving a little earlier, they would have been driven on to  $C$  by the then existing negative field; while a little later they would fail to cover the distance  $CD$  before the alternating field reversed its direction and drove them back to  $C$ . Since the fields between  $AB$  and  $CD$  are in phase, we see that the condition necessary for the ions to reach  $E$  are that the time taken for the ions to cover the distance  $BC$  shall be an exact integral multiple of the period of alternation  $\tau$  of the A.C. potential applied to the grids. The time taken to cover the distance  $BC$  is given by  $t = BC/kX$ , where  $k$  is the mobility of the ions. The electrometer will therefore collect a charge whenever  $t = n\tau$ , where  $n$  is an integer.

In practice the frequency of the alternations is gradually increased from zero, and a graph plotted between the frequency and the electrometer current. For ions of a single kind the graph shows, as theory suggests, a series of peaks of diminishing intensity (Fig. 9 c), corresponding to increasing values of  $n$ . If more than one kind of ion is present, each forms its own set of peaks. Owing to the sharpness of the peaks, the resolving power is very good.

In most of the earlier experiments, such as Rutherford's for example, the ions were actually formed in the gas in which the mobility was to be determined. In this way we measure the mobility, say, of the hydrogen ion in hydrogen gas, or the oxygen ion in oxygen gas. The results obtained gave values ranging from 21.4 for helium to 0.22 for

methyl iodide vapour, expressed in cm./sec. per volt/cm. It is also possible with the Tyndall technique to measure the mobility of an ion of one substance in a gas of a different kind. It is found, in general, that the mobility of an ion depends on the nature of the ion; the nature, pressure, and temperature of the gas through which it is travelling; and the sign of the ionic charge. Negative ions in general give rather higher mobilities than positive ions of the same substance. The mobility of an ion under ordinary conditions also decreases with age, particularly in the presence of traces of water vapour or other impurities.

**15. Approximate calculation of the mobility of an ion.** We have seen, (4), p. 17, that, assuming that the ion retains none of the velocity produced in it by the field after collision with another atom, its velocity through the gas is given by  $v = Xe/2m \cdot \tau$ , where  $\tau$  is the time which elapses between two collisions. Now since the ion forms part of the gas it will participate in the thermal agitations of the gas molecules, and the kinetic theory shows that the kinetic energy of the ions for this motion must be equal to that of the molecules of the gas in which it is formed. This velocity will therefore vary as the square root of the mass of the ion. At ordinary temperatures these thermal velocities are very large. In air, for example, the mean molecular speed at atmospheric pressure and 0° C. is 48,500 cm. per sec. Since these velocities are distributed equally in all directions the number of ions carried by the thermal agitations across any plane in the gas in one direction will be equal to the number carried across it in the opposite direction, and hence the total transference of electric charge across the plane will be zero. Thus the thermal agitation causes no transference of electricity, and hence no current in the gas. We may therefore neglect it in considering the transference of electricity through the gas. On the other hand it will be the controlling factor in determining the number of collisions made by the ion. If  $\lambda$  is the mean free path of the ion in the gas and  $V_1$  the velocity of the ion, then  $\tau$  the time occupied in describing the mean free path will be equal to  $\lambda/V_1$ . But since the velocity of the thermal agitation is very large compared with the velocity added by the field (which in the case of a field as great as 100 volts per cm. would only amount in air to about 160 cm. per sec.), the value of  $V_1$  will be practically equal to  $V$ , the velocity of the thermal agitation. Hence we have, on substituting for  $\tau$  in (4),

$$v = \frac{1}{2} \frac{e \lambda}{m V} X \quad \dots \dots \dots (8)$$

or the mobility of the ion is equal to  $\frac{1}{2} e/m \cdot \lambda/V$ .

Since, from the simple kinetic theory of gases,  $\frac{1}{2}mV^2 = \frac{3}{2}\alpha\theta$ , where  $\theta$  is the absolute temperature and  $\alpha$  is Boltzmann's constant, we can also express the mobility  $k$  of an ion in the form  $k = e\lambda/2\sqrt{3\alpha\theta m}$ .

This formula should enable us to calculate the mobility of an ion if the size of the ions is known, or conversely to deduce the size of the ion from its mobility. If we assume that the ion formed in, say, oxygen is simply a charged molecule of the gas, then  $V$  is the ordinary velocity of the oxygen molecule, or 461 metres per sec.: the value of  $e/m$  for a hydrogen atom, as we shall see later (§ 20), is  $10^4$  e.m.u. per gm., and for the oxygen molecule therefore, assuming that all ions carry the same charge, it is  $\frac{1}{32} \times 10^4$ . The mean free path,  $\lambda$ , of an uncharged molecule of oxygen at normal temperature and pressure is  $10^{-5}$  cm. Substituting these values we have the mobility in oxygen equal to  $3.2 \times 10^{-8}$  cm./sec. per absolute e.m.u. of field or 3.2 cm./sec. per volt/cm., since a volt is  $10^8$  absolute e.m.u. of potential. This is rather more than twice the measured mobility of the positive ion in oxygen.

**16. Effect of pressure on the mobility of the ions.** The mobility of the ion is represented by  $\frac{1}{2}e/m \cdot \lambda/V$ . But the mean free path  $\lambda$  is, by the kinetic theory of gases, inversely proportional to the pressure  $p$  of the gas. Hence, if the nature of the ion does not change, the mobility  $k$  of the ion should be inversely proportional to the pressure of the gas, or the product  $pk$  should be constant.

The experimental results obtained for dry air are represented by the curves in Fig. 10, where the mobility is plotted against the reciprocal of the corresponding pressure. The lower curve, which is straight, refers to positive ions, and the upper one to the negative. It will be seen that the product

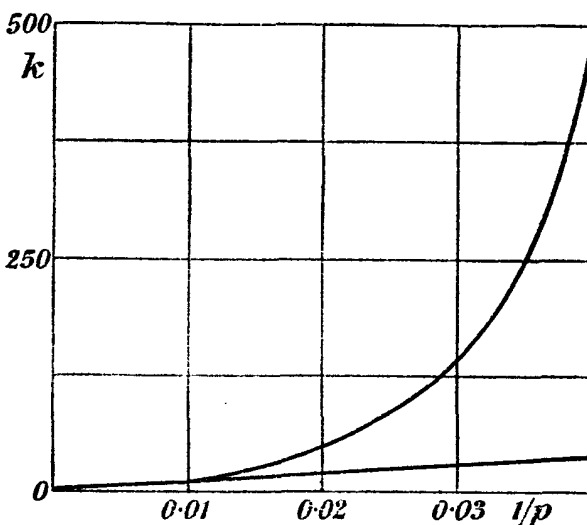


FIG. 10. Relation between mobility and pressure for gaseous ions

$pk_1$  for the positive ions remains practically independent of the pressure down to the lowest pressures used. With the negative ions however at pressures less than about one-tenth of an atmosphere the product  $pk_2$  increases rapidly as the pressure is reduced, thus indicating that the mobility of the negative ion is increasing more rapidly than

would be due merely to the alteration of the free path with pressure. In other words the negative ion is becoming smaller as the pressure is reduced. If the ion at atmospheric pressure consists of a single molecule it is clear that at low pressures the negative carriers must be much smaller than the molecule.

We have already seen (§ 1) that the atoms of all substances contain negatively charged particles, or electrons, as they are called, of a mass equal to about  $1/1839$  of that of a hydrogen atom, and that under certain circumstances these electrons may be ejected from the atom and appear in a free state. We must therefore regard the mechanism of gaseous ionization in the following way. The action of the ionizing agent brings about the expulsion of an electron from a molecule of the gas, leaving the latter with an excess of positive electrification. This positively charged molecule forms the positive ion. The negative electron is now, for a moment, in the free state, but being strongly charged it readily attaches itself to one of the neutral molecules by which it is surrounded, thus forming a negative ion of the same size as the positive.

It may be noted that whereas the positive ion is formed directly from a molecule, the negative carrier begins life in the form of an electron. Since the electron is very small its velocity under the action of an electric field will be very much larger in the free state than when attached to a molecule. Thus if the negative carriers pass any appreciable fraction of their time in the electronic state the velocity with which negative electricity passes across the gas will be materially increased, and thus the mobility of the negative ions which measures the average rate at which they are moving will be considerably increased. In many gases, such as air for example, the effect is quite appreciable even at ordinary pressures as shown by the fact that the measured mobility of the negative ion is greater than that of the positive. As the pressure is reduced the time spent by the charge in the free state is considerably increased and the mobility becomes rapidly greater. Increase of temperature causes a similar effect on the mobility of the negative ion.

More recently Loeb <sup>(3)</sup> has found extremely high mobilities for the negative ion, even at atmospheric pressure, for electropositive gases such as nitrogen, hydrogen, and helium if the gases are purified with the utmost care. Thus mobilities up to 30,500 cm./sec. per volt/cm. have been recorded in very carefully purified nitrogen, 22,000 in helium, and 7800 in hydrogen. In such cases the negative ion must spend most of its life in the electronic state, and it seems very probable

that molecules of these gases are incapable of attaching an electron to their structure. These abnormal values at once drop to a few cm. per sec. if the gas is allowed to be contaminated by a minute trace of oxygen, chlorine, or other electronegative gas.

Even for electronegative molecules the attachment of an electron is not instantaneous. Loeb has calculated that in chlorine or oxygen an electron will on an average make some 3000 collisions before becoming permanently attached to a molecule. It is this difficulty of attachment that gives to the negative ion its apparent greater mobility.

**17. The nature of gaseous ions.** We have seen that the measured mobility of an ion is appreciably smaller than that calculated from Equation 8 on the assumption that the ions are simply charged molecules of the gas. The equation itself is, of course, a rather rough approximation. We have, for example, neglected the fact that since the ions are charged, they will attract the neutral molecules by which they are surrounded, and thus make rather more collisions with them than an uncharged molecule would. This has the effect of reducing the mean free path. More important still, the assumption that the drift velocity given to the ion by the field is wiped out at each collision obviously cannot be justified, since the principle of the conservation of momentum must apply. A more complete solution was given by Langevin <sup>(4)</sup> in 1905 in which these and other matters were taken into account. Even so a discrepancy remained. The experimental values for the mobilities of the ions were definitely smaller than those calculated from theory on the assumption that the ions were single molecules of the gas.

It is now known that all the measurements made before 1930 were, in fact, vitiated by the presence in the gases used of minute traces of impurities, the possible effects of which had not, up to that time, been fully realized. An ion in a gas at atmospheric pressure and temperature makes some  $10^9$  collisions per second. Thus, if impurity is present only in the proportion of one molecule in a million, the ion will encounter, on an average, one of these foreign molecules every thousandth of a second. If the molecule is *polar* (that is, if its constituent atoms are electrically charged) there is a strong probability that it may become attached to the ion by the attraction of the ionic charge: a process which may be repeated at further encounters. The original ion thus becomes the centre of a cluster of molecules which move with it and by increasing its mass reduce its mobility. Water is a strongly polar substance, and as water vapour is extremely difficult

to eliminate, it is not surprising that the early measurements of mobilities gave results which were too low.

The decrease of ionic mobility with the age of the ion, studied in great detail by Erikson,<sup>(5)</sup> is an illustration of the clustering process in action. Working with air at atmospheric pressure, Erikson found that the mobility of the positive ion, initially 1.87 cm./sec. per volt/cm. was reduced after a few hundredths of a second to 1.36. The ratio of these two mobilities ( $=1.38$ ) is rather less than  $\sqrt{2}:1$ . It can be shown that the mobility of an ion of mass  $m$  moving through a gas consisting of molecules of mass  $M$  should be proportional to  $\left(\frac{1}{m} + \frac{1}{M}\right)^{\frac{1}{2}}$ .

For a very heavy ion for which  $m$  is much greater than  $M$ , this reduces approximately to  $(1/M)^{\frac{1}{2}}$ . On the other hand, for an ion consisting of a single molecule of the gas (for which  $m=M$ ), the expression becomes  $(2/M)^{\frac{1}{2}}$ . Thus the building up of a cluster round the original ionized molecule should reduce its mobility in a ratio which approaches  $\sqrt{2}:1$  as the size of the cluster increases. This, as we have seen, is in accordance with Erikson's observations.

A further possibility is that the original ion may actually hand over its charge to the foreign molecule with which it collides, thus forming an ion of a different kind. Instances of this have been recorded. The probability of such an exchange depends on the chemical nature of the impurity and the ion.

The uncertainties which existed in the subject until a dozen years ago were finally dispelled by the experiments of Tyndall<sup>(6)</sup> and his colleagues. A study of the electrochemical properties of the elements indicated that freedom from clustering and from exchange of charges might best be achieved by measuring the mobilities of the ions of the alkali metals in the group of inert gases helium, neon, argon, etc. The gases were scrupulously dried and purified, and the ions were produced by heating certain compounds of the metals concerned. A selection of Tyndall's results is given in Table I.

TABLE I  
Mobilities of the Alkali Ions in rare Gases (Tyndall)

Gas	Mobility of Ion (cm./sec. per volt/cm.)			
	Na <sup>+</sup>	K <sup>+</sup>	Rb <sup>+</sup>	Cs <sup>+</sup>
Helium . . . . .	23.1	22.3	20.9	19.2
Neon . . . . .	8.87	7.88	7.08	6.49
Argon . . . . .	3.21	2.77	2.37	2.23

It will be seen from the Table that for each gas it is possible to plot a graph showing the relation between the mobility and the mass of the ion. One of these curves is shown in Fig. 11. A smooth curve is obtained of the form predicted by Langevin's theory. The importance of this is that in order to determine the nature of an unknown ion it is only necessary to measure its mobility in the given gas, and its mass

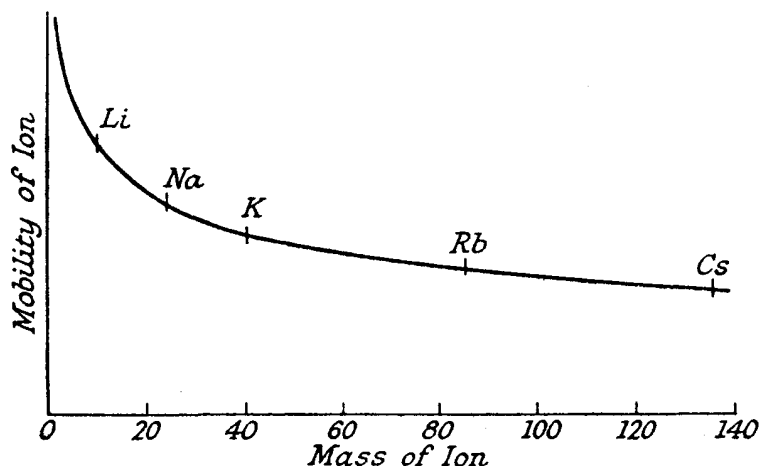


FIG. 11. Graph showing relation between the mobility and the mass of a gaseous ion

can then be read off on the graph. By purposely introducing traces of known impurities into the gas, Tyndall was in this way able to follow the process of cluster formation. The experimental method described on page 22 is particularly suitable for this purpose, since each set of ions present in the gas produces, quite independently, its own set of peaks on the current-frequency curve. Thus, with sodium ions traveling through pure nitrogen the introduction of traces of hydrogen into the gas led to the production of complex ions in which the sodium ion was attended by one or more molecules of ammonia. With nitrogen ions the introduction of traces of water vapour resulted in each ion collecting a cluster of six water molecules.

Substitution in the complete Langevin formula showed excellent agreement between the experimental and the calculated mobilities for the alkali metal ions in argon, krypton, and xenon, on the assumption that the ions were charged atoms of these elements. (The alkali metals have monatomic molecules.) The agreement was not quite so satisfactory for the other gases. This is probably due to the fact that the approximations which still remain in the theory do not hold so well for these gases. There is now no doubt that the gaseous ion begins life as a charged molecule of the gas in which it is formed, that it has a definite mobility depending only on the nature of the ion and



the gas through which it is moving, and which is the same for the positive and negative ion. Any change in mobility is due to a change in the nature of the ion.

**18. Measurement of the coefficient of recombination.** The coefficient of recombination of the ions can be measured directly by a method due to Rutherford.<sup>(7)</sup> A steady stream of gas is passed down a long tube (Fig. 12) at the entrance to which is placed a layer of uranium oxide. This supplies a constant source of ionizing rays so that the gas is all ionized to the same extent. At various points along the tube are placed electrodes which can be connected to an electrometer. These electrodes may conveniently take the form of thin rods of equal length

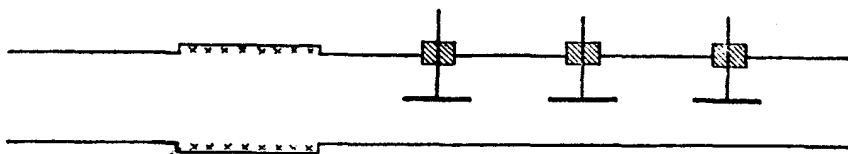


FIG. 12. Illustrating Rutherford's method of measuring the recombination of ions

parallel to the axis of the tube, while the tube itself, raised to a sufficiently high potential to produce the saturation current through the gas, forms the other electrode.

The saturation current is measured successively at each of the electrodes. Since the electrodes are all equal and similar, the saturation currents will obviously be proportional to the number of ions present in the gas per unit volume, at the given electrode. If the distance apart of the electrodes is known and the rate of flow of the gas along the tube, the time taken for the gas to flow from one electrode to the next can be calculated, and thus the number of ions left in the gas after various intervals of time is obtained. In this way Rutherford was able to verify experimentally the equation (3) for the rate of decay of ionization in the gas. In one of his experiments, for example, he found that the ionization amounted to as much as 10 per cent. of its maximum value as long as 4 seconds after removal from the ionizing rays, while in another experiment where the initial ionization was less intense the gas even after 16 seconds retained 10 per cent. of its initial conductivity.

The values found for  $\alpha$  vary very little in different gases. Air, oxygen, and carbon dioxide all give values of  $\alpha$  of about  $1.6 \times 10^{-6}$  at atmospheric pressure and temperature. The value for hydrogen is  $1.4 \times 10^{-6}$ . The coefficient decreases as the pressure is reduced below 1 atmosphere, and has only one-quarter of its normal value at a

pressure of  $\frac{1}{3}$  atmosphere. It also decreases if the pressure is increased above 3 atmospheres. An increase in temperature causes a rather rapid decrease in the coefficient. The value obtained also depends on the ionizing agent employed.

The number of recombinations per sec. per c.c. in air at atmospheric pressure is, as we have seen, about  $1.6 \times 10^{-6}n^2$ , where  $n$  is the number of ions of each sign per c.c. On the kinetic theory of gases the total number of collisions per c.c. per sec. in air at this temperature and pressure is  $1.2 \times 10^{10}n^2$ . Thus, even if we assume that every collision results in recombination, the number of collisions between ions is nearly ten thousand times greater than between the same number of molecules in the normal state. This must be largely due to the strong electrical attraction between the oppositely charged ions, which draws them into collisions which they would otherwise never have made.

Again, as we shall see later, ionization usually takes place along the track of some ionizing particle. Thus the actual density of the ions in the relatively small volume of gas in which recombination is taking place may be much greater than the average value for the gas as a whole. Recent experiments by more accurate methods, while confirming the order of magnitude of the coefficient obtained in earlier work, indicate that recombination may occur by more than one process, and that it is probably impossible to assign a definite value of  $\alpha$  to a given gas which will cover all possible experimental conditions.

**19. The diffusion of ions.** If the ions are not uniformly distributed throughout the gas they will tend to become so by the process of diffusion, just as a mixture of two gases becomes uniform. In fact, since we can identify the ions by their charges, we can regard them as a special kind of gas mixed with the main mass of the gas which is uncharged, and the ordinary laws of diffusion can be applied to them. If the gas is contained in a metal box the ions which come in contact with the walls will give up their charges, and cease to be ions. The concentration of the ions will thus be zero at the walls and increase as we pass out into the main volume of the gas. We shall thus get a pressure gradient set up for the ions, and there will be a steady diffusion drift of the ions towards the walls. Ions can thus be lost by diffusion. In an ordinary ionization chamber, however, the loss by diffusion is quite negligible compared with that due to recombination.

The problem is mathematically equivalent to that of the diffusion of water vapour to a surface of strong sulphuric acid, and the treatment follows similar lines. The coefficient of diffusion of the ions in different gases was measured by Townsend,<sup>(8)</sup> by drawing a measured stream of

ionized gas through a bundle of narrow tubes and measuring the fraction of the total number originally present in the gas which survived the process. The actual calculations required are lengthy and tedious, and need not concern us here. The coefficient of diffusion of the positive ions in air was 0.028, and that of the negative 0.043, as measured at atmospheric pressure. These coefficients are small compared with those for uncharged gases. The coefficient of diffusion of air through carbon dioxide, for example, is 0.134. Since diffusion is mainly determined by the mean free path of the molecules this would indicate that the mean free path of the ions is considerably smaller than that of the uncharged molecule, a conclusion we have already drawn from the measured mobilities of the ions.

#### 20. Determination of the total charge on one gram-molecule of ions.

The interest of these diffusion measurements is mainly theoretical. Let  $n$  be the number of ions per c.c. at a point in the gas. The number per second crossing unit area of a plane at right angles to a given direction  $x$  in the gas is thus, by definition,  $D \cdot dn/dx$ , where  $D$  is the coefficient of diffusion. We can thus regard the ions as drifting uniformly in the direction of  $x$  with a velocity  $1/n \cdot D \cdot dn/dx$ . Regarding the ions as a separate kind of gas they will have a partial pressure  $p$  which is at any point proportional to  $n$ , and we may write their velocity in the form  $1/p \cdot D \cdot dp/dx$ . But  $dp/dx$  is the force acting on unit volume of the ions owing to the pressure gradient. Assuming that the velocity is proportional to the force we see that the velocity of the ions under unit force is  $D/p$ .

Now let us suppose that an electric field of intensity  $X$  acts on the ions. The force on each ion is  $Xe$ , and the total force on all the ions in 1 c.c. of the gas is therefore  $nXe$ . Assuming that the resultant velocity  $u$  is proportional to the applied force per unit volume we have thus

$$u = \frac{D}{p} \cdot Xen.$$

But  $u = kX$ , where  $k$  is the mobility of the ion. Thus

$$k = \frac{n}{p} \cdot De.$$

But  $n/p$  is a constant for the gas, depending only on the temperature, and may therefore be written  $n_0/\pi$ , where  $n_0$  is the number of gas molecules in 1 c.c. of the gas at the temperature of the experiment and at standard atmospheric pressure  $\pi$ . Thus, finally,

$$\frac{k}{D} = \frac{n_0 e}{\pi}.$$

$n_0e$  is the total charge on 1 c.c. of ions at standard atmospheric pressure, that is it is the total charge which would exist in 1 c.c. of gas if all its molecules were ionized. Taking the mobility of the positive ions in hydrogen as 5.3 cm./sec. per volt/cm., atmospheric pressure as  $10^6$  dynes per sq. cm., and the coefficient of diffusion of the ions in hydrogen at  $0^\circ$  C. and atmospheric pressure as 0.12 we have, converting the volts into absolute e.m.u.,

$$n_0e = \frac{5.3 \times 10^{-8}}{0.12} \times 10^6 \text{ e.m.u.} = 0.44 \text{ e.m.u.}$$

This is the total charge in 1 c.c. of charged ions at normal temperature and pressure. If  $N$  is the number of molecules in a gram-molecule, an important constant which is known as Avogadro's number, then, since the volume of a gram-molecule of any substance when in the gaseous state is  $2.22 \times 10^4$  c.c. at normal temperature and pressure, the total charge on all the molecules in a gram-molecule, if they were all ionized, would be  $0.44 \times 2.22 \times 10^4$ , that is 9768 e.m.u. Townsend's mean value, for all his observations on different gases, was  $9.8 \times 10^3$  e.m.u.

It is instructive to compare this estimate for gaseous ions with that obtained from electrolysis experiments. The passage of 1 e.m.u. of electricity through acidulated water liberates  $1.04 \times 10^{-4}$  gm. of hydrogen, or in other words 1 gm. of hydrogen ions carries a charge of  $1/1.04 \times 10^{-4}$ , or 9649 e.m.u. of charge. If we regard the hydrogen ions as the molecules of a gas of molecular weight unity, then 1 gm. constitutes a gram-molecule of the hydrogen ions. Thus, if  $E$  is the charge on a monovalent electrolytic ion and  $N$  is Avogadro's number, the charge carried by 1 gram-molecule of electrolytic ions is given by  $NE = 9649$  e.m.u. This, however, is within the limits of experimental error identical with Townsend's value for the gaseous ions. Hence, since  $N$  is a constant for all gases, the charge on a gaseous ion is identical with that on a monovalent ion in solution. This was by far the earliest and is still in many ways the most direct proof of this very important fact.

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CHAPTER IV  
THE CHARGE ON AN ION

**21. Condensation of water drops on ions.** By applying a saturation voltage across a gas we can, as we have seen, sweep all the ions in the gas to one or other of the electrodes. The charge given to either electrode is then equal to the sum of the charges on all the ions of one sign present in the gas, that is, to  $ne$ , where  $n$  is the total number of ions of either sign and  $e$  the charge carried by each. To determine  $e$  it is necessary to find some method of counting the number of ions present in the gas.

A method of doing this was afforded by the discovery of C. T. R. Wilson that charged ions can serve as nuclei for the condensation of water vapour. It had long been known from the researches of Aitken that the water drops which form when a space becomes supersaturated with water vapour always collect about dust particles or similar nuclei present in the gas. If all nuclei are removed (either by filtration through cotton-wool or better still by repeatedly forming clouds in the same closed space and allowing the drops so formed to settle, thus carrying the nuclei with them), a very considerable degree of supersaturation may exist in the gas without the separation of any drops. If, however, the pressure of the aqueous vapour in the gas exceeds about eight times its saturation pressure for the temperature of the experiment a fine drizzle sets in even in the absence of all nuclei.

Wilson showed that if the gas was ionized deposition could take place in the absence of other nuclei, the ions themselves serving as nuclei about which the drops could form. It was found that deposition would take place on negative ions when the pressure of the aqueous vapour was four times the saturation value, and on the positive ions when it reached about six times that amount.

These effects are due to surface tension. It can be shown that the vapour pressure over a convex surface of liquid is greater than that over a plane surface by  $\sigma/(\rho - \sigma) \cdot 2T/r$ , where  $\sigma$  is the density of the vapour,  $\rho$  that of the liquid,  $T$  the surface tension, and  $r$  the radius of curvature of the surface. If  $r$  becomes indefinitely small, as would be the case for a drop starting from zero dimensions, this excess of pressure would be infinite, and the drop would immediately evaporate. The nuclei,

by providing surfaces of finite radius of curvature on which to condense, enable the drops to commence at a finite size, and hence to grow to visible dimensions even with a comparatively small degree of supersaturation. The drizzle which sets in when the supersaturation is eight-fold is probably due to deposition round molecules of vapour or gas.

The effect of giving a charge to the drop can most easily be discussed from the considerations of potential energy. The potential energy of a drop due to its surface tension is equal to its area multiplied by the surface tension, that is to  $4\pi r^2 T$ , and becomes smaller as the radius is decreased. Hence surface tension will tend to reduce the size of the drop, that is, to assist evaporation, since any system tends to reduce its potential energy to a minimum. On the other hand, the potential energy of a drop due to a charge  $e$  upon it is  $\frac{1}{2}e^2/r$ . This increases as  $r$  becomes smaller, and hence the effect of the charge is to tend to increase the size of the drop. The electric charge therefore acts in the opposite sense to the surface tension, and may be regarded as producing a diminution in it. Thus a charged drop may grow under conditions under which an uncharged drop of the same radius would evaporate. It is obvious, therefore, that a drop will be formed around a charged molecule, or ion, with a smaller degree of supersaturation than that required when the molecule is uncharged.

**22. C. T. R. Wilson's cloud experiments.** The method adopted for producing supersaturation in the air was based on the cooling of a gas by adiabatic expansion. The original apparatus used is shown in Fig. 13. The gas in the expansion chamber is shut in by a movable piston  $P$ , which is made as light as possible so as to move with ease and rapidity. The joint is made air-tight with water  $W$ , which also serves to keep the space saturated with moisture. The air space inside the hollow piston communicates by means of a wide tube  $T$  with the barrel  $B$ , and the piston can be raised to any required position in the outer glass cylinder  $D$  by forcing a little air in through the tap  $A$ . Thus by suitably adjusting the initial position of the piston an expansion of any desired amount can be made.

The expansion is made as follows. A wide tube  $H$  connects  $B$  with a large vessel of several litres capacity which is kept at a low pressure by the continuous action of a water pump.  $H$  is normally closed by a valve  $V$ , but by smartly pulling back the rod  $R$  the valve is opened, allowing the space beneath the piston to be exhausted very quickly through the tubes  $T$  and  $H$ . The pressure below  $P$  being thus reduced, it falls with great rapidity, producing a nearly instantaneous expansion of the gas in the expansion chamber.

The actual expansion can easily be deduced by reading the pressure gauge  $G$  before and after the experiment. Let  $\pi$  be the atmospheric pressure and  $p_1$  the difference of pressure in the two arms of the gauge before expansion. Then  $P_1$  the initial pressure of the gas in the chamber is  $(\pi - p_1 - \sigma)$ , where  $\sigma$  is the saturation pressure of aqueous vapour at the temperature of the experiment. The final pressure of the gas after the apparatus has again attained the temperature of the

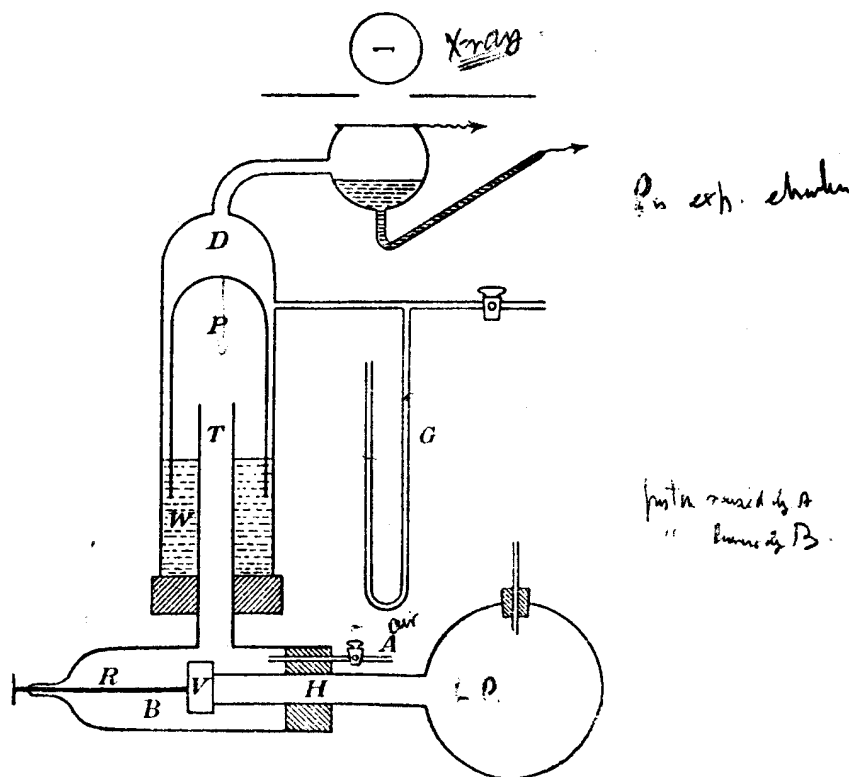


FIG. 13. Illustrating C. T. R. Wilson's early apparatus for depositing drops on ions

laboratory is  $(\pi - p_2 - \sigma)$ , where  $p_2$  is the new reading of the gauge. Hence by Boyle's law

$$\frac{v_2}{v_1} = \frac{\pi - p_1 - \sigma}{\pi - p_2 - \sigma},$$

which gives the expansion.

Since the expansion of the gas is adiabatic,  $\theta v^{\gamma-1}$  is a constant and

$$v_1^{\gamma-1} \theta_1 = v_2^{\gamma-1} \theta_2,$$

$$\log_e \frac{\theta_1}{\theta_2} = (\gamma - 1) \log_e \frac{v_2}{v_1} \quad \dots \dots \dots (9)$$

which gives  $\theta_2$ , since  $\gamma$  is known. The values of the saturation pressure of aqueous vapour at the temperatures  $\theta_1$  and  $\theta_2$  can be ascertained



from tables and hence the degree of supersaturation produced in the space by the expansion can be obtained.

Working with this apparatus Wilson<sup>(1)</sup> found that, after freeing the chamber from dust nuclei, no cloud was formed for values of  $v_2/v_1$  less than 1.375, which was the expansion necessary to produce eight-fold supersaturation. If, however, the gas was previously ionized by passing X-rays into it through an aluminium window, a dense cloud appeared when the expansion exceeded 1.26, showing that fresh nuclei are formed in the gas by the action of the rays. If these nuclei are gaseous ions they should be removable by the action of an electric field. This was found to be the case.

Later experiments showed that for an expansion of 1.26 condensation took place only on the negative ions in the gas. If positive ions alone are present a considerably greater expansion, 1.30, corresponding to a six-fold degree of supersaturation, is necessary to produce any condensation in the gas. The negative ions are thus more efficient in condensing drops of water than the positive.

**23. C. T. R. Wilson's improved cloud apparatus.**<sup>(2)</sup> In the original form of cloud chamber the gas expanded through a side tube, thus

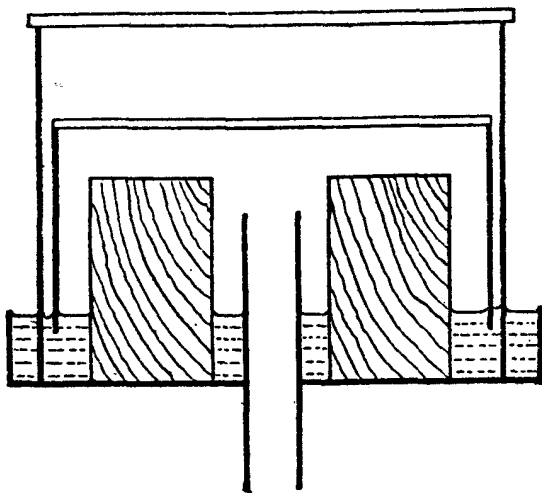


FIG. 14. C. T. R. Wilson's improved cloud chamber

setting up eddy currents in the chamber and producing a thorough mixing of the gas. A simple but great improvement in the apparatus consists in making the whole floor of the apparatus act as the piston producing the expansion (Fig. 14). The gas, after expansion, is perfectly at rest. If now one or two ionizing particles are fired through the chamber immediately after expansion, ions are formed along the actual track of the particles,

and since the gas is already supersaturated, deposition takes place with these ions still in situ. The actual track of the particle is thus made visible, as a string of shining beads of water, when the chamber is suitably illuminated from the side. Permanent records can be made by focusing a camera on the chamber through the upper face, which is made of plate glass. In practice the camera shutter is opened before the expansion is made, and the tracks are illuminated by an

PLATE I.



FIG. 15. TRACK OF AN  $\alpha$ -PARTICLE.

*C. T. R. Wilson.*



FIG. 16. TRACK OF A  $\beta$ -PARTICLE.

*C. T. R. Wilson.*

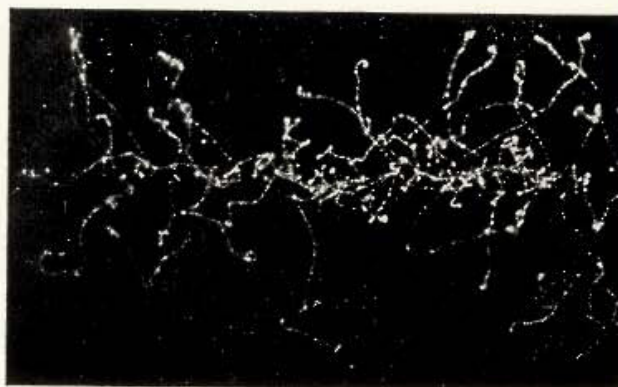


FIG. 17. X-RAYS.

*C. T. R. Wilson.*



FIG. 17A.  
TRANSFORMATION OF AN  
X-RAY.

*C. T. R. Wilson.*

intense electric spark, which is timed to take place immediately after the passage of the ionizing particle.

The photographs shown in Plate I were taken by C. T. R. Wilson in this way. Fig. 15 (Plate I) shows a photograph of the track made by an  $\alpha$ -particle from radium. The ionization caused by the  $\alpha$ -particle is so intense that the drops coalesce into a single continuous streak. Fig. 16 in the same plate shows the track left by a single  $\beta$ -particle. Here the ions formed per cm. are far less numerous than in the case of the  $\alpha$ -particle and the individual ions can be clearly seen. Fig. 17 is a photograph of the track left by a beam of X-rays passing from left to right. We shall refer to this later.

The "cloud chamber" is one of the most important appliances in modern research in atomic physics. In later forms of the apparatus, as developed by Blackett, Shimizu and others, the motion of the piston is controlled by a revolving cam which gradually raises the piston into the proper position and then, after a suitable interval to allow the gas to assume the temperature of its surroundings, suddenly releases it, the downward acceleration of the piston being assisted by strong springs. The resulting adiabatic expansion is hardly so rapid as in the original method, but for many purposes this is more than compensated for by the fact that the whole process can be made automatic. A large number of expansions can thus be made in a fairly rapid succession; the resulting tracks being photographed by a cinematic camera. In a still later form, the expansion is controlled by the particle whose track it is wished to observe (§ 129).

If the chamber is photographed simultaneously by two cameras inclined at a suitable angle a stereoscopic effect can be obtained; alternatively, if the angle between the cameras is fairly large, say  $90^\circ$ , the actual directions of the tracks in three-dimensional space can be calculated from the photographs.

**24. Application of the condensation experiments to the determination of  $e$ . Thomson's method.<sup>(4)</sup>** Suppose a cloud is formed in a cloud chamber of the form in Fig. 13, the expansion being arranged so that the drops condense only on the negative ions in the gas. These ions may be produced by any convenient ionizing agent. Let  $W_1$  be the weight of water vapour required to saturate the space inside the chamber completely at the initial temperature, and  $W_2$  that required to saturate it at the lowest temperature reached during expansion. Then the weight of water deposited is given by  $M = W_1 - W_2$  and is known, since  $W_1$  and  $W_2$  can be calculated from the volume of the chamber and the densities of the saturated vapour at the two

temperatures. It may be assumed that, providing that drops are formed, the whole of the vapour over and above that required to saturate the space will be deposited.

The small drops making up the cloud will fall under the action of gravity with a uniform limiting velocity given by Stokes' law. Thus if  $r$  is the radius of the drop,  $\eta$  the viscosity, and  $\sigma$  the density of air,  $\rho$  the density of water, and  $g$  the acceleration due to gravity, the drops will fall with a uniform velocity given by

$$v = \frac{2gr^2}{9\eta}(\rho - \sigma) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This velocity can be measured by observing the rate at which the surface of the cloud settles down by means of a cathetometer or reading microscope. Since all the constants are known the observed value of  $v$  will enable us to calculate the radius  $r$  of a single drop. It is found that the drops are all the same size, as can be seen from the fact that the cloud settles down as a whole without separating into layers. The mass of each drop is therefore  $\frac{4}{3}\pi r^3 \rho$  and if  $n$  is the total number of drops in the cloud

$$\frac{4}{3}\pi r^3 \rho n = M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which gives us  $n$  since  $M$  the mass of the cloud is known.

If the water surface is connected to an electrometer the quantity  $Q$  of electricity brought down by the cloud can be measured. The charge on a single ion is then equal to  $Q/n$ .

*Balancing method of determining  $e$ .* The previous method was first carried out by Sir J. J. Thomson. An interesting modification is due to H. A. Wilson.<sup>(5)</sup> In this the clouds were formed and the radius of the drops determined by the application of Stokes' law as in the previous experiment. An electric field  $X$  was then applied across the chamber so as to attract the drops towards an upper plate which was positively charged, the lower surface being earthed. The electric force  $Xe$  acting on the drops was thus in the opposite direction to that of gravity, and by suitably adjusting the strength of the field the two forces could be made to neutralize each other exactly so that the cloud remained stationary in the chamber, suspended in air "like Mahomet's coffin." When this state is reached we have

$$Xe = mg \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where  $X$  is the electric field and  $m$  the mass of the drop. But  $m$  can be calculated, as before, from the velocity of the free fall under gravity

and hence  $e$  can be obtained. Assuming that each drop forms round a single ion this is the charge on the ion.

Both these experiments are subject to a serious error. The gas after expansion begins to warm up again by conduction and radiation from the room, and consequently the space around the drops soon ceases to be saturated. Hence the drops begin to evaporate and their mass  $m$  is constantly decreasing during the course of the experiment. This defect has been overcome by a method due to Millikan.

**25. Millikan's experimental determination of  $e$ .** The question has been investigated very fully by Millikan,<sup>(6)</sup> using a method similar to that of Wilson, but free from the uncertainties caused by evaporation. Instead of condensing drops of liquid on the ions Millikan forms his drops mechanically by some form of sprayer in an atmosphere saturated with the liquid, and leaves them to acquire charges from the surrounding ionized air by collision with the ions. His apparatus is shown very diagrammatically in Fig. 18.

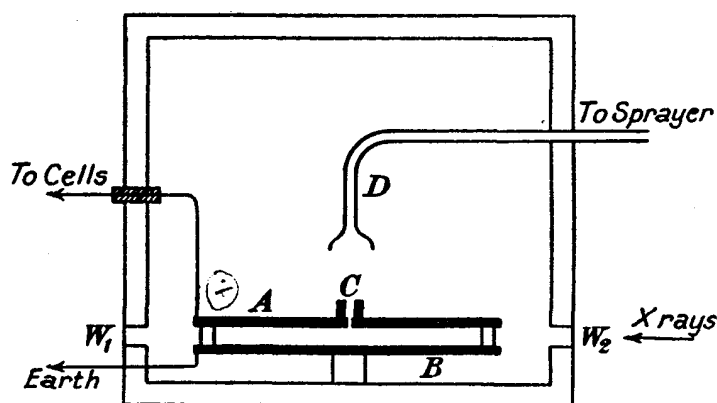


FIG. 18. Millikan's apparatus for determining the charge on an ion

It consists essentially of a pair of optically flat brass plates,  $A$  and  $B$ , each 22 cm. in diameter held perfectly parallel to each other by glass distance pieces optically worked to exactly the same thickness, 14.9174 mm. The lower plate  $B$  is earthed, while the upper plate  $A$  can be charged to a suitable positive or negative potential, usually of the order of 6000 volts from a large high-tension battery. A constant and very accurately measurable field can thus be applied across the central part of the space between the plates. The whole apparatus is enclosed in a large metal box, which itself is immersed in a thermostat to avoid errors which might arise from convection currents in the apparatus. Drops, usually of oil but sometimes of mercury, were sprayed into the space above  $A$  through the tube  $D$ , and some of them

fell through a small hole  $C$  into the space between the plates. The drops were illuminated by a beam of light introduced through a suitable window,  $W_1$ , and were viewed, in a direction nearly perpendicular to the light, by a microscope with a micrometer eye-piece. If required the air in the space between the plates could be ionized by X-rays, introduced through another window,  $W_2$ .

The drops were sometimes found to be charged by friction in passing through the sprayer. If not, they become charged by collision with the ions in the space  $AB$ . Since the diameter of the drop is large compared with that of an ion, the potential of the drop rises slowly, and it may accumulate a considerable number of charges of the same sign before its potential becomes sufficiently large to prevent other like charges from reaching it. On the other hand, if an ion of opposite sign strikes it, part of the charge will be neutralized, and the total charge on the drop decreased. Since the drop has no tendency to evaporate the same drop may be kept under observation several hours, being kept in the field of view of the microscope by adjusting the electrical field to balance the gravitational forces, as in H. A. Wilson's experiments.

Suppose now that the plates are connected together so that the field is zero.

The drop will begin to fall under the force of gravity upon it, that is,  $mg$ , where  $m$  is the effective mass of the drop. As it is falling through a viscous fluid it will, by Stokes' law, move with a uniform terminal velocity  $v_1$ , which is directly proportional to the force acting upon it. Thus  $mg = kv_1$ , where  $k$  is some constant which depends on the viscosity of the air and the dimensions of the drop, but will remain unaltered as long as we confine our attention to the same drop. If the plate  $A$  is now raised to a sufficiently high potential of opposite sign to the charge on the drop the electrical attraction will be greater than the gravitational force and the drop will move upwards with a velocity  $v_2$  given by  $XE - mg = kv_2$ , where  $X$  is the field and  $E$  the charge on the drop. The velocities  $v_1$ ,  $v_2$  are measured by timing the transit of the drop between two fixed hair lines at a known distance apart in the micrometer eye-piece. Combining our two equations we have

$$E = \frac{k}{X}(v_1 + v_2).$$

The same drop can be observed for many hours during which its total charge will undergo many changes owing to collision with either positive or negative ions. A large number of different values for  $E$  (in terms of the constant  $k$ ) can thus be obtained with the same drop.

As a result of many experiments Millikan found that when the drop changed its charge it did so always by exactly the same amount, and further that the total charge on the drop was always an exact integral multiple of this amount. This was the case whether the whole charge was due to ionic collisions, or whether it was derived from friction in the sprayer. Thus all the charges measured by Millikan could be represented by the relation  $E = ne$ , where  $e$  was a constant and  $n$  an integer which generally lay between 3 and 20.

It would be impossible to imagine a more direct proof of the fact that electricity, like matter, is not indefinitely indivisible. It consists of units, or "atoms," which cannot be further sub-divided, and all these atoms have exactly the same charge. This is the charge carried by a gaseous ion. A negative ion, however, is merely an atom which has gained an electron. The electron is thus the atom of electricity, and we shall refer to this natural unit of electricity as the *electronic charge*, and denote it by  $e$ . The fact that the frictional charges on the drops were also found to be integral multiples of  $e$  shows that ordinary frictional charges are also made up of electrons.

The proof of the atomicity of electricity requires no further measurements than those of the two velocities of the drop, and involves no assumptions except that the velocity of the drop is proportional to the force acting upon it. To find the value of  $e$  in absolute units it is necessary to determine the value of  $k$ . According to Stokes' law  $k = 6\pi\eta a$ , where  $a$  is the radius of the drop and  $\eta$  the viscosity of air. Since the mass  $m$  of the drop is equal to  $\frac{4}{3}\pi\rho a^3$  or, allowing for the buoyancy of the air, to  $\frac{4}{3}\pi(\rho - \sigma)a^3$ , where  $\sigma$  is the density of the air and  $mg = kv_1$ ,  $k$  can be calculated when the density  $\rho$  of the drop and the viscosity of air are known.

On evaluating his results, however, Millikan found that the value obtained for  $e$  varied with the size of the drop used, becoming progressively larger as the drop became smaller. When the radius of a drop becomes comparable with the average distance between the molecules of the air it is no longer possible to regard the air as a continuous homogeneous medium, as required in Stokes' law. To put the matter somewhat crudely, the small drop finds occasional gaps between the air molecules through which it can move freely under the action of the forces upon it, and it thus acquires a rather larger average velocity than it would if the medium were really homogeneous. It can be shown that the constant  $k$  in these circumstances is of the form  $6\pi\eta a \left(1 + \frac{A\lambda}{a}\right)^{-1}$ , where  $\lambda$  is the mean free path of a molecule in the

gas, and  $A$  is a numerical constant. The value of  $A$  can be deduced from the measurements of  $e$  made with drops of very different radii. Millikan found that on using this corrected form of Stokes' law, all his experimental data were completely consistent, and the value of  $e$  was exactly the same for drops of all sizes.

Corrections were also made for the effect of the finite size of his apparatus on the motion of the drops, and new determinations were made of the viscosity of air, the accepted value having been found to be in error by about 2 per cent. As a final result of a series of experiments made between 1911 and 1917 Millikan found that the electronic charge had the value  $4.774 \times 10^{-10}$  e.s.u. with a probable error of about 0.1 per cent.

**26. Perrin's determination of  $e$  from the Brownian movement.** We have already seen (§ 20) that the total charge on a gram-molecule of charged ions is given by  $N \cdot e = 9649$  e.m.u., where  $e$  is the charge on a single ion and  $N$  is Avogadro's number. Thus if we can determine  $N$ , we have an independent method of determining  $e$ . An interesting method of determining  $N$  is afforded by Perrin's <sup>(7)</sup> experiments on the Brownian movement.

Suppose a small particle, not too small to be visible with an ultra-microscope yet not too large in comparison with the size of the molecules to be affected by collision with them, is suspended in water. This particle will, by the kinetic theory, be bombarded on all sides by the molecules of the water moving under their velocities of thermal agitation. On an average the number striking the particle will be the same in all directions and the average momentum communicated to the particle in any direction will be zero. Since, however, the collisions are governed by the law of probability there will, if the interval of time considered be sufficiently small, be an excess of momentum in one direction or the other, and if the mass of the particle is not too large compared with the masses of the colliding molecules it will be given an irregular kind of motion which may be observed under a microscope. This motion has been known for many years as the Brownian movement and is always to be observed in suspensions of fine particles in a liquid.

The particles in Brownian movement are thus sharing the thermal agitations of the molecules of the surrounding liquid in exactly the same way that the molecules of a heavy vapour share the motion of the molecules of a light gas in which they may be placed. In other words, from the point of view of the kinetic theory the particles in a suspension behave like the molecules of a dilute gas of very high molecular weight,



each particle functioning as a single molecule. We can, therefore, apply the gas laws to them, and from observations on these visible particles determine the various unknown constants in the gas equations.

*Distribution of the particles in a suspension in a vertical direction when in equilibrium under gravity.* The most direct method of applying this result is to determine the way in which the particles of a suspension distribute themselves under the action of gravity. The case is similar to that of a gas under gravity.

Consider two horizontal planes separated by a distance  $dh$  in a vertical column of gas of unit cross section. If  $p$  and  $p+dp$  are the pressures at the two planes, the difference  $dp$  is equal to the weight of the gas enclosed between the two planes, since the cross section is unity. Hence if  $\rho$  is the density of the gas

$$\begin{aligned} dp &= mg = \rho g \cdot dh \\ &= \frac{M}{V} g \cdot dh \end{aligned}$$

where  $M$  is the molecular weight of the gas in grams and  $V$  is the volume of 1 gram-molecule. But  $V = RT/p$ , where  $T$  is the absolute temperature and  $R$  is the gas constant. Thus

$$\begin{aligned} dp &= \frac{Mgp}{RT} dh, \\ \therefore \log_e \frac{p_0}{p} &= \frac{Mg}{RT} H \quad \dots \dots \dots (13) \end{aligned}$$

where  $H$  is the vertical distance between the two points in the vertical column where the pressures are  $p_0$  and  $p$ .

But the pressure is directly proportional to the number of molecules per c.c. in the gas. Hence we may write (13) in the form

$$\log_e \frac{n_0}{n} = \frac{MgH}{RT} \quad \dots \dots \dots (14)$$

which gives the distribution of the molecules in a vertical column of the gas.

We can now apply this result to the case of the suspension. In this case if  $m$  is the mass of a single particle the "molecular weight in grams" will be equal to  $mN$ , where  $N$  is the number of molecules in 1 gram-molecule of a gas, which by hypothesis is the same for all gases. The apparent mass of a particle suspended in a liquid is, however, less

than its true mass  $m$  by the weight of the liquid it displaces; that is, it will be equal to

$$m\left(\frac{D-d}{D}\right),$$

where  $D$  is the density of the particle and  $d$  that of the liquid. Hence substituting in equation (14), we have

$$\log_e \frac{n_0}{n} = \frac{Nm}{RT} \left(1 - \frac{d}{D}\right) gH \quad . . . . . (15)$$

If the mass of the particles and their density are known a determination of the ratio of  $n_0/n$  for different depths will enable us to calculate  $N$ , the constant of Avogadro.

A suitable emulsion was found in gamboge, the particles being obtained of uniform size by centrifugal fractionation. The density could be determined by finding some solution in which the particles would remain suspended without falling; while the volume could be determined either by direct measurement of the radius of the particle or by calculating the latter from its rate of fall under gravity by Stokes' law.

To find the vertical distribution of the particles when in equilibrium a small drop of the emulsion was placed in a hollow cut out in a micro-

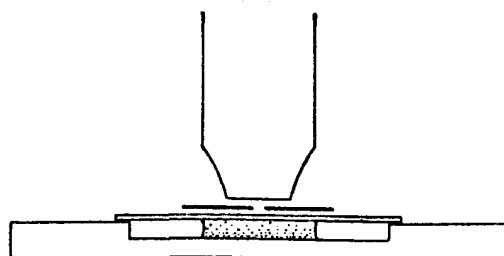


FIG. 19. Perrin's experiment.

scope slide (Fig. 19) and covered with a thin cover-slip to prevent evaporation. A high-power microscope was then focused on some definite layer of the emulsion and the field of view limited by a stop so that not more than five or six grains were in the field of view at the same instant.

Owing to the Brownian movement the particles are constantly appearing and disappearing, but if not more than five or six are visible at the same time a practised eye can immediately grasp the number present at any given moment. Readings were made at equal short intervals of time, and the average number present in view is taken as proportional to  $n$  for that layer. Thus for some particular layer  $A$  in one set of experiments the numbers visible at ten successive moments were

2, 2, 0, 3, 2, 2, 5, 3, 1, 2,

giving an average of 2.2. At some higher level  $B$  the first ten readings were

2, 1, 0, 0, 1, 1, 3, 1, 0, 0,

giving an average of 0.9. The ratio  $n_1/n_2$  for these two layers was therefore 2.2/0.9. In this way the average number of particles occurring at different depths in the liquid was determined and was found to agree very closely with the theoretical distribution, as calculated above. Substituting the experimental values in equation (15)  $N$  was found to be  $68.2 \times 10^{22}$ . A second method based on the distribution after a definite interval of time in a horizontal plane of a number of particles introduced at a given point in the liquid gave a mean value of  $N = 68.8 \times 10^{22}$ . Substituting these results in the equation  $NE = 9649$  we find that the value of  $E$ , the charge on a monovalent ion in electrolysis, is equal to  $1.40 \times 10^{-20}$  e.m.u. or  $4.2 \times 10^{-10}$  e.s.u.

**27. Summary of results.** Perrin's experiments were important as providing an entirely independent method of determining Avogadro's number, and thus, indirectly, the electronic charge. The method is clearly not capable of a very high order of accuracy, although the discrepancy between the values obtained by Perrin and Millikan is larger than would have been expected. This gave rise to a suggestion that Stokes' law might not be applicable to bodies as small as the oil drops used by Millikan. Experiments by Fletcher on the Brownian movement of minute oil drops in air gave a value for  $N$  of  $60.3 \times 10^{22}$ , and consequently a value for  $e$  of  $4.79 \times 10^{-10}$  e.s.u., in close agreement with that of Millikan. As, however, Fletcher used Stokes' law to determine the mass of his drops, his experiments hardly provide independent confirmation of Millikan's experiments.

On the other hand values for  $e$ , of rather less precision, can be deduced from entirely different considerations, such for example as the determination of the charge on an  $\alpha$ -particle (§ 98), which is known to be  $2e$ . These left no doubt that Perrin's value was definitely too low, and Millikan's final value was unhesitatingly accepted until 1928, and indeed later.

In that year Backlin, by reflecting X-rays of known wavelength from crystals of known structure, deduced a value for  $e$  which was substantially higher than the Millikan value. The method is described in § 94. Later, and more precise, observations by the same method made by Bearden <sup>(9)</sup> and others gave values as high as  $4.80 \times 10^{-10}$  e.s.u. Millikan's value had, however, become so firmly established in the subject, that for three or four years it was generally felt that some error, perhaps in the theory of X-ray reflection, might be discovered, which would account for the discrepancy. The most critical examination failed to reveal any flaw in the argument. In the meantime, a re-examination of the data used by Millikan was undertaken by several

observers. In particular, several redeterminations of the viscosity of air were made, all of which gave a value at least one half per cent. higher than that used by Millikan in his calculations. Combining the mean of the most recent values for the viscosity with the original oil-drop data, Millikan's value for  $e$  becomes  $4.81 \times 10^{-10}$  e.s.u., in close agreement with the X-ray findings. A summary of these results is given in Table II.

TABLE II  
The electronic charge

Observer	Method	$e \times 10^{10}$ e.s.u.
Rutherford (1908) . . .	Charge on $\alpha$ -particle	4.65
Regener (1909) . . .	" "	4.79
Millikan (1913) . . .	Oil and mercury drops	4.774
" (1917) . . .	" "	4.774
Matteuch (1925) . . .	" "	4.758
Fletcher (1913) . . .	Brownian movement	4.79
Bearden (1932) . . .	X-ray measurements	4.804
Millikan (1938) . . .	Recalculated from new viscosity data.	4.811
Laby and Hopper (1940) . .	Oil-drop method	4.802
Miller and Dumond (1940) .	X-ray measurements	4.801

Since the X-ray measurements are capable of the higher precision their results have now been generally adopted. The accepted value for  $e$  is  $4.803 \times 10^{-10}$  e.s.u. or  $1.60 \times 10^{-20}$  e.m.u.

Assuming this value for  $e$  the value of Avogadro's constant becomes  $60.2 \times 10^{22}$  molecules per gm. molecule. The Faraday constant is 9649 e.m.u. per gm. equivalent. The number of monovalent atoms per gm. of a substance of unit atomic weight would thus be  $9649/1.60 \times 10^{-20}$ , and the actual mass of an atom of unit atomic weight would thus be  $1.60 \times 10^{-20} \div 9649$  or  $1.658 \times 10^{-24}$  gm. The masses of atoms of other elements can be obtained by multiplying this number by the chemical atomic weight (oxygen=16). Thus the mass of a hydrogen atom is  $1.0081 \times 1.658 \times 10^{-24}$  gm. or  $1.671 \times 10^{-24}$  gm.

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## CHAPTER V

### IONIZATION BY COLLISION. THE SPARK DISCHARGE

**28. Ionization currents with large voltages.** We have already seen (§ 9) that the current through an ionized gas increases with increasing electric field up to a certain maximum value known as the saturation current. There is then a considerable range of electric fields greater than this saturation value for which increasing the field produces no appreciable increase in the current. It is found, however, that if the field is increased above a certain value the current again begins to increase, at first slowly but soon with great rapidity (Fig. 20), until finally a spark passes between the electrodes, accompanied by a temporary but very considerable current. The field  $X$  required to produce

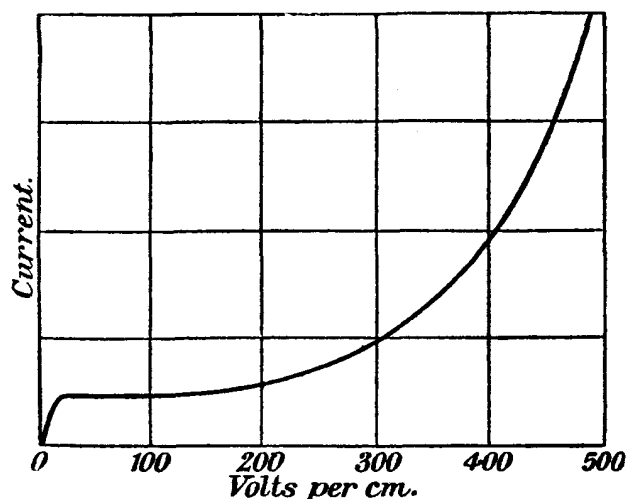


FIG. 20. Relation between current and field at high intensities

an appreciable increase in the current above its saturation value depends on the distance apart of the electrodes, and on the nature and pressure of the gas. For air at ordinary pressures, and with distances of half a centimetre or more between the electrodes it is very large. It diminishes, however, as the pressure of the gas is reduced, and the phenomena are most conveniently investigated in gases at the comparatively low pressures of from 1 to 10 mm. of mercury. With such pressures it is possible to produce a considerable increase in the current through the gas with potential differences of not more than 50 volts.

The effect is to be ascribed to collision between the ions generated in the gas and the molecules of the gas. We have already shown (§ 15) that the velocity acquired by an ion of mass  $m$  and charge  $e$  moving under the action of a field of strength  $X$  between two successive collisions is

$$\frac{Xe\lambda}{m\bar{v}},$$

where  $\lambda$  is the free path of the ion. Its velocity, on striking another molecule of the gas, is thus, for a given field, proportional to its free path.

The mechanism of ionization is a little obscure. Since however it involves the complete separation of a negative from a positive charge it is obvious that work must be done on the molecule so ionized, and the energy for this must be supplied by the source of ionization. In what follows we assume that this energy is supplied by the kinetic energy of the colliding particle. The kinetic energy of this particle must, therefore, reach a certain minimum value if fresh ions are to be produced by it in the gas by the process of collision.

The mean free path of a particle in a gas can be calculated from the kinetic theory of gases. The same theory, however, shows that in a considerable number of cases collision will not occur until the particle has described a free path considerably longer than the mean, while in others collision takes place before the mean free path has been described. Thus it can be shown that in 14 per cent. of the total number of collisions, collision will not occur until the particle has described a path at least twice as great as the mean, while in 10 per cent. of the cases it takes place before one-tenth of the mean path has been completed. Thus, assuming that the particle must describe a path  $\lambda$  before it has sufficient kinetic energy to produce fresh ions, ionization by collision will commence long before the mean path of the ions has reached this critical value, and will not be even approximately complete until the conditions in the gas correspond to a mean free path considerably greater than the critical value.

Kinetic theory shows that, if  $\lambda$  is the mean free path of a molecule in a gas, a molecule will in a centimetre of its path make  $1/\lambda \cdot e^{-x/\lambda}$  collisions for which its actual free path has been greater than  $x$ . The energy of an ion on collision, assuming it to be due entirely to the electrical field  $X$  acting upon it, is  $Xex$ , where  $e$  is the ionic charge. If we assume that ionization will occur if this energy exceeds some constant value  $T$ , the number  $\alpha$  of fresh ions formed per cm. by the given ion will be equal to the number of encounters terminating free

paths greater than the critical value  $x$  given by  $Xex = T$ , or  $x = T/Xe$ . Substituting this value of  $x$  in the equation and remembering that the mean free path is inversely proportional to the pressure, we have

$$\alpha = Cp e^{-CTp/Xe} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where  $C$  is a constant for the gas.

The value of  $\alpha$  thus depends on the pressure and nature of the gas and on the field. For the negative ion in air at a pressure of 4.1 mm. in a field of 200 volts per cm. Townsend found a value for  $\alpha$  of 0.50 per cm. In carbon dioxide at a pressure of 3.95 mm. and with a field of 972 volts per cm. the value was as high as 4330.

**29. Expression for the current through the gas when ionization by collision occurs.** Experiment shows that ionization by collision occurs with much lower fields for negative than for positive ions. Thus in circumstances where negative ions were found to be producing fresh ions by collision with a potential of 100 volts between the electrodes, the potential had to be raised to 350 volts before any appreciable increase in current was observed when the ions originally present were all positive. When the gas, as is usually the case, contains ions of both signs the initial rise in the saturation current is thus due entirely to the negative ions. Confining our attention for the moment to this case, let us consider the current between two parallel plates immersed in a gas. Consider a single negative ion situated at a distance  $d$  from the positive electrode. As it travels towards the positive electrode it will produce fresh negative ions, which, in turn, will also produce more ions on their journeying in the same direction. Let  $n$  be the total number of negative ions which owe their existence directly or indirectly to the original ion after it has travelled a distance  $x$  from its starting-point, including the original ion itself. In a small additional distance  $dx$  these  $n$  ions will produce  $\alpha n \cdot dx$  fresh pairs of ions, where  $\alpha$  is the number of ions produced per cm. of its path by a single negative ion, and is constant under the conditions of any one experiment. Thus the rate  $dn$  at which negative ions are increasing as the positive electrode is approached is given by

$$dn = \alpha n \cdot dx$$

Neglecting recombination, which in these circumstances will be small, the number of ions  $n_a$  reaching the positive electrode as the result of the activities of the single original ion will be obtained by integrating the equation from  $x=0$  to  $x=d$ ; i.e.

$$\int_1^{n_a} \frac{dn}{n} = \int_0^d \alpha \cdot dx$$



since  $n=1$  when  $x=0$ , and  $n=n_a$  when  $x=d$ . Thus integrating both sides we have

$$n_a = \epsilon^{ad} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Thus if  $n_0$  ions are produced per second in a plane at a distance  $d$  from the positive electrode the current  $i$  through the gas will be  $n_a e$ , since each ion carries the unit charge,  $e$ . The saturation current,  $i_0$ , in the absence of ionization by collision, is  $n_0 e$ . Thus

$$i=i_0 e^{\alpha d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In general, however, ionization is not confined to a single plane. If the gas between the electrodes is uniformly ionized we may proceed as follows. Let  $n_0$  be the number of pairs of ions produced by the ionizing agent per c.c. per sec. Consider a column of gas of unit cross sectional area stretching from one electrode to the other. The number of ions formed in this column is  $n_0 d$  per sec., where  $d$  is the distance between the electrodes, and the saturation current density is thus  $i_0 = n_0 d e$  per sq. cm. Consider now a thin section of the column between two planes distant  $x$  and  $x + dx$  from the positive electrode, the number of ions formed per sec. in this layer will be  $n_0 dx$  and the number reaching the positive electrode due to this layer, taking into account ionization by collision, will by (17) be  $n_0 e^{\alpha x} dx$ . The total number reaching the positive electrode per sec. will thus be the integral of this from  $x=0$  to  $x=d$ , or

$$\int_0^d n_0 e^{\alpha x} dx = n_0 \frac{e^{\alpha d} - 1}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The current density  $i$  is therefore given by

$$i = n_0 e^{\frac{\epsilon \alpha d - 1}{\alpha d}} = i_0 \frac{\epsilon \alpha d - 1}{\alpha d} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where  $i_0$  is the saturation current.

These equations have been very carefully tested by Townsend, to whom they are due. The apparatus consisted of a pair of parallel plates in an air-tight container, mechanical arrangements being made so that the distance between the plates could be varied and accurately measured. Experiments were carried out with various distances, usually of the order of a few millimetres, and at various electric fields. Uniform ionization was obtained by illuminating the gas with X-rays. When it was desired to confine the ionization to a single plane, the negative electrode, of zinc, was illuminated with ultra-violet light, from a quartz mercury lamp. This, while leaving the gas through

which it passes, un-ionized, causes a copious emission of electrons from the electropositive metal surface (see § 63). The experimental curves showed that both equations were of the correct form, and that for a given gas at a given pressure and field-strength  $\alpha$  was a constant. For varying pressures and fields Townsend found that  $\alpha/p$  was a function of  $X/p$ , as predicted by (16).

**30. Effect on the current of positive ions.** If the electric field across the gas increases beyond a certain limit the effect of the ionization produced by the positive ions becomes important.

It was originally suggested by Townsend that the increase in ionization was due to collision of the positive ion with the gas molecules. It is not difficult to show that, on this assumption, the combined effect of collisions by both positive and negative ions should produce a current density given by

$$i = i_0 \frac{(\alpha - \beta) \epsilon^{(\alpha - \beta)d}}{\alpha - \beta \epsilon^{(\alpha - \beta)d}} \quad \dots \quad (21)$$

where  $\alpha$  and  $\beta$  are the numbers of ions produced per cm. by a single negative and positive ion respectively. The expression was found to fit the experimental data quite well. When, however, it became possible to determine the actual minimum energy required to produce a pair of ions in a gas (see § 34) it was realized that the probability of a positive ion having a sufficiently long free path in which to build up the necessary energy was negligibly small. This does not apply to the negative ion, which at the pressures generally employed is almost certainly a free electron.

It is known that positive ions of comparatively small energy can cause the emission of electrons from a metal plate, and it is now thought that the increased current at higher fields is due to this cause. Let us consider the case where the ionization produced by the ionizing agent is confined to the negative electrode, and let  $n_0$  be the number of negative electrons produced by the source per sec. By (17)  $n_0 \epsilon^{\alpha d}$  negative ions reach the positive electrode of which  $n_0(\epsilon^{\alpha d} - 1)$  have been produced by collisions in the gas. Since each collision produces a pair of ions, this is also the number of positive ions formed, all of which (neglecting possible recombination) will eventually bombard the cathode. If  $\gamma$  is the number of electrons liberated when a positive ion hits the electrode, the total number liberated will be  $\gamma n_0(\epsilon^{\alpha d} - 1)$ , and the consequent number of negative ions reaching the positive electrode will be  $\gamma n_0(\epsilon^{\alpha d} - 1)\epsilon^{\alpha d}$ . As before,  $\gamma n_0(\epsilon^{\alpha d} - 1)(\epsilon^{\alpha d} - 1)$  positive ions are formed in the gas, which in turn liberate  $\gamma^2 n_0(\epsilon^{\alpha d} - 1)^2$

electrons from the cathode. Thus, summing up to infinity, the total number of negative ions reaching the positive electrode is given by

$$n = n_0 \epsilon^{\alpha d} + \gamma n_0 (\epsilon^{\alpha d} - 1) \epsilon^{\alpha d} + \gamma^2 n_0 (\epsilon^{\alpha d} - 1)^2 \epsilon^{\alpha d} + \dots$$

$$= n_0 \frac{\epsilon^{\alpha d}}{1 - \gamma(\epsilon^{\alpha d} - 1)} \dots \dots \dots (22)$$

This equation is also due to Townsend, and  $\gamma$  is known as the second Townsend coefficient. In terms of the current densities

$$i = i_0 \frac{\epsilon^{\alpha d}}{1 - \gamma(\epsilon^{\alpha d} - 1)} \dots \dots \dots (23)$$

Either (21) or (23) seems to fit the experimental data equally well.

**31. The spark discharge.** Equations (21) and (23) agree in predicting that the current density  $i$  will become infinite for some particular value of the distance  $d$  between the electrodes.

This corresponds to what is known as the spark discharge. A considerable amount of experimental research has been devoted to the spark discharge. If two electrodes are separated by a gas at any pressure and the difference of potential between them is gradually increased eventually a point is reached at which a bright spark accompanied by the well-known crackling noise will pass between them and the two conductors become completely discharged. If the difference of potential is continuously restored by connecting the two electrodes to an electrical machine a continuous torrent of sparks is obtained which may pass into the form of an arc. In this case the current may rise to many amperes.

The spark may also be made to pass by bringing the electrodes nearer together as in the case of the discharge of a Leyden jar by discharging tongs, or by reducing the pressure of the gas between the electrodes. This latter is, however, only effective down to a certain point. It is found that for any given distance apart of the electrodes there is a critical pressure at which the discharge passes most easily. Reducing the pressure below this critical value causes a very rapid rise in the potential necessary to produce a spark, so that the potential at very low pressures may easily exceed by many times that which would suffice to produce discharge at the same distance in air at atmospheric pressure.

The critical potential difference between the electrodes necessary to produce a spark is called the sparking potential. If we assume that the spark takes place between two parallel plate electrodes situated  $S$  cm. apart the value of the sparking potential is equal to  $XS$ , where  $X$  is the field between the plates.

Thus if  $\alpha$  and  $\beta$  are the two coefficients of ionization under the conditions of the experiment, sparks should pass between the electrodes when their distance apart is given by

$$\alpha = \beta \epsilon^{(\alpha - \beta)S} \quad . . . . . (24)$$

on the earlier theory, or by

$$1 = \gamma(\epsilon^{\alpha S} - 1) \quad . . . . . (25)$$

in the later version.

The sparking potential should be equal to  $XS$ , where  $X$  is the field between the plates.

Both these equations have been tested by Townsend.  $\alpha$  and  $\beta$ , or  $\alpha$  and  $\gamma$ , are measured for some definite field  $X$  and the value of  $S$  calculated from the equations. The plates were then set at this distance apart and the potential difference between them gradually increased until a spark passed. The minimum sparking potential as thus obtained was found to agree in every case with the product  $XS$  as previously determined, the divergence between the two values being less than 1 per cent. This agreement is strong proof of the substantial accuracy of the theory.

It will be seen that this theory requires the existence of a certain number of free ions in the gas between the electrodes if the spark is to pass. Under normal circumstances the air generally contains a few ions, especially if the electrodes are illuminated by sunlight which contains a small proportion of ultra-violet light. If, however, the ions present are very few, as, for example, if the sparking experiments are made in the dark, a considerable time may elapse before the few ions originally present have increased sufficiently in number to give rise to a spark discharge, and under these circumstances the difference of potential may be increased considerably beyond the critical sparking potential without a spark passing. This phenomenon is well known and is called the retardation of the spark. Under certain circumstances this retardation may amount to several minutes. It is reduced to zero if the negative electrode is exposed to a weak source of ultra-violet light. Many early experiments were vitiated by the neglect of this precaution.

**32. Sparking potential for parallel plate electrodes.** The conditions are most easily interpreted when the electrodes take the form of a pair of parallel plates. Experiments of this kind have been made by Carr,<sup>(4)</sup> using apparatus as shown in Fig. 21. If the discharge is taken between two metallic plates in the usual way it is found that the spark always passes between rough points or small projections on the edges of

the disks. To avoid this the electrodes  $E, E$  were completely embedded in ebonite except for a portion near the centre which was carefully planed and polished. The plates were separated by ebonite rings  $R$  of varying thickness so that spark lengths of different values could be used. The gas to be experimented on could be introduced by fine tubes  $t, t$  running through the ebonite, and the whole apparatus was made air-tight.

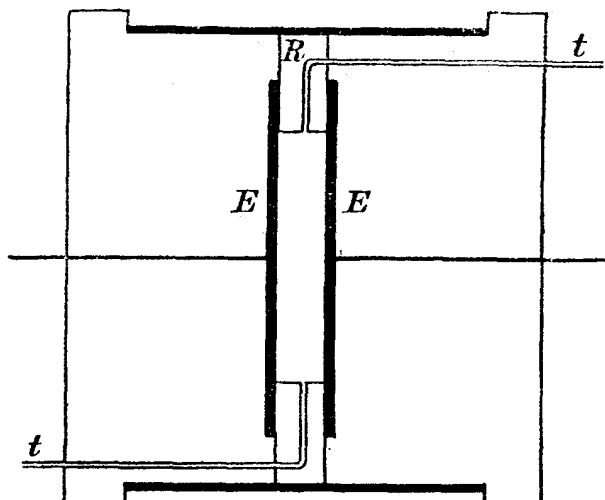


FIG. 21. Apparatus for investigating the spark discharge

The experiments showed that for a given gas there was a minimum potential difference depending only on the nature of the gas below which a spark would not pass. For any given distance between the plates it was to always possible find some pressure at which the spark passed at this minimum potential. Either increasing or decreasing the pressure from this critical

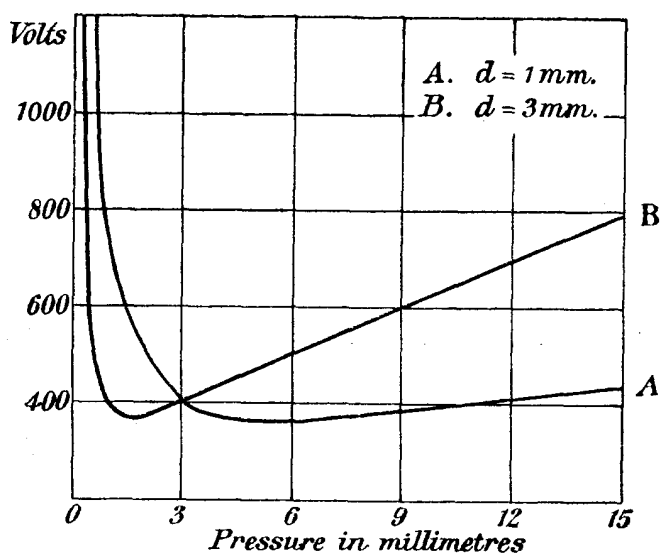


FIG. 22. Variation of sparking potential with pressure in air

value increased the sparking potential. These results are shown in Fig. 22, which represents some of Carr's observations on air at various pressures and for various distances between the electrodes. Similar

results were obtained for other gases. Within the limits of experimental error the product of the sparking distance into the critical pressure is a constant for a given gas. For different gases this product is approximately proportional to the mean free path of the molecules of the gas.

**33. Paschen's law.** As the result of a large number of experiments Paschen<sup>(5)</sup> came to the conclusion that for a given gas the sparking potential was a function only of the product of the sparking distance into the pressure of the gas, that is to say, of the mass of gas between unit area of the electrodes. This law applies also to the case of the discharge between two spheres if the distance between the spheres is appreciably less than the radius of curvature. This result follows from the first form of Townsend's theory. Let  $V$  be the sparking potential. Then  $V = SX$ , and if  $M$  represents the product  $pS$  we have from (16),

$$\alpha = pf\left(\frac{X}{p}\right) = \frac{M}{S}f\left(\frac{V}{M}\right),$$

writing  $f(X/p)$  for  $C\epsilon^{-CT/peX}$  and similarly

$$\beta = p\phi\left(\frac{X}{p}\right) = \frac{M}{S}\phi\left(\frac{V}{M}\right).$$

Hence, substituting in Equation (24),

$$\alpha = \beta\epsilon^{(\alpha-\beta)S}$$

we have

$$(\alpha - \beta)S = \log_{\epsilon} \left( \frac{\alpha}{\beta} \right),$$

$$\therefore M \left[ f\left(\frac{V}{M}\right) - \phi\left(\frac{V}{M}\right) \right] = \log \left[ f\left(\frac{V}{M}\right) / \phi\left(\frac{V}{M}\right) \right] \quad (26)$$

thus showing that the sparking potential  $V$  depends only on  $M$ , that is, the product  $pS$ . Carr has shown that this law is true for all pressures both above and below the critical pressure. It is thus a result of great importance. It follows that if  $p$  and  $S$  are varied in such a way that the product  $pS$  remains constant the sparking potential will be unaltered. The sparking potential is not affected by temperature provided that the mass of gas between the electrodes remains constant.

**34. Minimum energy required to produce ionization.** The determination of the energy necessary to produce an ion in a given gas can be made more conveniently and more accurately if electrons are substituted for the ions of the gas itself as the bombarding particles. The

simplest arrangement, due to Lenard, is shown diagrammatically, in Fig. 23.  $Z$  represents a metal filament, which when heated to incandescence by the passage of an electric current, serves as a source of electrons (§ 57). The electrons, which are emitted with very small initial velocities, are accelerated in an electric field applied between the filament  $Z$  and a wire gauze, or grid,  $G$ , which is at a positive potential with respect to the filament. The distance between the filament and the grid is small compared with the mean free path of an electron in the gas at the low pressure at which the experiments are conducted, and the majority of the electrons pass through the grid without having made a collision in the gas, and thus with the full velocity  $v$  produced by a free fall through the potential difference  $(V_G - V_Z)$  existing between the filament and the grid. They thus pass into the space above the grid with a definite kinetic energy  $\frac{1}{2}mv^2 = (V_G - V_Z)e$ , where  $m$  is the mass and  $e$  the charge on the electron.

If this energy is sufficient to produce ionization by collision in the molecules of the gas, positive ions will be formed in the large space between  $G$  and the collecting plate  $E$  which is connected to an electrometer, and will be directed to the plate by the field  $(V_G - V_E)$  acting from the grid to the plate. The electrometer will thus charge up with positive electricity. The potentials are so ordered that  $(V_G - V_E)$  is greater than  $(V_G - V_Z)$  so that any electrons which make no collisions in the gas will be unable to penetrate to the plate, and will be returned, together with any negative ions formed in the gas, to the grid. The electrodes are, of course, in a sealed enclosure so that the experiment can be carried out at suitable low pressures, usually of the order of a millimetre of mercury.

It will be seen that if the filament is heated and  $(V_G - V_Z)$  is gradually increased from zero no current should be registered by the electrometer until this potential difference reaches the value necessary to give the electrons the kinetic energy required to ionize the gas. This critical value is known as the ionization potential of the gas. The actual energy is obtained by multiplying this potential by the electronic charge. In Fig. 24, which represents a set of experiments carried out in this way in hydrogen, it will be seen that no current reaches the collecting plate until the accelerating potential on the

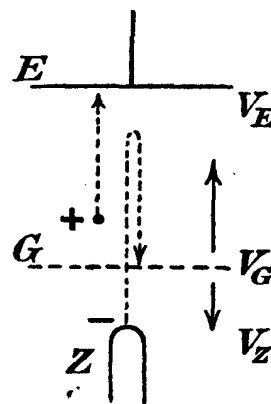


FIG. 23. Illustrating Lenard's method of measuring critical potentials

electrons reaches 10 volts. A current then begins to flow which increases with increasing voltage. It was natural to identify this critical potential with the ionization potential of the gas, and this was the interpretation at first put upon it. Further work, however, threw doubt on this interpretation.

When the P.D. between  $G$  and  $Z$  is still further increased other critical points make themselves manifest on the current-voltage curve. The curve, in fact, instead of rising continuously, shows a series of

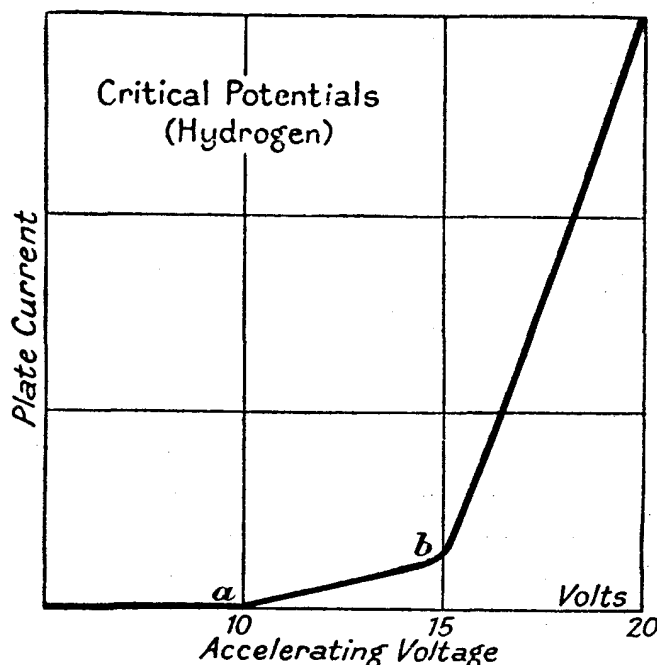


FIG. 24. Critical potentials in hydrogen

points ( $a$ ,  $b$ , Fig. 24) at which the slope of the curve changes abruptly, showing that new sources of current are coming into action. There is thus not one but a series of *critical potentials* for the gas (as many as fourteen have been recorded for mercury vapour) and it becomes necessary to interpret these.

**35. Radiation and Ionization Potentials.** A closer investigation of what is going on in the gas shows that the lower critical potentials are not ionization potentials; no ions are formed in the gas by electrons with the corresponding energies. The absence of ions in the gas can be demonstrated quite simply by substituting a fine wire for the solid plate collector,  $E$  (Fig. 23). Since the field would be directed towards the wire, this would collect ions from the gas as efficiently as the plate. It is found, however, that the current collected by the wire at the lower critical potentials is vanishingly small (it is, in fact, proportional



to the area of the collector), and is due to some action taking place at its surface, leading to the emission of negative electrons from it.

This emission of negative electricity, which simulates the collection of a positive charge, has been shown to be due to ultra-violet light (Chapter IX) emitted by the gas in the tube under the bombardment of the negative electrons from the filament. The radiation can, in fact, be detected spectroscopically when the electron stream is sufficiently intense. The light emitted at the lowest potential is monochromatic and consists of a single line from the normal spectrum of the gas in the tube. Further lines appear at other perfectly definite higher critical potentials. The complete spectrum is only evoked when true ionization takes place in the gas.

Since the lower critical potentials correspond to the emission of radiation, they are called radiation or, less happily, "resonance" potentials. They are of great importance in the study of spectra, and we shall return to them later. For the present we may say that the ionization potential measures the work which must be done to remove an electron completely from the atom; the radiation potential measures the energy necessary to transfer an electron from its normal position in the atom to some other possible state, where its energy is greater than normal. The fact that the critical potentials are quite few shows that the positions which the electron can occupy in the atomic structure are strictly limited; they can most easily be specified by the energy necessary to move the electron from its normal to its new position, that is to say by the corresponding critical potential.

The atom which has been thrown into one of these "excited" states, as we may call them, is unstable, as the excited electron tends to return to its normal position. In so doing the extra energy which the excited atom possesses is given out as we have seen in the form of monochromatic radiation.

**36. Accurate determinations of critical potentials.** The original Lenard method is not capable of much accuracy, as it is very difficult to fix with precision the position of a kink in a rapidly rising experimental curve. Other sources of inaccuracy are the Volta potential difference between the filament and the grid, and the finite velocity of emission of the electrons from the filament. A very accurate method, due to Franck and Hertz, is available when the gas has no electron affinity (§ 16). The grid  $G$  is removed from its position near the filament and placed just in front of the collecting plate,  $E$  (Fig. 25a), which is at a potential about half a volt lower than that of the grid. Negative electrons passing through the grid must therefore have

sufficient energy to enable them to travel against an adverse P.D. of this amount if they are to be collected by the plate. An accelerating field is applied, as in the earlier form of apparatus, between the filament and the grid.

The electrons make numerous collisions in the space between  $Z$  and  $G$ , but if the molecules have no electron affinity the collisions are perfectly elastic, and cause no loss of energy in the electrons, some of which consequently ultimately reach  $G$  with energy corresponding to  $V_A$ , the P.D. between the filament and the grid. If this is greater than half a volt, or if the electrons already have the necessary energy on emission, a small negative current will flow into  $E$ , and can be measured by an electrometer. This current steadily increases, as  $V_A$  is increased, until  $V_A$  reaches a critical potential for the gas. Since now the electrons have the energy necessary to throw the atom into an excited state, the

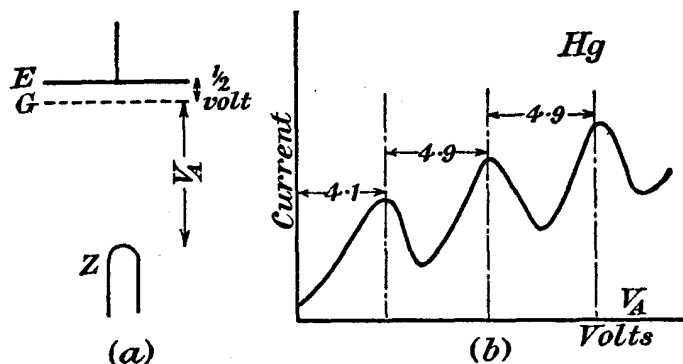


FIG. 25. Experiment of Franck and Hertz on critical potentials

collisions become inelastic, the electron handing over the whole of this energy to the atom with which it collides. The electron is thus left immediately in front of  $G$  with practically zero energy. It is thus unable to penetrate the adverse field between the plate and the grid, and the current begins to fall. The value of  $V_A$  for which the current is a maximum thus corresponds to the critical potential of the gas.

As  $V_A$  is further increased the inelastic collisions occur nearer the filament, and ultimately at such a distance from the grid that the electrons in falling to the grid from the point of collision gain sufficient energy to reach  $E$ . The current then rises again.

If, however,  $V_A$  is increased to about twice the critical potential, the inelastic collisions occur half way between the filament and the grid and the acceleration of the electrons in the remaining half of the field will be sufficient to give them the necessary critical energy again just before reaching  $G$ , and the current will again fall. The graph relating

the plate current with the potential  $V_A$  thus shows a series of peaks, as in Fig. 25b. It is clear that the distance between two peaks gives the critical potential. If the gas has a series of critical potentials each will produce its own set of peaks.

It will be noticed that the distance of the first peak from the origin is distinctly less than that between successive peaks. This is due partly to the finite velocity of emission of the electrons, and partly to the Volta effect, the spontaneous P.D. which is established when two dissimilar metals are in contact.

This method is not applicable to gases such as oxygen which have a strong electron affinity. The critical potentials for such gases are not known with great certainty.

A few values are given in Table III. Some gases give two ionization potentials. The higher value corresponds to the simultaneous ejection of two electrons from the atom. Where only one radiation potential is listed in the Table it is the first or lowest of the critical potentials for the gas.

TABLE III  
Some Critical Potentials (in volts)

Atom or molecule	Ionization potential	Radiation potentials
H <sub>2</sub>	15.9	10.15
He	24.5	20.5
N <sub>2</sub>	16.3	8.1
O <sub>2</sub>	12.8	7.9
Ne	21.5	16.6
Na	5.13	2.1
Hg	10.4	4.9, 5.76, 6.73, 7.73, 8.64, 8.86

The ionization potential measures, as we have seen, the potential difference through which an electron must fall to acquire sufficient energy to ionize the molecule. Since 1 volt is  $1/300$  e.s.u. and the electronic charge is  $4.803 \times 10^{-10}$  e.s.u., the energy of an electron which has fallen through a P.D. of 1 volt is  $4.803 \times 10^{-10}/300$  ergs or  $1.601 \times 10^{-12}$  ergs. This forms a very convenient unit of energy in atomic physics, and is known as the *electron-volt*. Thus the energy required to ionize a molecule of hydrogen is 15.9 electron-volts (ev).

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## CHAPTER VI

### THE PHENOMENA OF THE DISCHARGE TUBE

37. **Phenomena of the discharge in a gas at low pressure.** If the discharge takes place between electrodes enclosed in a glass tube containing gas at low pressure, say, from a few millimetres downwards, some very beautiful and interesting effects are observed. If the potential difference across the tube is not much more than the minimum necessary to maintain a current through the tube the luminosity is at first confined to the region of the two electrodes, the rest of the discharge being dark, as indicated in Fig. 26. As the pressure of the

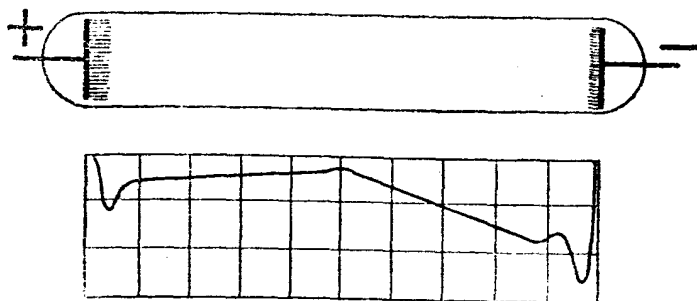


FIG. 26. Distribution of field strength in the dark discharge

gas is reduced, say, to about 1 mm. or so of mercury, the two glows extend outward, especially that at the positive electrode, which now occupies the major portion of the tube. The potential necessary to maintain a current across the tube has now its minimum value.

As the pressure is still further reduced it is seen that the glow near the cathode consists of two parts separated by a dark space. The glow nearest the cathode, covering its surface with a velvety light, is called the *cathode glow*, the other, which is much more extensive, stretching at low pressures some considerable distance into the tube, is known as the *negative glow*. The space between them which is comparatively non-luminous is the *Crookes dark space*, while the similar region between the negative glow and the positive column is known as the *Faraday dark space*. At pressures of 1 mm. or so of mercury the positive column presents a uniform luminosity, but as the pressure is reduced it generally breaks up into a series of bright and dark striae. The typical

appearance of a discharge tube at this stage is represented in Fig. 27, which corresponds to a pressure of about half a mm. in the case of air.

The length of the tube occupied by the phenomena near the negative electrode depends principally upon the nature and pressure of the gas and is independent of the length of the discharge tube. The rest of the tube, however long, is filled by the positive column.

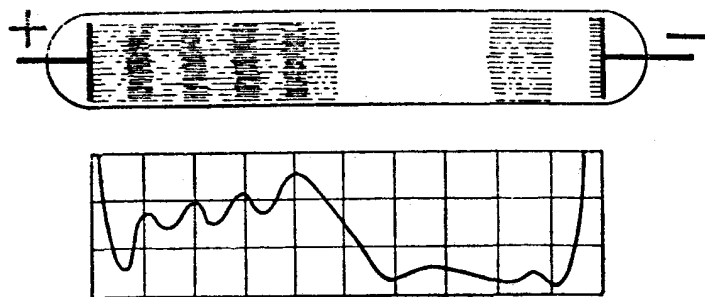


FIG. 27. Distribution of electric field in the striated glow discharge

As the pressure is reduced still further the negative glow and the Crookes dark space increase in length while the positive column contracts towards the anode and at very low pressures is represented only by a feeble glow on the surface of the anode (Fig. 28); at this stage there is no region of constant intensity in the tube. Finally, if the exhaustion is pushed to extreme limits as in an X-ray bulb, the Crookes

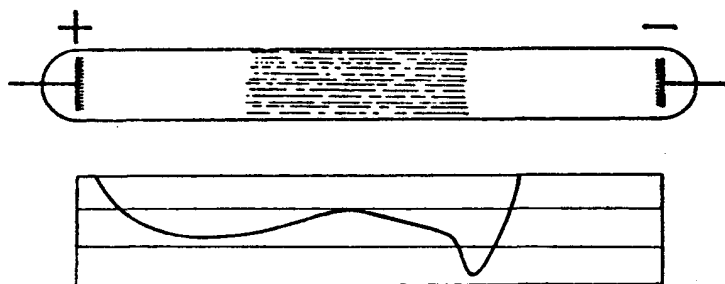


FIG. 28. Electric field strength in the low pressure discharge tube

dark space expands until it fills the whole of the discharge tube, which is then entirely dark. At this stage the walls of the glass vessel fluoresce brightly, the fluorescence being bluish with soda glass and greenish with German glass. The greenish fluorescence is probably due to traces of manganese.

**38. Investigation of the intensity of the field in various parts of the discharge.** The electric potential in various parts of the discharge has been investigated by many observers. The most usual way is to

insert in the tube a small subsidiary electrode or "probe" of fine pointed platinum wire. If there are free ions in the tube and the potential of the wire is less than the potential of the space in its immediate neighbourhood there will be a field in the gas tending to drive ions of the appropriate sign up to the wire. For example if the wire has a smaller potential than the surrounding gas the field will drive up positive ions until the potential of the electrode becomes equal to that of the gas.

The accuracy of the method obviously depends on there being a plentiful supply of ions of both signs in the gas around the wire. If ions of one sign only are present the results may be very misleading. For example supposing only negative ions are present, which is practically the case at points in close proximity to the anode, these negative ions will strike the surface of the wire and continue to do so until its negative potential is so high as to prevent by its electric repulsion any further ions from reaching it. As there are no positive ions to neutralize it, it will thus acquire a negative potential which may be considerably higher than that in any part of the original space before the introduction of the wire. Langmuir has shown that these uncertainties can be eliminated if, instead of allowing the probe to reach a steady potential in the discharge, the current to the probe is measured for different applied potentials. The modified probe method is found to give reliable results and is in general use.<sup>(1)</sup>

Using a single probe, the field in different parts of the discharge must be deduced from the variation of the potential along the tube. A direct method of determining the field strength, due to H. A. Wilson,<sup>(2)</sup> is illustrated in Fig. 29. The electrodes *C* and *A* were kept

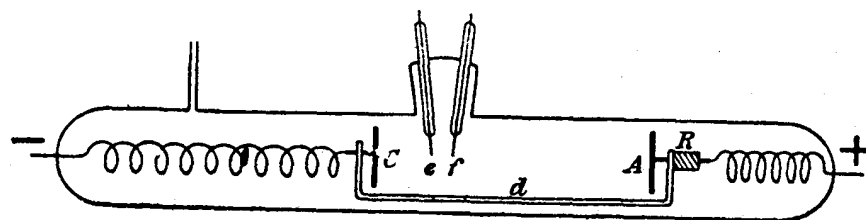


FIG. 29. Illustrating H. A. Wilson's apparatus for the field in a discharge tube

at a fixed distance apart by means of the glass rod *d* to which they were both attached. The glass tube containing the electrodes was much longer than the distance between the electrodes, which were connected to terminals passing through the ends of the tube by long wire spirals. A piece of soft iron *R* attached to one electrode enabled the electrodes

to be moved about from one end of the tube to the other by means of a magnet. Two electrodes,  $e, f$ , were sealed into the tube, the line joining them being along the axis of the tube, and their distance apart about 1 mm. If the two electrodes are connected to opposite quadrants of an electrometer the deflection indicates the difference of potential between the two points: that is, since  $ef$  is constant, it is proportional to the field in the gas in the region of  $e, f$ . By sliding the electrodes along, this region could be made to coincide with any desired part of the discharge.

An ingenious method which does not involve the uncertainties of the probe method is due to J. J. Thomson.<sup>(3)</sup> In this the fact that a beam of cathode rays is deflected by an electric field is made use of. It will be shown later (§ 44) that the deflection of the rays is proportional to

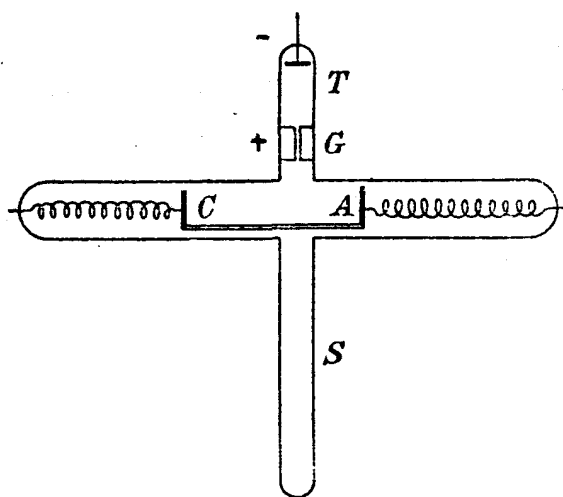


FIG. 30. Thomson's method

the field through which they pass. The main discharge passes between  $C$  and  $A$  (Fig. 30), the experimental devices for moving the discharge electrodes being the same as in the apparatus just described. The cathode rays are generated in a side tube  $T$  at right angles to the discharge, and pass across the main discharge through a fine hole in the metal ring  $G$  which serves as anode. The deflected beam passes down a long

tube  $S$  falling on a screen at the end of the tube where the deflection of the beam is readily measured. This method has been applied to investigate the field very near the cathode.

The results obtained vary somewhat with the state of the discharge, and are shown in Figs. 26–28 immediately below the diagrams of the discharge to which they refer. It will be seen however that they present certain constant features.

Starting at the cathode there is a very strong field in the Crookes dark space which drops with considerable rapidity as we approach the negative boundary of the negative glow, where indeed it reaches its minimum value for the tube. The field in general rises again slightly in the negative glow and falls again in the Faraday dark space, rising gradually as the edge of the positive column is approached.



If the positive column is uniform or unstriated the field has a constant value until the neighbourhood of the anode is reached, when there is again a sharp increase in the field accompanied by rapid increase of potential up to the anode. This anode fall of potential is, however, always much less than that at the cathode.

If the column is striated the field shows variations superimposed upon the steady field, being a maximum where the striation reaches its maximum brightness. This is shown in Fig. 27.

**39. Number of ions in various parts of the discharge.** The apparatus of Fig. 29 can be modified so as to give the relative number of ions in different parts of the discharge. If the two points, *e, f*, are replaced by two small plates, their planes being parallel to the discharge, the current between the plates when some small constant difference of potential is maintained between them (say, for example, that of a Clark cell), will, by (5), § 13, be roughly proportional to the sum of the ions of each kind present in the gas at the given point. In this way by moving the discharge so that various parts of it come between the plates the distribution of ions in the discharge can be

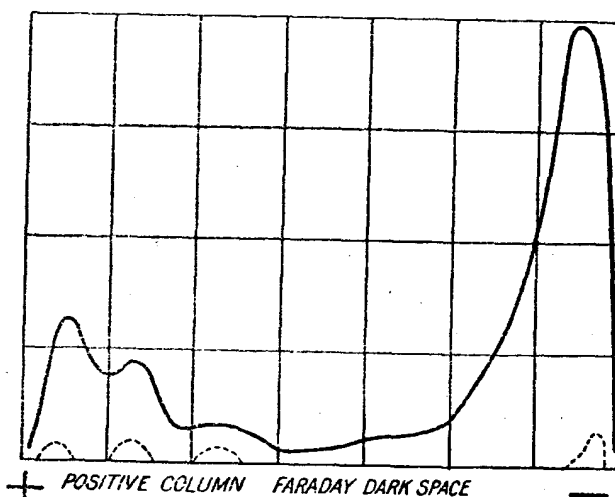


FIG. 31. Numbers of ions in various parts of the discharge

found. The results obtained by H. A. Wilson <sup>(2)</sup> are shown in Fig. 31. It will be noticed that the number of ions is very small near the anode and again in the Crookes dark space. It rises to a maximum in the negative glow, falls again in the Faraday dark space to rise again in the positive column. If the column is striated the ions are most numerous in the luminous parts and less numerous in the dark parts.

Later, however, van der Pol <sup>(4)</sup> has investigated the matter again by a method not involving the use of subsidiary electrodes, with their attendant uncertainties. The discharge tube is placed between the plates of a small parallel-plate condenser which forms part of an oscillating circuit, the current in which can be measured by a Duddell galvanometer. The current in this circuit diminishes as the conductivity of the medium between the plates increases. By bringing

different parts of the discharge tube successively between the condenser plates the relative conductivities of the different portions of the discharge can be estimated. It was found that if the positive column was unstriated the conductivity along it was constant and somewhat higher than that in the dark space. If the column was striated the conductivity was found to be a minimum at the bright edge of the striation. This is exactly the reverse of Wilson's result. It is, however, what might be expected from J. J. Thomson's observation that the field is a maximum at this point. A large field would tend to remove the ions from the space more rapidly than a small field, and the ionization and hence the conductivity should be smaller, as found by van der Pol.

**40. The cathode fall of potential.** The potential difference between the cathode and the boundary of the negative glow is called the cathode fall of potential. Owing to the very large field in the Crookes dark space the cathode fall is usually a large fraction of the whole potential difference across the tube. For small currents the discharge passes only from a small area of the cathode covered by the cathode glow. The area of this glow increases with increasing current so that the current density at the cathode remains constant until the glow covers the whole surface of the electrode. The cathode fall of potential remains constant until this stage is reached. If the current is then still further increased the cathode fall increases with the current.

The cathode fall varies with the nature of the electrode and of the residual gas, and varies from 64 volts for a potassium cathode in argon to 370 volts for a platinum cathode in oxygen. It is small for the electropositive metals such as magnesium, sodium, or potassium, but increases for the less electro-positive metals such as platinum or silver. The effect is probably connected with the work which must be done to extract an electron from the metal (§ 60). The cathode fall is smaller for the inert gases, helium, neon, and argon, than for oxygen or hydrogen.

If the current through the tube is not too large, so that the whole of the cathode is not covered by the cathode glow, Aston has shown that the cathode fall is directly proportional to the square of the length of the Crookes dark space.

The abrupt change of potential near the anode is much less than that at the cathode and varies little either with the pressure or nature of the gas or the nature of the electrodes. It lies between 18 and 24 volts.

**41. Theory of the discharge.** The phenomena of the discharge tube can be grasped most readily if we suppose that a steady discharge is

already established in the tube, and consider the conditions requisite for its maintenance. Since luminosity in a gas results from the recombination of ions (§ 35) the negative glow must contain a plentiful supply of ions of both signs. Let us take the positive ions first. Some of these will be directed towards the cathode by the existing electric field and, since the intensity in the Crookes dark space is very high, will strike the cathode with very high velocities, and with energy corresponding in some instances to a free fall through the full cathode fall of potential. The impact of these particles on the cathode is sufficient to liberate electrons from the surface of the cathode which, in turn, falling through the strong field in the dark space acquire more than sufficient energy to produce fresh ionization in the negative glow. Since the cathode fall of potential is many times greater than the ionization potential of a gas, they will, in fact, attain the necessary energy at quite a short distance from the cathode. The Crookes dark space thus represents the distance an electron travels before making a collision with a gas molecule, that is to say it is the mean free path of the electrons at the pressure existing in the tube.

The negative electrons, owing to their small mass and consequent high acceleration, are rapidly removed from the Crookes dark space, leaving a positive space charge due to the much more slowly moving positive ions from the negative glow. This accumulation of free positive charges in proximity to the cathode, which is of course negatively charged, maintains the high value of the electric field which is characteristic of this part of the discharge.

The positive ion bombardment of the cathode can be verified by using a perforated cathode. A stream of positive particles then emerges from the hole into the space behind the cathode, where it is visible, if the gas pressure is not too low, by the luminosity it excites in the gas. The investigation of this positive stream is described in a later section (§ 49).

The negative glow is produced by the recombination of the ions produced by the electrons from the cathode. As, however, the cathode is screened by the large positive space charge in the Crookes dark space, the field in the negative glow is very small. Thus when the initial energy of the electrons is spent they are no longer able to gain fresh energy from the field to produce ionization by collision. Ionization thus ceases, luminosity disappears, and we have the commencement of a new dark space, the Faraday dark space.

The residual field is, however, sufficient to direct the electrons across the Faraday dark space, and these electrons form a negative space

charge which increases the field between the dark space and the anode. The field thus increases steadily through the dark space and finally attains a sufficiently high value to enable the negative ions again to ionize by collision. Hence luminosity is restored and we have the beginnings of the positive column. As the field in the positive column remains practically uniform, ionization by collision of the negative ions will occur along its whole length and we shall get a uniform column of light. If, however, there happens to be an accumulation of positive ions at any point, this may produce a local fall of field and the discharge will cease to be luminous at that point. Just as in the case of the Faraday dark space this loss of ionizing power will be attended with an accumulation of negative electricity which will restore the field to its normal value. Thus a series of striae may be set up in the positive column, as is often found to be the case. The non-luminous parts of the striae on this hypothesis are thus a repetition of the phenomena of the Faraday dark space.

The theory outlined above accounts satisfactorily for the main features of the low pressure discharge, but requires considerable amplification when we come to details. We have, for example, seen that the discharge is maintained by the large field in the Crookes dark space, but this field is itself a consequence of the discharge. One result of this is that a discharge when once started can be maintained with a much smaller potential difference than that required to start the discharge. For a very complete summary of the subject the reader may be referred to Thomson's *Conduction of Electricity through Gases*.

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## CHAPTER VII

### CATHODE RAYS AND POSITIVE RAYS

**42. Cathode rays.** In addition to the glows described in the previous chapter close observation of the discharge tube will usually reveal another phenomenon, having the appearance of a stream of faint blue light proceeding normally from the cathode and passing along the axis of the tube. At moderate pressures this ends in the negative glow, but if the pressure is reduced until the Crookes dark space fills the whole tube, the stream impinges on the opposite wall of the discharge tube, its point of impact being marked by a greenish fluorescence on the surface of the glass. The fluorescence can be much increased by coating the glass with a layer of zinc blende, or willemite. The bluish light which marks the track of the rays through the gas is also a fluorescence effect and disappears when the gas pressure is greatly reduced.

The cathode rays are actually a stream of electrons proceeding from the cathode, as already described in § 41. The nature of the rays was, however, the subject of a long controversy. Goldstein, to whom the name is due, regarded them as some kind of disturbance in the ether. Crookes suggested that they were material in nature. As it was the researches made in connection with this divergence of opinion which resulted in the discovery of the electron and the atomic nature of electricity, it will be worth while to discuss the evidence in some detail.

(1) *The rays travel in straight lines.* This can readily be shown by constructing a discharge tube of the form of Fig. 32, in which some

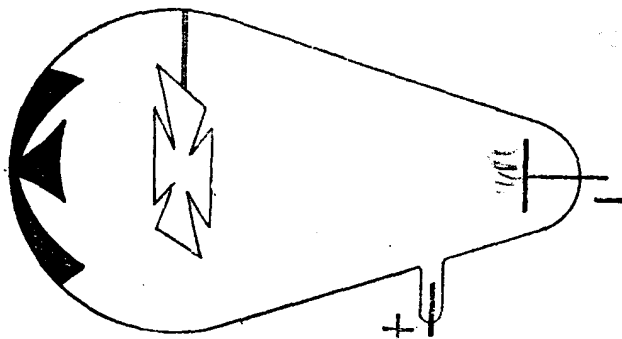


FIG. 32. Shadow formation by cathode rays

obstacle, generally in the form of a mica cross, is placed in the path of the rays. A shadow of the obstacle appears in the centre of the fluorescence caused by the rays on the further boundary of the tube.

(2) *The rays emerge normally from the cathode.* If a tube such as that just described is constructed with a large plane cathode and a small obstacle such as a wire is placed near it a sharp shadow of the wire appears on the further wall of the tube without any sign of penumbra around it. If the rays were given off in all directions from the cathode (as, for example, light is emitted from a uniformly luminous disk) no shadow of the wire would be seen. Similarly if the cathode is concave the cathode rays come to a focus at some point along the axis. Owing to the mutual repulsion of the rays the focus is generally somewhat beyond the geometrical centre of the concave surface. This property of the rays is made use of in the construction of X-ray tubes.

(3) *The rays can penetrate small thicknesses of matter* such as sheets of aluminium foil or gold leaf without producing perforations in the metal. This discovery, which was due to Hertz, can be demonstrated by constructing a "window" of thin aluminium leaf in the end of the tube struck by the rays. The passage of the rays through the foil is demonstrated by luminous blue streamers in the air on the far side of the leaf. These are sometimes known as *Lenard rays*.

(4) *The cathode rays are deflected by a magnetic field.* If an ordinary bar magnet is held near a discharge tube the deflection of the rays by the magnet is made evident by the movement of the fluorescent spot which they produce on the further boundary. We shall return to this important point later. The direction of the deflection shows that the cathode rays experience the same deflection as would be experienced by a flexible conductor coinciding with the path of the rays and carrying an electric current, the direction of the current being towards the cathode.

(5) *The rays carry a negative charge.* This was demonstrated by Perrin <sup>(1)</sup> in 1895. A modification of the original experiment, due to Thomson, is shown in Fig. 33.

The cathode *A* is contained in a small side tube which also contains the anode *B*. The rays starting from the cathode pass through a small slit in a brass plate *C* and cross the larger bulb *D*, where they manifest themselves by phosphorescence of the walls of the bulb. Enclosed in this bulb but out of the direct line of fire of the cathode stream is a small cylindrical vessel, *E*, having a small aperture facing the centre of the bulb. This vessel is carefully insulated and connected by the electrode, *F*, to an electrometer. The vessel is screened from the powerful electrostatic disturbances in the discharge tube by surrounding it with a nearly closed metal vessel, *G*, which is connected to earth. This arrangement forms what is known as a *Faraday cylinder*.

When the discharge was passed the inner cylinder received a very small negative charge. If, however, the cathode rays were deflected by a magnet so as to fall on the openings in the Faraday cylinder a very large negative charge was recorded by the electrometer. This charge increased for some little time but finally became constant owing to the fact that the cathode rays turned the residual gas between the cylinders into a partial conductor. The charge on the vessel increased until the loss by conduction through the ionized gas (which

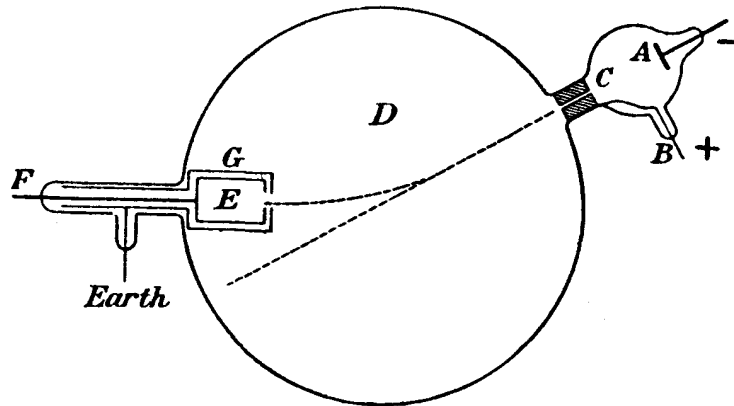


FIG. 33. Illustrating Thomson's proof of the negative charge on cathode rays

is roughly proportional to the potential of the Faraday cylinder) was equal to the gain from the cathode stream. This experiment proves conclusively that a negative charge is a necessary accompaniment of the cathode stream.

(6) *The rays are deflected by an electrostatic field.* This effect, which was a necessary consequence of the theory that the rays consist of charged particles, was for some time sought for in vain. The results just described gave a clue to the cause of the failure. We have seen that the cathode rays ionize the gas through which they pass, thus making it a partial conductor. Thus they move down the discharge tube through a sort of conducting cylinder of their own construction, which screens them from the action of any electric field applied to the tube. By exhausting the tube very completely of the residual gas J. J. Thomson <sup>(2)</sup> in 1897 succeeded in reducing this conductivity sufficiently to allow him to observe the direct deflection of the cathode rays by an electrostatic field. The direction of the deflection showed that the particles in the rays were negatively charged.

(7) In addition to these effects the cathode rays can exert mechanical pressure, and convey very considerable amounts of kinetic





of the particle the speed will remain unaltered. Let  $\rho$  be the radius of curvature of the path of the particle under the action of the field, and  $m$  its mass. The centrifugal force is equal to  $mv^2/\rho$  and for equilibrium this must be balanced by the mechanical force due to the field. Hence

$$\frac{mv^2}{\rho} = Hev \quad . . . . . (28)$$

$$\rho = \frac{mv}{eH} \quad . . . . . (29)$$

As  $v$  is constant the radius of curvature is constant, that is, the particle describes a circle of radius  $m/e \cdot v/H$  in a plane at right angles to the magnetic field.

If the velocity of the particle has a component parallel to the magnetic field this component will remain unaltered since there is no mechanical force acting on the particle along this direction, and the particle thus moves forward through equal distances in equal times along this line, while at the same time describing a curved path around it. Its path is thus a helix, the axis of which is parallel to the field. If  $\theta$  is the angle between the direction of projection of the particle and the field we have

$$\begin{aligned} \frac{mv^2}{\rho} &= Hev \sin \theta, \\ \rho &= \frac{mv}{eH \sin \theta} \quad . . . . . (30) \end{aligned}$$

Since  $v$  and  $\theta$  are constant  $\rho$  is constant and the path of the particle is a helix wound on a circular cylinder with its axis parallel to the magnetic field and of radius  $r = \rho \sin^2 \theta = (mv \sin \theta)/eH$ . When  $\theta = \pi/2$  this helix contracts into a circle, the case we have already considered.

In the general case, when the field is not uniform, the particles will describe helices of varying amplitudes about the lines of the magnetic field.

**44. Motion of a charged particle under the joint action of electric and magnetic forces.** If the particle is moving in a uniform electric field of intensity  $X$  it will experience a force equal to  $Xe$ , and in the absence of any resisting medium, as for example when it is freely moving through a vacuum, it will have an acceleration of  $Xe/m$  in the direction of the electric field. Since the acceleration is constant in magnitude and direction, the problem is identical with that of a particle in a gravitational field, and the particle describes a parabolic path with increasing velocity.

If there is also a magnetic field it will also have an acceleration  $H(e/m)v \sin \theta$  due to the magnetic field  $H$ , and its actual acceleration at any instant is the resultant of these two. The path will evidently depend on the magnitudes of the velocity, the magnetic, and the electric fields, and on their relative directions. The general solution has been obtained. It will, however, be sufficient to consider the cases which are of practical importance.

CASE 1. *The magnetic and electric fields act in the same line and the particle is projected at right angles to them.* In this case since the acceleration due to the magnetic field is at right angles to the field while that due to the electrostatic field is parallel to the field the two accelerations are at right angles to each other, and similarly the two deflections produced will also be at right angles to each other and to the path of the particle. Hence if the original velocity of the particle is so great that we may neglect the small change in its speed produced by the action of the electrostatic field on the particle, the deflection of the particle produced by one of the fields in the direction in which it acts will be independent of the action of the other. The final displacement of the particle will thus be the resultant of the two displacements which each would produce separately. For example if the fields are arranged with their lines of force vertical, the electric field if acting alone would produce a vertical displacement of the particle of, say,  $y$  cm., while the magnetic field alone would produce a horizontal displacement of, say,  $x$  cm. Then if they are both acting together the particle will by their joint action be displaced to a position the co-ordinates of which will be  $x, y$ . This is the arrangement most generally employed in experimental work.

CASE 2. *The electric and magnetic fields are at right angles to each other, and the particle is projected at right angles to the two fields.* In this case the mechanical force due to the two fields acts along the same line, and at right angles to the original velocity  $v$ . Thus the resultant force on the particle is equal to

$$Hev - Xe,$$

assuming that the fields are so arranged that the forces are in opposite directions, and acts along the direction of the lines of force of the electric field. This force will be zero if

$$Hev - Xe = 0$$

$$\text{or } v = X/H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The motion of the particle will in this case be unaltered by the joint

action of the electric and magnetic fields, an important relation, which has often been used to determine  $v$ .

CASE 3. *A charged particle, initially at rest, is exposed to the joint action of electric and magnetic fields at right angles to each other.* Suppose the electric and magnetic fields coincide with the axes of  $X$  and  $Z$  respectively. The particle, being charged, will begin to move in the direction  $OX$ . As soon, however, as it has acquired a velocity it will be acted upon by the magnetic field, and its path will therefore be modified.

Suppose that at any instant the component of its velocity parallel to the electric field is  $dx/dt$ . There is then a mechanical force acting on the particle at right angles to the magnetic field and to the electric field, that is, in the direction of  $OY$ , and equal by (27) to  $He dx/dt$ . Hence

$$m \frac{d^2 y}{dt^2} = He \frac{dx}{dt} \quad . . . . . (32)$$

The electric force along  $OX$  is equal to  $Xe$ , and there will also be a mechanical force in this direction due to the magnetic field equal to  $He dy/dt$ , where  $dy/dt$  is the component of the velocity of the particle along  $OY$ . Hence

$$m \frac{d^2 x}{dt^2} = Xe - He \frac{dy}{dt} \quad . . . . . (33)$$

The solution of equations (32) and (33) is

$$\left. \begin{aligned} x &= \frac{X}{\omega H} (1 - \cos \omega t) \\ y &= \frac{X}{\omega H} (\omega t - \sin \omega t) \end{aligned} \right\} \quad . . . . . (34)$$

where  $\omega = \frac{He}{m}$ .

These are the equations to a cycloid, the curve traced out by a point on the circumference of a circle when the latter rolls along a straight line. The path therefore consists of a series of loops as shown in Fig. 34, where the electrons are supposed to be liberated from the surface of the plate  $CD$ . It is obvious that the particle can never reach more than a certain distance in the direction of the electric field, and cannot penetrate beyond the plane  $LL$  which is the common tangent to the curves. Since the minimum value of  $\cos \omega t$  is  $-1$  this maximum distance is equal to  $\frac{2X}{\omega H}$  or  $\frac{2Xm}{H^2 e}$ .

Thus if  $AB$  and  $CD$  are two parallel plates, and an electric field of strength  $X$  is established between them, while a uniform magnetic field  $H$  is applied at right angles to the plane of the paper, then if ions are formed on the surface of the negative plate  $CD$ , either by the

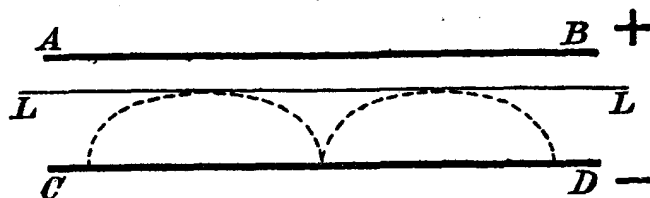


FIG. 34. Illustrating the path of a charged particle in uniform electric and magnetic fields

action of ultra-violet light, or by raising it to incandescence or otherwise, no charge will be received by the upper plate until it reaches the position  $LL$  in the diagram, at a distance  $d$  from the lower plate given by

$$d = \frac{2Xm}{H^2} \frac{e}{e} \dots \dots \dots (35)$$

At any smaller distance the whole of the ions will reach the upper plate, which will thus receive a charge.

The arrangement has been used to determine the ratio  $e/m$  for photo-electrons (§ 66) and thermions (§ 58).

**45. Thomson's discovery of the electron.** The first satisfactory evidence that in the cathode rays we have a type of particle hitherto unknown was given by J. J. Thomson in April, 1897.<sup>(2)</sup> Using the apparatus illustrated in Fig. 33, a thermocouple of known heat capacity was mounted inside the cylinder,  $E$ , so that all cathode particles entering the cylinder impinged upon it. A magnetic field  $H$  of the right magnitude to deflect the rays on to the aperture was applied for about 1 second; and the charge  $Q$  entering the cylinder and the heat generated by the impact of the rays on the thermocouple were measured. If  $W$  is the mechanical equivalent of this heat,  $N$  the number of particles entering the cylinder and  $v$  their average velocity,  $W = \frac{1}{2}mv^2N$  and  $Q = Ne$ ; whence  $W/Q = mv^2/2e$ , where  $m$  and  $e$  are the mass and charge of a particle. But by (29) the radius of curvature  $\rho$  of the path of the rays in the field  $H$  (which can be calculated from the dimensions of the apparatus) is given by  $\rho = mv/He$ . Combining the two equations  $e/m = 2W/(\rho^2H^2Q)$ ; and can thus be calculated from the experimental observations.

The result was startling. The value of  $m/e$  for the cathode particles was found to be  $1.6 \times 10^{-7}$  gm./e.m.u. The value of the same ratio for

the hydrogen ion in solution (i.e. the electrochemical equivalent of hydrogen) is about  $10^{-4}$ . Thus, if the particles are atomic they must carry a charge at least 600 times that on a monovalent ion in solution. Moreover, since the value of  $m/e$  is independent of the materials in the discharge tube, we should have to imagine some mechanism which made the charge on the particle directly proportional to its mass. The alternative is to assume, with Thomson, that the charge on the cathode particle is the same as that on the hydrogen ion, in which case the mass is only  $1/600$  of that of a hydrogen atom. Thomson called these new particles "negative corpuscles"; they are now known as *electrons*.

The experiment, though historically important, is obviously not susceptible of any great accuracy (actually it overestimated the mass about three-fold). A more accurate method, also due to Thomson, is described in the next section.

**46. Thomson's method <sup>(3)</sup> of determining  $e/m$  and  $v$  for cathode rays.** The apparatus used is shown in Fig. 35. The cathode  $E$  is a small aluminium disk, the anode  $F$  being placed in a side tube. A brass disk pierced with a small slit along the axis of the tube limits the rays to a narrow pencil, and to narrow the pencil still further a second brass disk similarly pierced but with a still finer slit is placed at  $B$  some distance from  $A$ . A very fine flat pencil of rays is thus obtained which in the absence of any deflecting fields falls on a fluorescent screen of barium platino-cyanide or powdered willemite at the far end of the tube.

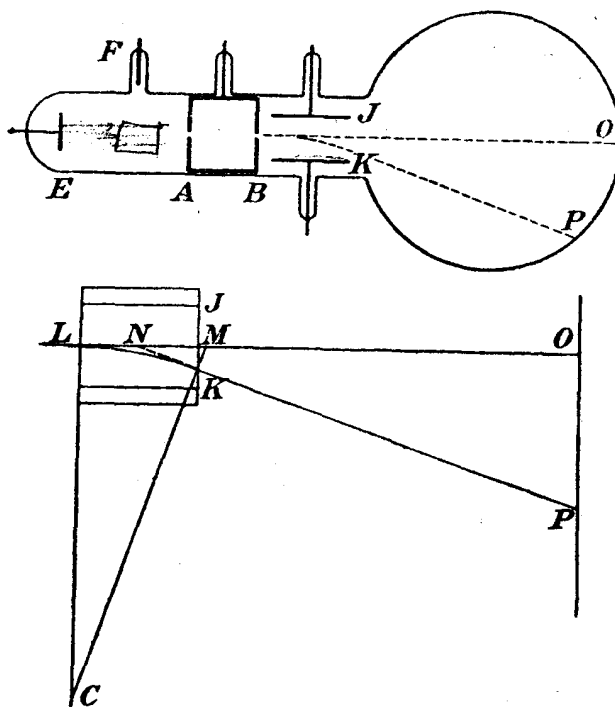


FIG. 35. J. J. Thomson's method of determining  $e/m$  for cathode rays

The electric field is applied by two parallel plate electrodes  $J$ ,  $K$ , their planes being parallel to the path of the rays. The lower plate is earthed while the upper is charged to a suitable potential by means of a

large number of small accumulator cells. The field can thus be calculated if the distance between the plates is known. The deflection produced by the field on the negative particles of the rays is in the direction of the field, that is, in the plane of the paper.

The magnetic field is applied by a small electromagnet placed with its poles at right angles to the plates  $J, K$ . Since the magnetic deflection is at right angles to the magnetic field the magnetic deflection is also in the plane of the paper.

Suppose now that both the magnetic and the electric fields are uniform and coterminous being applied over a short length  $LM$  of the path of the particles. The two fields produce deflections in the same straight line and by properly adjusting the sign and intensity of the electric field these two deflections may be made to neutralize each other, a condition the fulfilment of which can be ascertained by the return of the spot of light on the fluorescent screen to its undeflected position. Under these conditions we have by (31)

$$v = X/H.$$

To find  $e/m$  for the particles we must now measure the deflection produced either by the magnetic or the electric field acting alone. Taking the former case the path of the particles while in the uniform field is bent into the arc of a circle of radius  $\rho$  given by

$$\frac{1}{\rho} = \frac{He}{mv}.$$

On emerging from the field at  $K$  the particles will continue to move along the tangent to the circle at  $K$ . The angle  $PNO$  through which the rays are deflected is by the geometry of the figure equal to the angle  $LCM$ . Thus  $LM/\rho = OP/ON$ , from which  $\rho$  can be determined when the deflection and the dimensions of the apparatus are known. Since the magnetic field and the velocity  $v$  are both known,  $e/m$  can be calculated.

The electrostatic deflection can also be calculated. The acceleration produced in this case is parallel to the field and equal to  $Xe/m$ . If the time during which the particle is in the field is  $t$  this produces a velocity  $X(e/m)t$  or  $Xe/m \cdot LM/v$  in a vertical direction. The horizontal velocity,  $v$ , of the particle is unaltered by the field. On leaving the field the particle moves with these two velocities. Hence if  $P'$  is the deflected position of the ray,  $OP'/OM$  is equal to the ratio of the horizontal to the vertical velocity, that is, to

$$\frac{Xe}{m} \frac{LM}{v^2} \dots \dots \dots (36)$$

which can be evaluated for  $e/m$  since the field  $X$  and the velocity  $v$  are known.

It is obvious that under the conditions of the experiment neither of the fields can be perfectly uniform owing to the disturbances near the edges of the plates and the poles. A correction can be applied as follows.

Since the curvature of the path is small we have approximately, if  $x$  is the co-ordinate of the particle measured along the undisturbed path  $LO$  and  $y$  the co-ordinate at right angles to it,  $1/\rho = d^2y/dx^2$ . Hence

$$\frac{d^2y}{dx^2} = \frac{He}{mv},$$

$$\therefore OP = \frac{e}{mv} \int_0^{BP} [\int_0^x H dx] dx \quad . . . . . (37)$$

Similarly the electric deflection  $OP'$

$$= \frac{e}{v^2 m} \int_0^{BP'} [\int_0^x X dx] dx \quad . . . . . (38)$$

If these integrals are evaluated, which can be done when the distribution of the two fields is known, we have two equations for determining  $e/m$  and  $v$ .

**47. Direct measurements of the velocity of cathode rays.** *Wiechert's method.* A direct determination of the velocity of the cathode stream

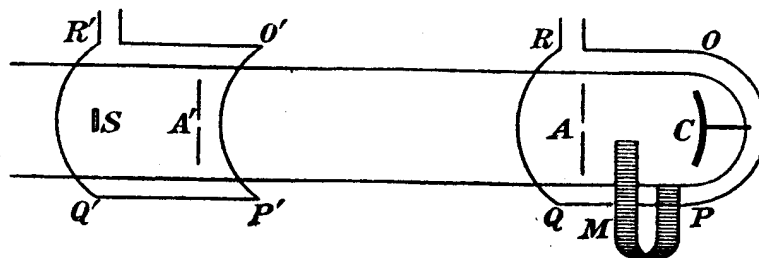


FIG. 36. Wiechert's apparatus for measuring the velocity of cathode rays

was made by Wiechert.<sup>(4)</sup> The principle employed, which has since found many important applications, was to compare the time taken by the rays to travel a known distance with the period of a high-frequency alternating current. A stream of cathode rays from a cathode  $C$  (Fig. 36) is passed through two small holes  $A$  and  $A'$  in a pair of metal plates situated some distance apart in the discharge tube, and is received on a narrow fluorescent screen at  $S$ . The plate  $A$  serves as the anode. A permanent magnet  $M$  is then brought up to the cathode, which deflects the rays from the aperture  $A$  on to the opaque part of

the plate. The screen  $S$  thus becomes dark. A circuit  $OPQR$  carrying a high-frequency current of known period is brought near  $AC$ . The cathode stream is thus acted upon by an alternating magnetic field which sets it swinging like a pendulum in a plane at right angles to the field. The circuit is adjusted so that the rays just reach and pass through the aperture  $A$  when the current is at its maximum. Intermittent flashes of rays thus pass down the tube and illuminate  $S$ .

A second circuit  $O'P'Q'R'$  carrying the same high-frequency current is now brought up near  $A'S$ , thus subjecting the rays in this part of their path to an alternating magnetic field in phase with that applied across  $AC$ . Since the rays only emerge through  $A$  when this field is a maximum, it is obvious that, if the time taken by the rays to travel the distance  $AA'$  is negligible, the field across  $A'S$  will also be at its maximum and the rays will be deflected away from the screen  $S$ , which will thus remain dark. If, however, the time taken by the rays to cover the distance  $AA'$  is just equal to one quarter of the period of the current, the field at  $A'S$  will have fallen to zero, when the rays arrive; they will therefore be undeflected and the screen will light up. The condition for luminosity of the screen is thus that the time taken by the rays to travel a distance  $AA'$  shall be one-quarter of the period of the high-frequency current, or, in general, an odd multiple of this quarter period.

The adjustment can be carried out most conveniently by varying the frequency of the current; the measurement of the frequency can be carried out by standard H.F. methods. The values obtained by Wiechert were of the order of  $3 \times 10^9$  cm. per sec., about one-tenth of the velocity of light. The result was important at the time as a demonstration that the cathode rays do not travel with the velocity of light, and are therefore not electromagnetic disturbances.

The velocity is a function of the potential  $V$  applied to the discharge tube. If we assume that the electron falls through this full difference of potential, and that the energy lost by collisions with the residual gas molecules is negligible, the loss of potential energy,  $Ve$  can be equated to the kinetic energy gained. Thus on reaching the anode the velocity of the electron should be given by  $Ve = \frac{1}{2}mv^2$ , or

$$v = \sqrt{(2V \cdot e/m)}$$

By measuring the magnetic deflection of the rays, and substituting the value of  $v$ , calculated from this equation, in Equation (29) Kaufmann, in 1897, obtained a value for  $e/m$  of  $1.77 \times 10^7$  e.m.u. per gm., which is quite near the accepted value.



**Dunnington's method.** A particularly elegant experiment in which a direct determination of the velocity of the particles was combined with an accurate measurement of the magnetic deflection was carried out by Dunnington.<sup>(5)</sup> Two pairs of plates,  $AA'$  and  $DD'$  (Fig. 37), each with a narrow slit in it are connected to the same source of high-frequency alternating potential;  $A'$  and  $D'$  being connected together so that they are always at the same potential. The whole apparatus is placed in a uniform magnetic field  $H$  at right angles to the plane of the paper.  $F$  is a source of electrons (in this case a heated filament) and  $C$  a device for detecting any electrons which emerge from the slit  $D$ .

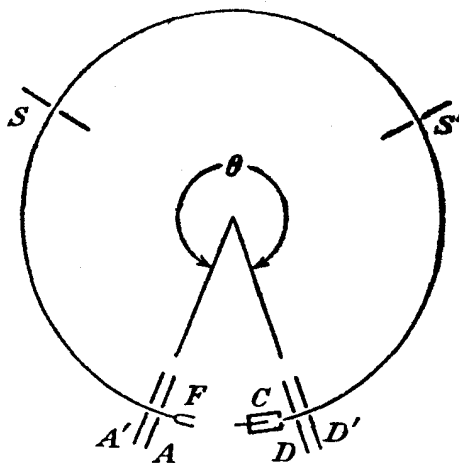


FIG. 37. Dunnington's method of determining  $e/m$  and  $v$

The electrons passing through  $A$  are accelerated in the alternating field between  $AA'$  and their velocity on leaving  $A'$  depends on the instantaneous value of the p.d. existing between  $A$  and  $A'$  at the time of transit. Thus electrons with a wide range of velocities emerge through  $A'$ , but only those whose velocity  $v$  satisfies the relation  $v = H\rho e/m$  can describe the circular path of radius  $\rho$  marked out for them by the slits  $A'SS'D'$ , and so reach the slit  $D$ .

Suppose that the time taken by the electron to travel from  $A'$  to  $D'$  along its circular track, of angle  $\theta$ , is exactly one period  $T$  of the alternating potential wave. The decelerating potential between  $D$  and  $D'$  will then be precisely equal to the accelerating potential which gave the electron its velocity. The electron will, therefore, lose the whole of its energy and be unable to escape from the slit in  $D$ . Thus, if the field  $H$  is adjusted until the electrons just fail to reach the collector we have,  $vT = \rho\theta$ , or  $v = \rho\theta f$ , where  $f$  is the frequency of the H.F. potential. But by (29)  $v = H\rho e/m$ . Equating these two expressions for  $v$  we have finally

$$e/m = f\theta/H.$$

**48. Numerical value of  $e/m$ .** Experiments have been made using discharge tubes with electrodes of many different metals, and containing residual gases of many kinds and at different pressures. All give substantially the same value for  $e/m$ , showing that identical negatively charged particles can be produced from any kind of matter and are

thus presumably a constituent part of all kinds of matter. Experiments, to be described later, have also been made on negative particles from other sources, such for example as a hot wire (§ 58), the action of ultra-violet light on metals (§ 65), and from radio-active substances (§ 114). These also gave identical values for  $e/m$ . Modern theory enables us also to calculate from spectroscopic data values for  $e/m$  for the particles concerned with light emission. The values are identical with those obtained in other ways. A few of these numerous determinations are collected in Table IV. These results leave no doubt of the existence of the electron as a universal particle common to all kinds of matter, the mass and charge of which are independent of its mode of origin.

TABLE IV  
 $e/m$  for electrons from different sources

Observer	Date	Source of electrons	$e/m \times 10^7$ , e.m.u. per gm.
Classen . . .	1908	Cathode rays	1.775
Classen . . .	1908	Hot lime	1.776
Bucherer . . .	1909	Slow $\beta$ -rays (§ 114)	1.763
Alberti . . .	1912	Photo-emission (§ 65)	1.756
Houston . . .	1927	H and He spectra (§ 143)	1.760
Perry and Chaffee .	1930	Cathode rays	1.761
Houston . . .	1937	Zeeman effect (§ 151)	1.760
Dunnington . . .	1937	Hot filament	1.759
Bearden . . .	1938	Refraction of X-rays	1.760

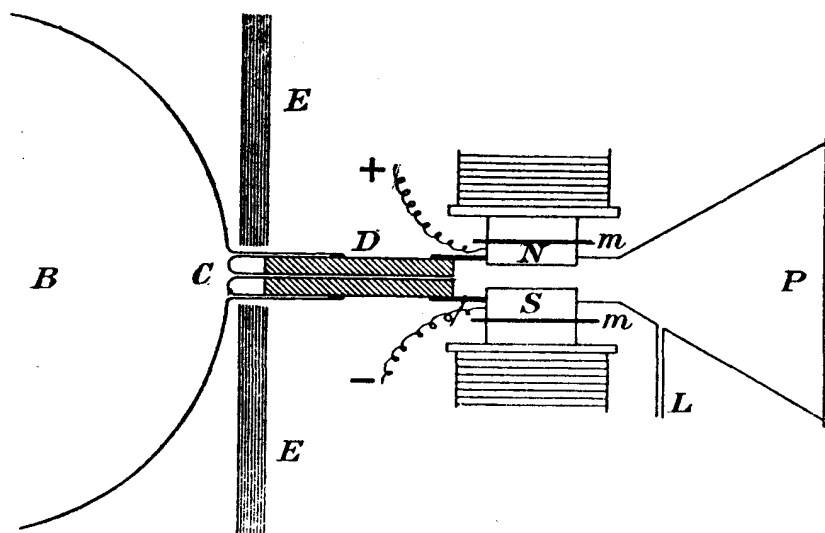
The mean of recent observations, made with the utmost refinements, give a value of  $e/m$  for the electron of  $1.759 \times 10^7$  e.m.u. per gm. to an accuracy of about 1 in 2000. Taking  $e$  as  $1.601 \times 10^{-20}$  e.m.u., the mass of the electron is  $9.107 \times 10^{-28}$  gm. The ratio of the mass to the charge for a hydrogen ion (or proton), i.e. the electrochemical equivalent of hydrogen is  $1.044 \times 10^{-4}$  gm. per e.m.u. The ratio of the mass of the proton to that of the electron is thus  $(1.044 \times 10^{-4}) \times 1.759 \times 10^7$  or 1834. The electron thus has a mass which is very small compared with that of the lightest known atom.

49. **The positive rays. Thomson's experiments.** We have already seen that if the cathode of a discharge tube is pierced by a narrow hole, streamers appear behind the cathode. These streamers are formed by particles which travel from the discharge towards the cathode, and which are, therefore, positively charged. They consist of atoms and

molecules of the various gases present in the discharge tube, carrying positive charges which are either numerically equal to, or some small integral multiple of, the electronic charge.

The investigation of these particles proved to be a matter of some difficulty. Positive rays, owing to their relatively large mass, produce intense ionization in the residual gas in the tube. By combination with the negative electrons so liberated the particles may lose some or all of their charge, or may even acquire a negative charge. Also in any collision between a positive particle and a molecule of the residual gas it may easily be that it is not the original particle but the molecule struck which continues in the course of the rays. Should these effects occur when the particle is already in the deflecting fields, the deflection obtained will correspond neither to one kind of particle nor the other, and some very puzzling and misleading results may be obtained. The effect could be reduced by working at a very low gas pressure; on the other hand, if the pressure is too low no discharge can pass at all.

The difficulties were overcome by J. J. Thomson <sup>(6)</sup> with an apparatus one form of which is shown in Fig. 38.



**FIG. 38. J. J. Thomson's positive ray apparatus**

The discharge takes place in the bulb *B* of some 4 litres capacity as the discharge passes more readily in a large volume than in a confined space. The cathode *C* is an aluminium rod pierced by an axial copper tube of  $\frac{1}{16}$  mm. or even  $\frac{1}{100}$  mm. diameter.

The tube is surrounded by a soft iron cylinder  $D$ , to screen it from stray magnetic fields which would deflect the rays and so cause them to strike the sides of the tube. The main discharge is similarly shielded

by soft iron screens  $E, E$ . The anode is not shown; its position is immaterial. The cathode is water cooled, to remove the large quantities of heat developed there by the discharge, and so preserve the numerous wax joints in this part of the apparatus from being melted.

The fine copper tube serves a double purpose. In the first place it produces a very fine pencil of rays, thus giving a minute and well defined spot on the fluorescent screen or photographic plate at  $P$ . In the second place it acts as a fairly efficient separator of the discharge tube from the measuring apparatus or camera. Diffusion takes place very slowly along a fine tube, particularly at the low pressures employed in these experiments. Thus by feeding the gas very slowly into the bulb, and removing it as rapidly as possible from the camera through the tube  $L$ , it is possible to have the camera at a really high vacuum, while the pressure in the bulb is sufficiently large to enable the discharge to pass. This device is essential to accurate measurement of positive ray deflections, and is embodied in the apparatus of later experimenters.

To measure the ratio of the mass to the charge for the mixed beam of positive rays, the method of parallel fields (§ 44) was employed. The pole pieces,  $N, S$ , of a small electromagnet are let into the sides of the apparatus, and are insulated from the core of the magnet by thin strips of mica,  $m, m$ . The pole pieces can thus be charged, by a battery of accumulators, to any required potential difference, and can be used for applying the electrostatic as well as the magnetic field. The electrostatic deflection is in the plane of the diagram, the magnetic deflection is at right angles to it.

The deflections can be observed visually on a willemite screen, at  $P$ , or can be made to record themselves on a photographic plate which can be let down, without breaking the vacuum, in front of  $P$ . Positive particles, like cathode rays and X-rays, affect a photographic emulsion in the same way as light, and a point struck by the particles shows as a black dot after development.

**50. Positive ray parabolas.** The equations for the deflection of a particle of charge  $E$  and mass  $m$  in an electric or a magnetic field have been developed in § 44. Taking either the approximate calculations or the more elaborate formulae of (37) and (38) we have

$$\left. \begin{array}{l} \text{Electrostatic deflection } x = k_1 \frac{E X}{m v^2} \\ \text{Magnetic deflection } y = k_2 \frac{E H}{m v} \end{array} \right\} \dots \dots \dots (39)$$

where  $k_1$  and  $k_2$  are constants which depend only on the geometry of the apparatus, and can be evaluated if its dimensions are accurately known. Combining these equations we have

$$\frac{y}{x} = \frac{k_2 H}{k_1 X} v \quad \dots \quad (40)$$

$$\frac{y^2}{x} = \frac{k_2^2 H^2 E}{k_1 X m} \quad \dots \quad (41)$$

If the fields are kept at a constant value and if  $E/m$  is constant, then by (41)  $y^2/x = \text{constant}$ , which is the equation to a parabola. Thus all particles, no matter what their velocity, which have the same value of  $E/m$  will lie on a single parabola.

The maximum velocity which a particle can acquire in the discharge is that corresponding to a free fall through the full potential difference  $V$  between the anode and cathode, and is given by  $VE = \frac{1}{2}mv^2$ . Since the electric deflection is given by  $x = k_1 \cdot E/m \cdot X/v^2$  we see that the minimum deflection  $x_0$  is given by

$$x_0 = \frac{1}{2} k_1 \frac{X}{V},$$

and is the same for particles of all kinds. If every particle entered the fields with the maximum energy each set of particles in the beam would be deflected to some definite spot on the ordinate through  $x_0$ . As, however, the particles do not all originate at the anode, but in parts of the discharge nearer the cathode, many will not have fallen through the full potential difference  $V$ , and will thus have less than the maximum velocity; others will have lost energy in collisions with the residual gas molecules. The spots will thus be drawn out into a series of parabolic arcs, terminating on the ordinate through  $x_0$ .

Owing to the large number of particles which lose their charges before entering the deflecting fields, there is always intense blackening and diffusion round the point corresponding to the undeflected beam,

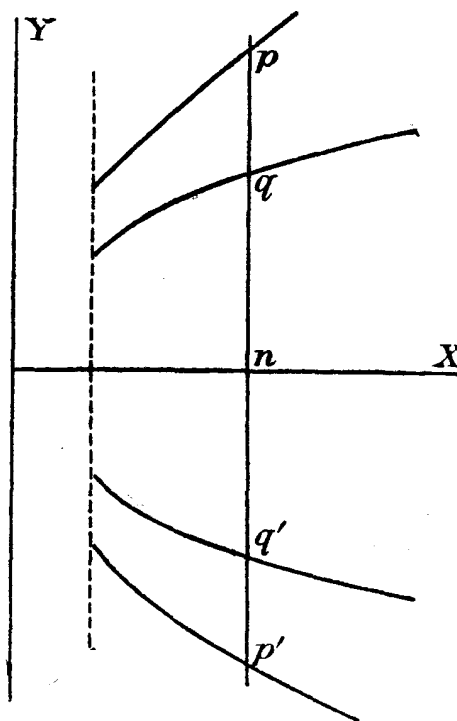


FIG. 39. Illustrating the positive ray parabolas

and it is impossible to determine its position accurately. The magnetic field is therefore reversed half way through the experiment. This reverses the magnetic deflection, and gives a repetition of the parabolic arcs on the lower half of the picture. On development the plate will present the appearance shown in Fig. 39, which represents the case of two sets of particles. This theoretical diagram may be compared with Fig. 40 (Plate II), which is reproduced from one of J. J. Thomson's plates.

**51. Positive Ray Analysis.** Equations (39) show that the lightest particles will be the most deflected. Evaluating the constants  $k_1, k_2$  for the apparatus it was found that the value of  $E/m$  for the most deflected parabola was  $10^4$  e.m.u. per gm. This is practically identical with the value for a hydrogen ion in solution. We may identify the corresponding particles as hydrogen atoms with a single electronic charge. Other particles may be identified most conveniently by direct comparison with this line. Draw any ordinate cutting the two parabolas (Fig. 39) in  $p, p'$ , and  $q, q'$ . Then

$$\frac{(pp')^2}{(qq')^2} = \frac{(2y_1)^2}{(2y_2)^2} = \frac{y_1^2/x}{y_2^2/x}$$

(since  $x$  is the same for both)

$$= \frac{E_1/m_1}{E_2/m_2}.$$

Thus, if all the particles carry the same charge, we have

$$m_2/m_1 = (pp')^2/(qq')^2 \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Matters are not quite as simple as this. A particle may lose more than one electron in the discharge and thus enter the fields with more than one positive charge. A particle which carries  $n$  times the normal charge will behave in the same way as a particle with a single charge and  $1/n$ th of the mass. Thus a doubly charged carbon monoxide molecule would give a parabola identical with that of a singly charged nitrogen atom, and it is not always possible to discriminate between such cases. In other instances the positive particles are not merely neutralized in passing through the ionized gas in the copper tube, but may even take up an extra electron, thus passing into the deflecting fields with a negative charge. These give rise to parabolas on the negative side of the  $Y$  axis. These negative parabolas are very common with elements of an electronegative character, such as oxygen, and the halogens. Strangely enough, they are also formed by hydrogen atoms. They have never been observed with nitrogen or helium.

The positive rays are always made up of atoms or molecules of the elements and compounds contained in the discharge tube. The method can, therefore, be employed to make a qualitative analysis of gases in the tube. The method is, for some gases, more sensitive than the spectroscopic method; it is possible for example to detect the helium present in 1 c.c. of air, an amount of the order of four millionths of a cubic cm. Some substances, however, seem to enter the positive rays only with difficulty. The metals, with the exception of mercury, are noticeably absent. On the other hand, owing to the fact that the particles register themselves on the plate in considerably less than one millionth of a second, temporary combinations of atoms of extreme instability are often detected. Thus with methane ( $\text{CH}_4$ ) in the tube parabolas are formed not only by methane itself, and its constituents, carbon and hydrogen, but also by the unstable compounds  $\text{CH}$ ,  $\text{CH}_2$ , and  $\text{CH}_3$ , which are not otherwise known.

The uniqueness of the method, however, lies in the fact that it measures directly the masses of individual atoms. Chemical methods give only the average mass of the atoms of an element, the assumption being made that all atoms of the same element are identical. Thomson showed that the neon line (atomic weight 20) is always accompanied by a fainter line corresponding to an atomic weight of 22. The substance forming this line is inseparable from neon by any chemical treatment. Radio-active theory (§ 197) had already suggested the possibility of the existence of substances identical in chemical properties, but differing in atomic weight. Such substances are called *isotopes*. Thomson's experiments suggested strongly that the existence of isotopes was not confined to the radio-active elements but that neon consisted of a mixture of two isotopes having atomic weights of 20 and 22.

Thomson's method is not sufficiently sensitive to separate atoms of higher atomic mass which differ by only one or two units. An ingenious modification of the method, due to Aston,<sup>(7)</sup> enabled the necessary precision to be attained.

**52. The mass-spectrograph. Aston's experiments.** In the experiments already described the positive particles having the same value of  $e/m$  but differing in velocity are scattered along a parabolic curve of considerable length. The intensity at any point on the curve is therefore small, and if the original beam of rays is at all fine, very prolonged exposures are required to obtain a measurable trace on the plate. If the particles of different velocity but the same mass could all be focused on the same spot it is obvious that a much finer pencil of rays could be

employed, together with much greater dispersion, without at the same time making the necessary exposure unduly long.

This was achieved by Aston in the following way. A narrow pencil of positive rays is passed first through the electric field between a pair of parallel plates and is thus spread out into what may be called an electric spectrum. The deflection  $\theta$  of a given particle is given by Equation (36), § 46. If the deflections are sufficiently small we may write  $OP'/OM = \theta$ , whence

$$\theta v^2 = X \cdot LM \cdot \frac{e}{m} = Xl \cdot \frac{e}{m} \text{ if } LM=l.$$

The deflected rays then pass through a magnetic field, arranged so as to produce a deflection of the rays in the opposite direction to that of the electrostatic field. The deflection  $\phi$  produced is obviously equal to  $L/\rho$ , where  $L$  is the length of the path in the magnetic field and  $\rho$  the resulting radius of curvature (see Fig. 36). Hence by equation (29) we have

$$\phi v = L \cdot H_m^e.$$

Now in a given experiment  $X$ ,  $H$ ,  $L$ , and  $l$  are constant. Hence for all particles which have the same value of  $e/m$ , we have, by differentiating, since  $e/m$  is constant,

$$\therefore \frac{\partial \theta}{\partial \phi} = 2 \frac{\theta}{\phi} \quad \dots \dots \dots (43)$$

Now  $\delta\theta$  and  $\delta\phi$  represent, respectively, the change in the electrical and the magnetic deviations corresponding to a change,  $\delta v$ , in the velocity of the particles. They will be equal if the mean electric deflection  $\theta$  is one-half the mean magnetic deflection  $\phi$ . Since  $\delta\theta$  and  $\delta\phi$  are equal and in opposite directions particles having the same  $e/m$ , but differing in velocity will emerge parallel to each other. Owing to the dispersion produced by the electric field, however, the beam will have a finite width. It can be brought to a focus by increasing the magnetic deviation.

Let us suppose for simplicity that the electric field can be supposed concentrated at a point  $Z$  (Fig. 41); the magnetic field acting at  $O$ . Then  $ZOP$  will be the path of one ray and  $ZO'P$  that of another of the



same mass but of different velocity. Then since  $\theta$  and  $\phi$  are both very small angles in practice, we have

$$OO' = OZ \cdot \delta\theta = OP \cdot \delta(\phi - \theta),$$

$$\therefore \frac{OP}{OZ} = \frac{\delta\theta}{\delta(\phi - \theta)} = \frac{2\theta}{\phi - 2\theta}$$

by Equation 43. Now in all cases the particles are most numerous near the heads of the different parabolas. The mean kinetic energies

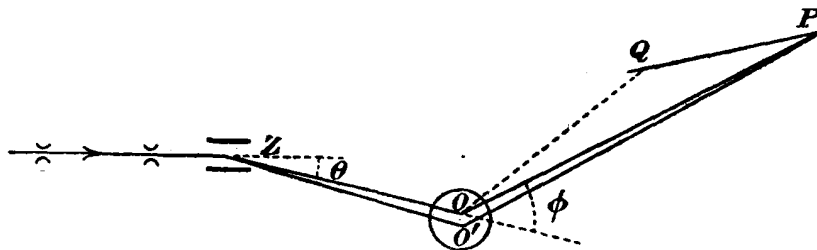


FIG. 41. Illustrating the theory of the mass-spectrograph

of the different kinds of particles will be therefore very much the same (§ 50) and the average value of the electrostatic deflection will be the same for all kinds of particles. Thus  $\theta$  is approximately constant and

$$OP(\phi - 2\theta) = OZ \cdot 2\theta = \text{constant} \quad . . . . (44)$$

This gives the locus of the foci of rays of different masses. This is approximately a straight line making an angle of  $2\theta$  with the direction  $ZO$  and passing through a point  $Q$  such that  $OQ = OZ$  and the angle  $Q\hat{O}Z$  equals  $\pi - 4\theta$ . A photographic plate placed along this line will be in focus for a very fair range of different masses. If a very fine slit is substituted for the circular tunnel in the cathode of the earlier experiments the different sets of particles will produce a corresponding set of fine lines on the photographic plate. The fineness of the focusing and the dispersion obtainable are shown in Fig. 42 (Plate II), which is reproduced from one of Aston's plates.

**53. The whole number rule.** The first mass spectrograph gave an accuracy of the order of 0.1 per cent. Aston found that, to this order of accuracy, every particle registered on the photographic plates had a mass which could be represented by a whole number, if the mass of the oxygen atom was taken as 16.00. Thus chlorine, which has a chemical atomic weight of 35.4, appears as a mixture of particles of atomic mass 35 and 37 respectively. Neon (20.2) is a mixture of particles with masses 20 and 22 respectively. Xenon consists of a mixture of no less than 9 different isotopes with masses ranging from

124 to 136. All elements whose chemical atomic weights differ from whole numbers are found to be mixtures of isotopes of integral masses, and even elements whose atomic weights appear to be integral are sometimes found to contain minute proportions of some other isotope. Thus hydrogen contains about 0.02 per cent. of an isotope of mass 2. This "heavy" hydrogen, or deuterium, has been isolated in a comparatively pure state by the fractional electrolysis of water. Oxygen (16) is accompanied by 0.04 per cent. of an isotope of mass 17 and by 0.20 per cent. of an isotope of mass 18. Helium and fluorine so far appear to be pure.

Since the positive rays may contain compound molecules, as well as atoms, and since very small traces of impurities register themselves on the plates, the mass spectra, as we may call them, are usually very rich in lines, and often nearly every integral position is occupied. Fig. 42 (Plate II), which is reproduced, by permission, from Aston's *Mass Spectra and Isotopes*, is a typical example. The gas in the tube was a mixture of bromine and carbon dioxide. The doubly charged carbon dioxide molecule (giving an effective mass of 22) appears at the top of the plate, and the singly charged molecule (44) at the bottom. In the middle we have the two chlorine isotopes 35 and 37, and their corresponding hydrogen compounds HCl (35) and HCl (37) at 36 and 38 respectively. The oxygen molecule appears at 32, and several other lines can also be identified on the plate.

Of the lines on the plate which are not represented by integers, 39.5 and 26.3 are the doubly and trebly charged atoms of bromine (79). The line 40.5 is the doubly charged atom of a bromine isotope (81). Its trebly charged atom is seen at 27. The numbers on the right of the diagram refer to a second spectrum taken on the same plate with rather less dispersion.

Results of this kind afford the strongest possible proof of the unitary structure of matter. Prout's suggestion that all the elements were built out of hydrogen atoms was negatived, for the time, by the fact that the chemical atomic weights were not whole numbers, and were thus not integral multiples of any chemical unit. The possibility that the atoms of a given element might not all be alike was overlooked. These results have cleared the way for a theory of matter in which the nuclei of the different elements are regarded as being structures made up of integral numbers of some fundamental kind of particle.

**54. The packing effect.** The one exception to the whole number rule was hydrogen. The experiments showed quite definitely that on the scale  $O=16.00$ , the mass of the hydrogen atom is 1.008. This is so

nearly equal to unity that it is difficult to avoid the assumption that the hydrogen nucleus (that is, the atom of hydrogen from which its single electron has been removed) is, in fact, the fundamental unit from which other atomic nuclei are constructed. That is why it has been given the name of *proton*. On the other hand, it is undoubtedly slightly, but definitely, too large for the structure.

A possible solution is offered by the electromagnetic theory of mass, which is discussed in a later chapter (§ 118). It can be shown that an electrical particle exhibits mass in virtue of the energy of its electromagnetic field. It can also be shown that if two electrical particles are brought very close together, the energy of their combined fields, and hence their total mass, is somewhat less than the sum of the energies of the individual fields. If we assume that the mass of the proton is electrical in origin we should expect that, when a number of protons are packed together within the very small volume of an atomic nucleus, the radius of which is of the order of  $10^{-12}$  cm., the total mass would be less than the sum of the masses of the individual particles. This loss of mass is known as *the packing effect*.

To investigate the packing effect with greater accuracy Aston<sup>(8)</sup> has constructed a new mass spectrograph with larger distances between

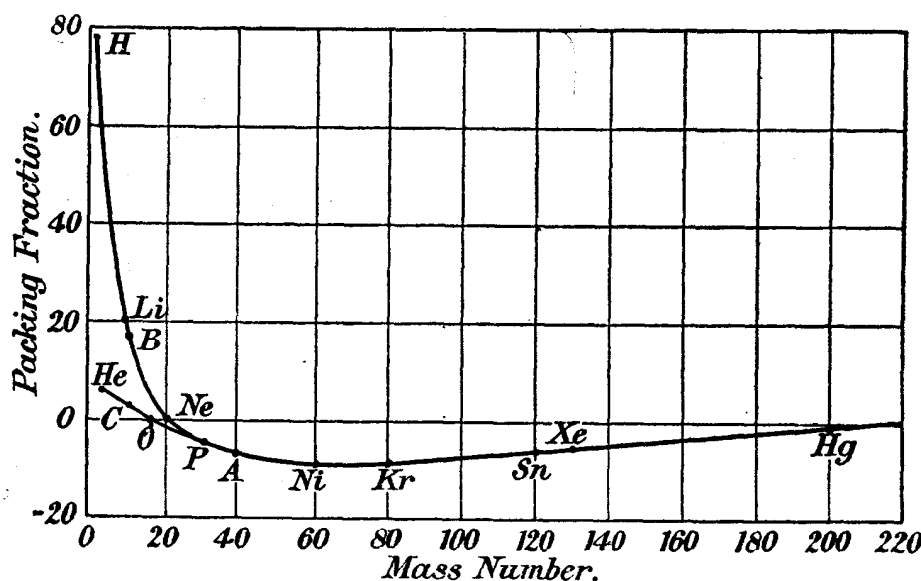


FIG. 43. Aston's packing fraction curve

the fields and the photographic plate, which is capable of giving an accuracy of 1 in 10,000. He finds that when measured to this degree of accuracy all the atoms show slight departures from the whole number law, thus showing, as might have been expected, that the packing effect is not quite identical in magnitude for all atoms. Thus if oxygen

is taken as 16.0000, helium has a mass 4.0039, nitrogen (14) a mass of 14.007, and argon (40) a mass of 39.975. To express such numbers conveniently Aston has defined a "packing fraction" which is the divergence of the atom from the whole number rule divided by the whole number which most nearly represents the mass of the atom (on the scale  $O=16.0000$ ). It is usually expressed in parts per 10,000. The relation between the packing fraction and the mass number of the element is indicated in Fig. 43.

As will be seen from the diagram, the packing effect is a maximum for elements in the neighbourhood of nickel. On the electromagnetic theory the packing effect, or mass defect, as it is often called, represents the energy difference between the actual energy of the nucleus, and the sum of the energies of its constituent particles. Thus while elements of low atomic weight should give out energy if condensed into atoms of higher atomic weight, elements of high atomic weight should yield energy on disintegration. The energy given out by radio-active substances (§ 200) is probably to be ascribed to this cause.

**55. Geometrical focusing. Bainbridge's method of positive ray analysis.** A difficulty met with in high-precision measurements on positive or cathode rays is that the rays emerge from the limiting slit,

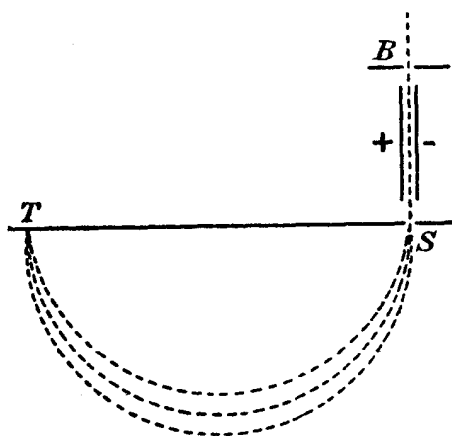


FIG. 44. Bainbridge's method of positive ray analysis

not in a parallel pencil but in a slightly divergent cone. Thus when the rays are deflected in a magnetic field, although all particles of the same  $E/m$  and the same velocity describe circular arcs of the same radius their actual paths diverge somewhat widely, so that, when intercepted by a photographic plate we get not a sharp line but a rather indefinite band. It can be shown, however, that if conditions are so arranged that the median ray is deflected through exactly  $180^\circ$ , all the

other paths converge surprisingly closely on to this same point, at the opposite end of the diameter of the semicircle described by the median ray. This is illustrated in Fig. 44, where the dotted lines show the paths of rays diverging from the slit  $S$  and converging again on  $T$  after describing semicircles in the magnetic field. The reader can easily convince himself of the effect by describing a few circles of equal radius all passing in slightly different directions through some common point.

The principle was first applied by Classen in 1908 in his measurements on  $e/m$  for electrons. Its first application to positive rays was made by Dempster<sup>(9)</sup> in experiments to be described in the next section. By combining this geometrical focusing principle with what may be termed a velocity selector, Bainbridge<sup>(10)</sup> has produced a high-precision instrument of accuracy and resolving power in no way inferior to the mass spectrograph of Aston. The positive rays pass through a fine slit in the cathode  $B$  (Fig. 44) and enter an electric field  $X$  between a pair of parallel plates placed very close together. A magnetic field  $H'$  is applied at right angles to  $X$ , as in the original Thomson experiment (§ 45). Thus only particles which are undeviated in the crossed fields can traverse the narrow gap between the plates, and all particles reaching  $S$ , irrespective of their mass, have this same critical velocity  $v = X/H'$ .

These particles now enter the main magnetic field, where they are deflected through a semicircle, all particles of the same  $E/m$  being brought to a focus at the same point in the plane  $ST$ , where they are received on a photographic plate. Since the rays have been deflected through a semicircle  $ST$  is the diameter of the circle and the radius  $\rho$  is therefore  $\frac{1}{2}ST$ . Since  $\rho = mv/EH$  and all particles have the same velocity  $v$ , the mass of the particle is directly proportional to the distance of its trace from  $S$  and the scale is linear. The method requires a large and powerful magnet to produce the necessary field. The one used by Bainbridge was capable of producing a uniform field of 15,000 gauss over a semicircle of 40 cm. diameter. With this apparatus atomic masses could be compared to an accuracy of 1 part in 10,000.

Further developments in mass-spectrography have increased the accuracy up to two or three parts in 100,000. A few recent values for some of the lighter elements are given in Table XVII (p. 313). These are measured on the basis that the most abundant isotope of oxygen is assigned the mass 16.0000. Chemical atomic weights are assigned on the basis that the mean atomic weight of oxygen is 16.0000. As oxygen contains traces of two isotopes, one of mass 17 and the other of mass 18, the chemical unit is obviously slightly greater than the one adopted by physicists. According to the most recent determinations the discrepancy amounts to about 27 parts in 100,000.

**56. Relative abundance of isotopes. Electrical methods of measurement.** The relative abundance of the different isotopes of a given element can be estimated from the relative intensity of the corresponding lines on the mass spectrogram. The method is only approximate, as the degree of blackening depends not only on the number but also

on the nature of the particles. The hydrogen line, for example, will often be the most intense line on the plate when the hydrogen atoms form no more than 1 per cent. of the whole beam. Since the isotopes of a single element do not differ, usually, by more than a few per cent. in mass the error is probably not great.

A more certain method is to allow the positive particles to pass through a fine slit into a small Faraday chamber, the inner electrode of which is connected to an electroscope. The charge collected by the electroscope is directly proportional to the number of particles passing through the slit.

This method has been applied by Dempster,<sup>(9)</sup> using the geometrical focusing device described in the previous section. Dempster worked with the positively charged metallic atoms which are emitted when a filament coated with the appropriate metallic salts is electrically heated (see § 57). These particles are given out with negligible velocities, but were accelerated by applying a known potential difference  $V$  between the filament and a perforated cathode  $B$  (see Fig. 44). Since no gas discharge is required to produce the particles, the apparatus could be exhausted to the highest possible vacuum, and it may be assumed that the particles pass through the slit in the cathode with kinetic energy  $\frac{1}{2}mv^2$  equal to the loss in potential energy  $VE$ .

A slit  $T$  is cut in the diaphragm  $ST$  and a Faraday chamber connected to an electroscope is placed behind the slit to receive the rays. The potential  $V$  is adjusted until the particles under observation are focused on  $T$  and have thus been given a radius of curvature in the magnetic field equal to  $\frac{1}{2}ST$ . By varying the potential  $V$  different sets of particles can, in turn, be focused on  $T$  and their intensity measured by the electroscope.

The particles, after giving up their charges, are deposited on the walls of the Faraday chamber. The method can therefore be used to separate any particular isotope of an element in a pure state. The rate of production is very slow, as the current carried by the positive rays is usually only a few microamperes. It is interesting to note, however, that the first appreciable-sized pure samples of the uranium isotope of mass 235 were produced in this way.

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## CHAPTER VIII

### EMISSION OF ELECTRICITY BY HOT BODIES

**57. Loss of charge from a hot body.** The fact that heated bodies are unable to retain electric charges has been known for nearly two centuries. The early experiments were not made under very definite conditions and most of our knowledge of the phenomena is due to experiments made since 1900, and very largely to the work of O. W. Richardson, who has given the name *thermionics* to this branch of the subject. The current from a heated body is thus known as the *thermionic current*, while the carriers by which the discharge is carried are known as *thermions*.

The phenomena are in general very complex, the current depending on the nature and pressure of the surrounding gas and on the nature and previous treatment of the heated substance. To simplify matters as far as possible we will take the case of a metallic wire heated in a high vacuum so as to eliminate all effects due to the presence of gas. The effect can be studied conveniently with the apparatus shown in Fig. 45. The wire *AB* to be heated is surrounded by an outer metal cylinder *CC* which remains cool during the experiment, the whole being enclosed in a glass vessel which can be evacuated. The wire is heated by a current from an insulated battery of cells and the temperature of the wire can be estimated by measuring its electrical resistance and thus making it serve as its own resistance thermometer. If the wire is raised to a suitable small potential a current flows from the hot wire to the cool cylinder. This current can be measured by allowing it to flow to earth through a sensitive galvanometer, or, under some circumstances, even through a milliammeter.

Initially with a new wire the current will flow through the tube whether the wire is negatively or positively charged, showing that thermions of both signs are emitted. If, however, the wire is kept glowing for some time and the gases evolved from the heated wire are continually removed by pumping, or better still by sweeping them out with pure oxygen, it is found that the positive current rapidly decreases and finally becomes negligibly small. The negative emission is also much reduced by this process, but finally settles down to a steady value. The temperature to which the wire must be raised for an appreciable



current to flow depends on the nature of the substance. For platinum a temperature of about  $1000^{\circ}\text{C}$ . is required; for sodium, on the other hand, there is a very considerable thermionic emission at temperatures as low as  $300^{\circ}\text{C}$ .

Other conditions being constant, the thermionic current increases with increasing potential difference until a maximum, or saturation, value is reached. The potential difference required to produce the

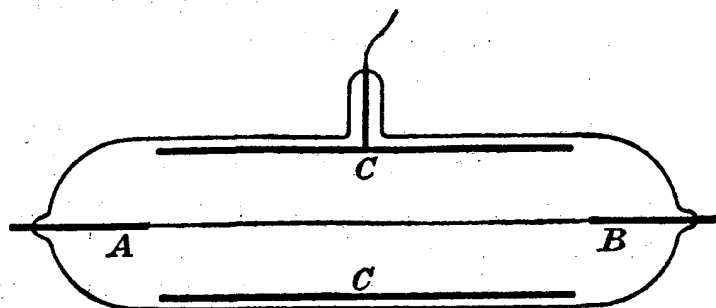


FIG. 45. Richardson's apparatus for investigating thermionic emission

saturation currents depends on the intensity of emission of the heated wire. If the emission is large the saturation potential difference may rise to several hundred volts.

**58. Nature of the thermions.** The nature of the thermions can be determined by measuring the value of  $e/m$  for the particles. The positive thermions are invariably of atomic dimensions. A metal wire heated for the first time *in vacuo* gives a fairly strong emission of positively charged atoms of potassium and sodium. These are undoubtedly due to the presence on the surface of the wire of minute traces of alkali salts deposited thereon from the air of the laboratory. The emission can be greatly increased if the wire is coated with the appropriate salts, and this method was used by Dempster (§ 56) as a means of introducing metallic ions into the positive ray beam. Certain phosphates also give large initial positive emission; in this case the carriers appear to be molecules of gas occluded in the salt.

If the measurements are made in a gas at appreciable pressure a steady positive emission is obtained from the hot wire. The carriers in this case are molecules of residual gas which becomes ionized by collision with the heated filament. In a high vacuum the positive emission gradually ceases as the traces of salt to which it is due are used up, and only the negative thermions are emitted. When this stage is reached current can only pass across the tube if the hot wire is negatively charged. The system thus acts as a rectifier, or valve. This is the principle of the well-known thermionic valve or "diode,"

now extensively used both in radio and in power circuits. Diodes capable of handling 500 kilowatts are now in use.

The negative emission in high vacua is entirely in the form of electrons, as shown by the values obtained for  $e/m$  for these particles. Some of these values are included in Table IV. In the presence of residual gas, however, negative ions may be formed from the gas molecules which impinge on the hot wire, and may cause a substantial increase in the current. In the rest of the chapter we shall deal only with the case where the wire has been heated for some time *in vacuo* and the emission is entirely electronic.

**59. Theory of thermionic emission.** It can be shown that the conduction of electricity through metals is due to the motion of electrons through the metal, and the fact that the current is directly proportional to the applied E.M.F., however small, shows that these electrons are free. If the electrons were bound to the atoms an electrical field of finite magnitude would be required to remove them from the atom, and until this field was reached no current could flow.

The electrons, however, though moving freely through the metal are retained within it by forces at the surface of the metal. An electron escaping from the surface of a metal will, for example, induce a positive charge immediately beneath it which will tend to attract it back to the surface. There may also be other intrinsic fields acting in the same direction. To separate an electron from the surface, a certain amount of work  $w$  must be done against these forces, and an electron will only be able to escape if its kinetic energy normal to the surface is greater than  $w$ .

If we could assume that the free electrons in the metal behaved like the molecules of a perfect gas at the temperature of the surrounding metal, we could calculate the probability of an electron reaching the surface with energy greater than  $w$ . Richardson developed a theory of the effect on these assumptions, which agreed satisfactorily with the experimental results.

Measurements on the specific heats of metals are, however, inconsistent with the assumption that a metal contains any appreciable number of free electrons sharing the thermal energies of the molecules. According to Sommerfeld, the electrons must be regarded as forming a "degenerate system" (§ 186). Fortunately we can evade these uncertainties by resorting to thermodynamical considerations.

Suppose we have an evacuated enclosure at a constant temperature containing a body emitting electrons. Electrons will accumulate in the vacuous space, forming an electronic gas, until the electron density

becomes so great that the number of electrons returning to the metal from the electronic atmosphere becomes equal to the rate of emission of electrons from the metal. A state of dynamical equilibrium will thus be reached, similar to that between a liquid and its saturated vapour.

The analogy holds in another respect. Since work is done when an electron escapes from the surface heat will be absorbed in the process. The absorption of heat from an incandescent filament during thermionic emission has actually been measured by Richardson,<sup>(4)</sup> as has also the reverse effect, the emission of heat when thermions are re-absorbed. We can thus speak of the latent heat of evaporation of the electrons, and apply the usual latent heat equation to the problem.

Thus, if  $p$  is the pressure of the electron atmosphere when equilibrium is established,  $V$  and  $U$  the volume of one gram-molecule of electrons in the free space and in the metal itself, and  $L$  the energy absorbed during the evaporation of one gram-molecule of electrons,

$$L = \theta \frac{dp}{d\theta} (V - U) = \theta \frac{dp}{d\theta} V$$

approximately, since the volume of the electrons in the metal is very minute compared with that outside.

Now the energy absorbed is used up in two ways: (a) in providing the work necessary to allow of the escape of the electrons from the surface; this is equal to  $Nw$ , where  $N$  is the number of electrons in a gram-molecule, that is Avogadro's number, (b) in overcoming the pressure of the electronic atmosphere into which the electrons evaporate. This is equal to  $pV$ , where  $p$  is the pressure. Since the electrons in the free space behave as a gas  $pV = N \cdot k\theta$ , where  $k$  is the gas constant for one molecule, i.e. Boltzmann's constant. Thus

$$L = Nw + Nk\theta \quad . . . . . (45)$$

$$\therefore V \theta \frac{dp}{d\theta} = N(w + k\theta),$$

or substituting for  $V$  from the relation  $pV = Nk\theta$ ,

$$Nk \frac{\theta^2 dp}{p d\theta} = N(w + k\theta),$$

$$k \frac{dp}{p} = \frac{w + k\theta}{\theta^2} d\theta,$$

$$p = K \theta \epsilon^{\int \frac{w}{k\theta^2} d\theta} \quad . . . . . (46)$$

If  $n$  is the number of electrons per unit volume in the free space, we can write  $p = kn\theta$ . Thus

$$n = C \epsilon \int \frac{w}{k\theta^2} d\theta \quad \dots \dots \dots (47)$$

where  $C$  is a constant.

The electrons in the free space will behave as molecules of a rarefied gas, and the ordinary kinetic theory of gases can thus be applied. It is true that as they are negatively charged they will repel each other and thus possess a negative intrinsic pressure. The actual pressure of the electronic gas is, however, so minute under ordinary conditions that the effect may be neglected. The root mean square velocity of the free electron is thus given by  $\frac{1}{2}mv^2 = \frac{3}{2}k\theta$ , where  $\theta$  is the absolute temperature and  $k$  is the Boltzmann constant, or

$$v = \sqrt{\frac{3k\theta}{m}} \quad \dots \dots \dots (48)$$

If we make the usual elementary assumption that the electrons may be regarded as travelling in six streams normal to the six faces of a cube, and that all the electrons possess the same velocity  $v$ , the number  $n_0$  striking unit area per second will be all the electrons in the stream normal to the surface whose distance from it is less than  $v$ , that is  $nv/6$ . The correct value, allowing for the distribution of velocities among the electrons, can be shown to be  $\frac{nv}{\sqrt{6\pi}}$ . Substituting for  $v$  its value as given by (48) we have

$$n_0 = \frac{n}{\sqrt{6\pi}} \sqrt{\left(\frac{3k\theta}{m}\right)} = C'n\theta^{\frac{1}{2}} \quad \dots \dots \dots (49)$$

where  $C'$  is a constant.

If all the electrons which fall upon the metal surface are absorbed by it,  $n_0$  is the number entering, and thus, when equilibrium has been established, the number leaving the surface per sq. cm. per sec. Experiment shows that a certain number (possibly 50 per cent.) of the returning electrons are reflected back from the surface. The number emitted will thus be  $n_0$  multiplied by an appropriate fraction; e.g. one half if the reflection is 50 per cent. This fraction can be included in the constant  $C'$ , so that the emission is still given by an equation of the form of (49).

Substituting for  $n$  from (47) we have finally

$$n_0 = A\theta^{\frac{1}{2}} \epsilon \int \frac{w}{k\theta^2} d\theta \quad \dots \dots \dots (50)$$

If we assume that the electrons within the metal have the same energy of thermal agitation as when in the free space outside the metal

we can regard  $w$  as independent of the temperature, as it is unlikely that the electrical forces will vary with temperature. Taking  $w$  as constant, Equation 50 obviously reduces to

$$n_0 = A \theta^{\frac{1}{2}} e^{-\frac{w}{k\theta}} \quad \dots \dots \dots (51)$$

which was the form first given by Richardson.

If, on the other hand, we assume that the electron has no energy in the metal, then the total work done in extracting the electron from the metal will include the work done in giving it the energy  $\frac{3}{2}k\theta$  which it has in the space outside the metal, and  $w$  will be of the form

$$w = w_0 + \frac{3}{2}k\theta.$$

It can be shown that this is equivalent to assuming that the "specific heat of electricity" in the metal, i.e. the Thomson effect, is zero. Substituting this value of  $w$  in (50) and integrating we have

$$n_0 = A_1 \theta^2 e^{-\frac{w_0}{k\theta}} \quad \dots \dots \dots (52)$$

Since each electron carries a charge  $e$ , the saturation current is  $n_0 e$  per square centimetre of surface. The experimental curve for platinum is shown in Fig. 46.

At first sight it might appear to be an easy matter to distinguish experimentally between the relations (51) and (52). In practice, however, the variation in the exponential term produced by small changes in  $\theta$  completely overshadows any variation due to changes in  $\theta^{\frac{1}{2}}$  or  $\theta^2$ , and the experimental results can be expressed almost equally well, though of course with different constants, by either relation. Recent experiments have decided in favour of the second form of Richardson's law.

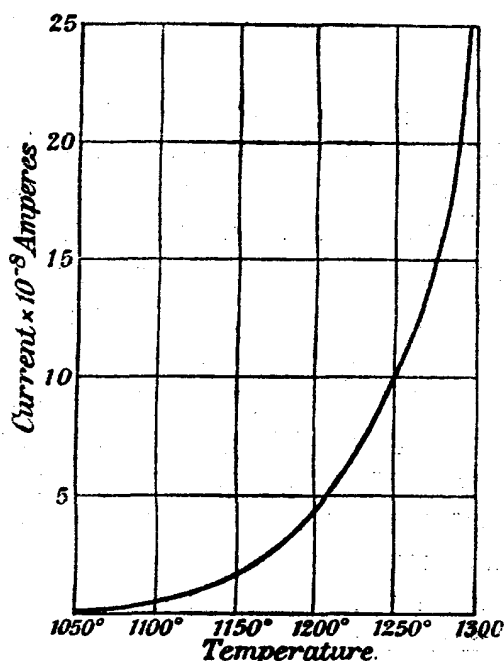


FIG. 46. Curve showing the relation between the temperature and the thermionic emission for a platinum wire

**60. The thermionic work function.** The thermionic emission is very largely controlled by the value of  $w$ . This quantity, which measures the work which must be done to extract an electron from the metal, is known as the thermionic work function. Like other electron energies, it is most conveniently measured in electron-volts. Its value can be

deduced from the experimental curve relating temperature and emission, since  $k$ , the gas constant per molecule, or Boltzmann constant, is known.

Table V gives the values of  $w$  for a few commonly used metals and refers to wires which have been specially purified and cleaned.

TABLE V  
Emission of pure metals

Metal	$A$ $\frac{\text{amp.}}{\text{cm.}^2 \cdot \text{deg.}^2}$	$w_0$ $\frac{\text{electron volts}}$
Molybdenum . . . . .	60.2	4.38
Platinum . . . . .	—	6.27
Tantalum . . . . .	60.2	4.07
Tungsten . . . . .	60.2	4.52
Thorium . . . . .	60.2	3.35
Thoriated tungsten . . . . .	—	2.6

The presence of minute traces of an impurity which has a small work function will increase the emission many fold. Thus the carbon filament of an electric lamp gives a very large negative emission—the well-known Edison effect adapted by Fleming to rectify wireless signals. The effect is, however, mainly due to impurities, as pure carbon does not differ very markedly from platinum in its emissive power. The occlusion of gases, particularly oxygen, on the surface of the metal also produces marked changes in the experimental values.

Owing to the very low value of the work function the emission from wires coated with oxides of calcium, barium, strontium or thorium, or mixtures of these oxides, may be many thousand times greater than that from a bare tungsten wire at the same temperature. Conversely for the same emission the coated wire may be run at a much lower temperature, with a corresponding saving in electrical energy. Coated filaments are, therefore, now used almost exclusively in thermionic valves. The emission follows Richardson's law, the increased effect being due solely to the low value of the work function.

Similarly a strip of platinum carrying a speck of barium oxide (such as can easily be obtained by placing a speck of sealing wax on the strip and burning off the lacquer) forms a convenient point source of electrons, the emission from the platinum being negligible in comparison with that from the oxide. The electrons are emitted with small energies, of the order of a volt, and can be speeded up to any desired

velocity by applying a suitable negative potential to the strip. The arrangement is known as a Wehnelt cathode.

The thermionic emission is generally increased by the admission of gas at low pressures, or even by its previous absorption in the wire. Hydrogen at a pressure of no more than 0.0006 mm. was found to increase the emission from platinum at 1350° C. no less than 2500 fold. The emission still follows Richardson's law, but with a smaller value for the work function.

The effect is probably due to the condensation on the platinum of a layer of hydrogen. Hydrogen is a very electro-positive element, and if we assume that the hydrogen atoms in the condensed layer are positively charged their presence will produce a field between the gas and the metal tending to assist the escape of the electrons from the metal. Less work will thus be required to extract an electron from the metal. Since  $w_0$  is a measure of this work, the value of  $w_0$  will be decreased, as is actually found to be the case.

**61. Distribution of velocities among the thermions.** The distribution of velocities among the electrons emitted from a hot wire has been investigated by Richardson <sup>(4)</sup> by a method based on the following principle. Let the cool electrode be maintained at a definite negative potential  $V$ . The work done by the negative electron in passing from the surface of the hot strip to the negative electrode will thus be  $Ve$ , and the electron will only succeed in reaching the electrode and imparting its charge to it if the kinetic energy  $\frac{1}{2}mv^2$  is greater than  $Ve$ .

Thus the current reaching the electrode is a measure of the number of thermions whose kinetic energy perpendicular to the strip is greater than  $Ve$ . In this way the distribution of normal velocities of the thermions can be studied. A slight extension of the method can be used to measure the distribution of the velocities of the thermions in a direction parallel to the surface of the hot strip. In both cases the distribution found agreed very closely with that given by the Maxwell-Boltzmann law for the distribution of velocities among the molecules of a perfect gas. The electrons thus escape from the metal not only with the mean velocity but also with the velocity distribution of gas molecules at the same temperature. These results are very interesting as they supply a very direct experimental verification of the Maxwell-Boltzmann distribution law.

**62. The current through a thermionic valve. Langmuir's law.** Richardson's law (52) gives the relation between the temperature and the saturation current from a heated filament. In the earlier

experiments where the currents measured were small saturation was easily obtained with potential differences of a few volts only. This is by no means the case in a modern thermionic valve. Langmuir<sup>(5)</sup> has shown that if a constant potential difference is maintained between the electrodes and the temperature of the hot wire is gradually increased the current at first increases in accordance with the law of Richardson. Soon however the rate of increase with temperature becomes rapidly smaller than that given by the law and eventually the current reaches a maximum value which then remains constant however much the temperature may be increased, as illustrated in Fig. 47. This constant current, however, increases with the potential difference employed. If  $V$  is this potential difference the value of the constant current  $i$  is given by

$$i = kV^{\frac{3}{2}} \quad . . . . . (53)$$

This is known as Langmuir's law.

This limitation of the current is due to the presence between the electrodes of the negative electrons which are carrying the current.

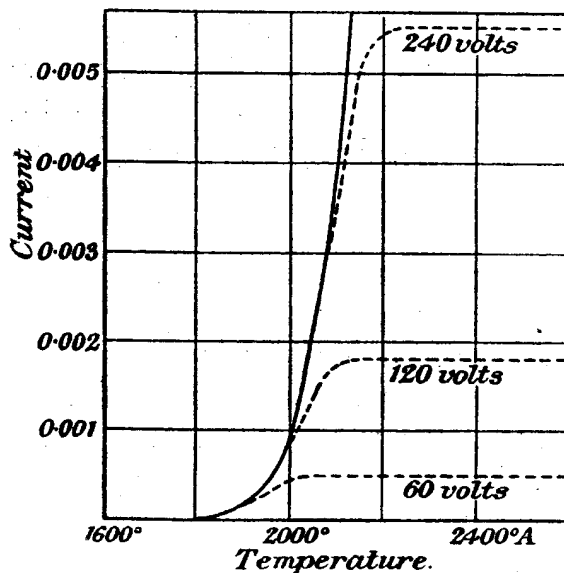


FIG. 47. Relation between current and temperature for a diode

These negative charges will repel the negative electrons which are tending to emerge from the hot cathode and, as the current grows bigger and hence the number of electrons in the space becomes more numerous, this repulsion increases until finally it counterbalances the applied field, and the actual field at the surface of the cathode becomes zero or may even be reversed. At this point further emission from the hot wire ceases. The maximum current which can pass for a given potential

difference is thus limited by the space charge due to the electrons between the electrodes.

We may take as a simple case a pair of parallel plates at a distance  $d$  apart, the heated plate being at potential zero and the cold plate at a positive potential. Then if  $\rho$  is the *negative* space charge (charge per



cubic cm.) at some point between the electrodes,  $V$  the potential at that point, then by Poisson's equation

$$\frac{d^2V}{dx^2} = 4\pi\rho.$$

Also the current-density  $i$  is clearly equal to the charge per unit volume into the velocity,  $v$ , with which the charges move, i.e.

$$i = \rho v,$$

where  $v$  is given by the equation

$$\frac{1}{2}mv^2 = Ve.$$

Hence

$$\frac{d^2V}{dx^2} = 4\pi i \sqrt{\left(\frac{m}{2Ve}\right)}.$$

Now if we neglect the small intrinsic velocities with which the electrons are emitted from a hot cathode (usually a fraction of a volt), we have seen that the maximum current will be attained when the field at the surface of the cathode has been reduced to zero by the space charge effect. Hence for the maximum current we have  $dV/dx=0$  when  $x=0$ , if we measure our distances from the surface of the cathode; and integrating our equation with this condition we have

$$\left(\frac{dV}{dx}\right)^2 = 8\pi i \sqrt{\left(\frac{2Vm}{e}\right)}.$$

Integrating again, and remembering that  $V$  is zero when  $x$  is zero, i.e. at the surface of the cathode, we have finally

$$i = \frac{\sqrt{2}}{9\pi} \sqrt{\left(\frac{e}{m}\right)} \cdot \frac{V^{3/2}}{d^2} \quad \dots \quad (54)$$

If  $i$  and  $V$  are measured and the distance apart of the plates is known this equation can be used to determine the ratio  $e/m$  for the thermionic carriers. This experiment has actually been carried out,<sup>(6)</sup> the result obtained being  $1.76 \times 10^7$ , in very satisfactory agreement with the accepted value for electrons.

**63. Relation between current and potential for a diode.** In the ordinary thermionic valve the filament is maintained at a constant temperature, and owing to its high thermionic emissivity saturation conditions are not approached with the potentials normally applied to the valve. The current is thus governed by the space charge effect and should thus vary with the applied potential difference in accordance with (54). A correction, however, is required on account of the intrinsic velocity of emission of the thermions. An electron emitted

from the hot cathode with an energy of, say, 1 electron-volt can, owing to this kinetic energy, penetrate some distance into an adverse field and will, in fact, reach the cold plate, or anode, unless the latter has a negative potential greater than 1 volt. The current through the valve will, therefore, continue to flow until the plate is at some negative potential,  $V$ , which is proportional to the energy of escape of the electrons. If all electrons escaped with the same energy this would merely involve displacing the origin from zero to  $V$ . The energies of the electrons are, however, not constant, but are distributed about the mean in accordance with the Maxwell-Boltzmann relation. Thus negative potentials considerably greater than the mean value  $V$  will

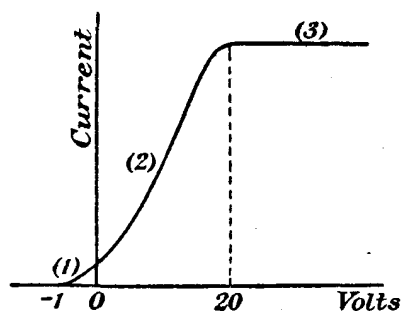


FIG. 48. Relation between current and voltage for a diode

be required to stop the most energetic electrons, while emission does not become complete until the retarding field approaches zero.

The first portion of the current-voltage curve (Fig. 48 (1)) will thus be complex, and will be governed mainly by the Maxwell-Boltzmann relation. As the applied potential difference is increased, however, and becomes large in comparison with the retarding voltage necessary to stop the emission of

electrons, the preliminary effects become comparatively negligible and the curve merges into that given by Langmuir's equation (Fig. 48 (2)). Finally if the potential is sufficiently increased it may be sufficient to produce saturation for the temperature at which the filament is being run. The current then remains constant with further increase in voltage (Fig. 48 (3)) and its value is given by Richardson's law.

**64. Conductivity of flames.** If two platinum plates are placed a few centimetres apart in a wide Bunsen flame and one of them is raised to a moderately high potential by a battery of accumulators a current passes between the plates through the flame. The current is usually large enough to be measured by a galvanometer, one terminal of the instrument being connected to the insulated plate and the other to earth. The current is maintained by the thermionic emission from the heated electrodes. In these circumstances both positive and negative thermions are produced. The relation between the current and the applied potential difference is complicated by the presence of the gas molecules, and if the potential is large ionization by collision

may also come into play. The current is approximately proportional to the square root of the applied potential difference. The current is greatly increased if salts, particularly those of the alkali metals, are vaporized in the flame, the effect being the greatest when the salt vapour comes into contact with the negative electrode. A theory of the effect has been worked out by J. J. Thomson,<sup>(7)</sup> but the effect is not of much practical importance.

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## CHAPTER IX

### PHOTO-ELECTRICITY

**65. The photo-electric effect.** If a metal plate is illuminated by the light from an arc or spark it emits negative electricity. If the plate is carefully insulated the emission continues until the plate has acquired a small positive potential, usually of the order of a volt. When this potential is reached the emission stops. If, however, the plate is connected to earth through a sensitive galvanometer so that its potential is always less than this critical value, a continuous negative current flows from the plate to earth, which continues as long as the plate is illuminated. The effect, which was discovered by Hallwachs in 1888, is known as the photo-electric effect, or, more rarely, as the Hallwachs effect.

For most metals the photo-electric emission is only excited by ultra-violet light, and is stopped if a glass plate is interposed between the source of light and the metal plate. The alkali metals are, however, peculiarly sensitive and respond to rays from the visible part of the spectrum. They can be excited even by the light from a candle.

If the experiments are conducted in air, or other gases, the rate at which electricity escapes from the illuminated plate depends on a large number of conditions, such as the pressure and nature of the gas, the state of polish of the surface and even on the time which has elapsed since the plate was last polished. A very considerable amount of experimental work was expended on the problem before it was finally realized that these variations were mainly due to condensed gas films on the surface of the metal which retarded the escape of the negative electricity. It was only when experiments began to be made on surfaces not only maintained but actually prepared in vacuo, and which had never been exposed to gas at appreciable pressure that consistent results were obtained.

The photo-electric effect can be shown to be due to the emission of comparatively slowly moving negative electrons, which are often spoken of as photo-electrons, to indicate their mode of production. The value of the ratio  $e/m$  for the photo-electrons has been determined by Thomson, Lenard, and others, the experiment being most conveniently carried out by applying the relations (34) and (35) of § 44. The value of the ratio  $e/m$  for the photo-electrons is the same as that for electrons produced in other ways. The photo-electric emission is,

therefore, simply an emission of negative electrons under the stimulus of the light.

It has been found that for each substance there is a definite wavelength at which the photo-electric emission commences; rays of wavelength longer than the critical value producing no effect. This critical wavelength is greater according as the element is more electro-positive and shorter as the element becomes more electro-negative. Thus for the alkali metals, which are extremely electro-positive, the maximum or *threshold* wavelength which will excite photo-electricity is actually in the visible spectrum, being greater for caesium than for potassium and sodium; for other metals the critical wavelength lies between  $4000 \times 10^{-8}$  and  $2500 \times 10^{-8}$  cm. For the non-metals light of still shorter wavelength is necessary.

**66. Velocity of emission of the photo-electrons.** It is found that a plate illuminated by ultra-violet light gives electrons of all velocities from a certain maximum downwards. The velocity is most conveniently measured by measuring the difference of potential necessary to prevent the escape of electrons from the plate. Suppose the metal plate emitting the electrons is at a positive potential  $V$ . The negative electron will experience a force attracting it backwards to the plate and by the time it reaches a place of zero potential it will have lost energy equal to  $Ve$ , where  $e$  is its charge. If the original energy with which it left the plate was  $T$  it will now have energy equal to  $T - Ve$ . If this is equal to or greater than zero the electron will escape being dragged back to the plate. If its initial energy is less than  $Ve$  its motion will be reversed and it will return to the plate again. Thus the minimum velocity an electron can have to escape is given by

$$\frac{1}{2}mv^2 = Ve,$$

$$v^2 = 2V \frac{e}{m}.$$

If the active plate is surrounded by conductors at zero potential and carefully insulated it will gradually acquire a positive potential owing to loss of negative electricity. This potential will grow until it is just sufficient to prevent the escape of the fastest electrons which the plate emits. Thus a measurement of the potential acquired by a plate exposed under these circumstances to ultra-violet light will enable us to estimate the velocity of the swiftest electrons emitted. Similarly the current flowing from the plate when its potential is maintained at some value  $V$  less than the maximum is a measure of the number of electrons which leave the plate with initial energies greater than  $Ve$ .

On this account the objectionable practice has grown up of stating the velocity in terms of the voltage acquired by the plate, and thus phrases like "a velocity of 1 volt" have become common. As will be seen from the formula the velocity is not proportional to the potential but to its square root. The actual electronic velocity can readily be calculated from the potential, since  $e/m$  is equal to  $1.76 \times 10^7$  e.m.u. Thus a "velocity of 1 volt" or  $10^8$  e.m.u. is equal to about  $5.9 \times 10^7$  cm. per sec. The objection does not apply to the measurement of energy in volts, since the energy acquired by an electron is directly proportional to the potential through which it has fallen, and, conversely, the minimum potential which will prevent its escape is a measure of its energy.

Experiments on the maximum velocity of emission of electrons from a metal under the action of light yielded unexpected results. It was shown, in the first place, that the maximum velocity was absolutely independent of the intensity of the light, even when the intensity varied in the ratio of  $10^8$  to 1. It varied with the nature of the metal and, in particular, it depended on the frequency of the radiation. If  $V$  is the positive potential difference necessary to prevent the escape of electrons, and  $\nu$  the frequency of the exciting radiation,

$$V = k\nu - V_0 \quad \dots \dots \dots (55)$$

where  $V_0$  depends on the nature of the substance, but  $k$  has the same value for all substances. Both  $k$  and  $V_0$  are independent of the intensity of the light. This result was first stated by Lenard. The first accurate proof of it was provided by Millikan.<sup>(2)</sup>

**67. Millikan's experiments on the photo-electric effect.** In order to be able to employ as wide a range of frequencies as possible Millikan worked with the alkali metals, for which the photo-electric effect begins in the visible spectrum, and his observations extended over a range of frequencies of nearly three octaves. The experiments were carried out in an exceedingly high vacuum, and in order to obtain really clean surfaces, the surfaces were actually turned up in the vacuum itself, by means of a tool which could be operated from outside by an electro-magnet.

The clean metal surface  $M$  (Fig 49 *a*) was illuminated in turn with various monochromatic beams of light, picked out spectroscopically from among the lines in the spectrum of a mercury arc. The escaping electrons were collected in a Faraday cylinder  $C$  of oxidized copper; this material being chosen because it does not itself emit photo-electrons for light of the frequency employed. The current collected by the

cylinder for different values of the applied potential difference  $V$  between the cylinder and the illuminated metal was measured by an electrometer, for a series of different wavelengths, and a series of curves plotted (Fig. 49 *b*) from which the value of  $V$  at which the emission ceased could be determined with accuracy, for each frequency  $\nu$  of the light employed. Plotting this critical value against the corresponding frequency  $\nu$ , Millikan obtained a series of points which lay accurately on a straight line. His results thus confirmed the relation (55) to an accuracy of at least 1 per cent. The value of  $k$  is

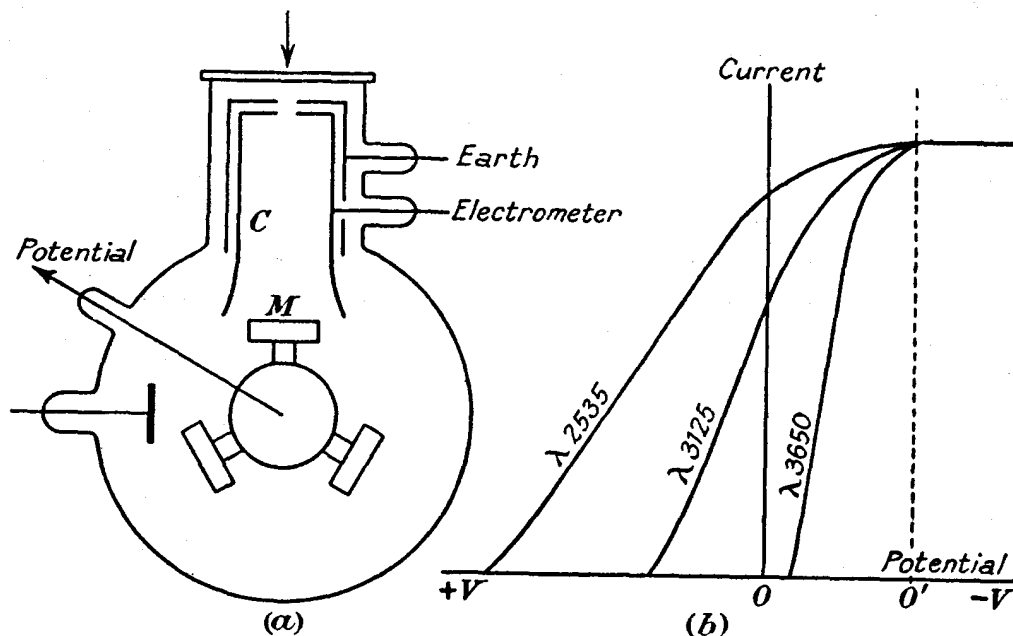


FIG. 49. Millikan's experiments on the photo-electric effect for the alkali metals

given by the slope of the line, and was found to be  $4.128 \times 10^{-15}$ , when  $V$  is measured in volts. If a second metal was substituted for the first (e.g. potassium for sodium) a second straight line was obtained parallel to the first. The value of  $k$  is thus identical for both metals, while that of  $V_0$  changes from metal to metal.

It will be seen from Fig. 49 *b* that the maximum photo-emission was only obtained when a negative, i.e. an accelerating potential was applied to the metal. The actual potential difference between the metal  $M$  and the collecting cylinder differs from the potential difference applied. If the plates of a condenser are of different metals and are joined by a wire of one of the metals there will actually be a field across the space between the plates owing to what is known as the contact potential difference. In Millikan's experiments it acts from

the alkali metal to the copper oxide, and thus tends to prevent the escape of electrons from the former, even when the two are connected together and the applied potential difference is zero.

The existence of this effect does not change the form of the relation (55), as the contact potential, unless corrected for, will be included in  $V_0$ . By subsidiary experiments Millikan was able to measure the contact potential difference, and so the actual potential difference through which the electron escaped. The effect of the correction is to move the zero from  $O$  to  $O'$  (Fig. 49 *b*) where  $OO'$  is the contact p.d. It was found that the photo-emission was complete when the actual p.d. between the plate and cylinder was zero. In the discussion which follows we shall suppose that this correction has been applied and that  $V$  measures the real potential difference between the illuminated metal and the collector.  $V_0e$  is thus the work which must be done by an electron in passing from the metal to the collector, and is thus the energy with which the fastest electrons leave the surface.

The significance of the experimental relation (55) can be made clear by multiplying the equation throughout by the electronic charge  $e$ , and transposing. Thus

$$Ve + V_0e = kev \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

Now  $Ve$  is the energy with which the fastest electron escapes from the illuminated surface.  $V_0e$  has also the dimensions of energy and can only be interpreted as measuring work done by the electron in escaping from the metal. This work may conceivably take two forms, work done in escaping from the parent atom, and work done in escaping through the surface. By ingenious experiments and arguments Millikan was able to show that, for a metal, no energy is used in extracting the photo-electron from the atom. The electrons ejected are those which are already in the free state in the metal.  $V_0e$  should thus be identical with the thermionic work function,  $w_0$ , and  $V_0$  should have the same value as  $w_0$  if the latter is stated in electron volts. Comparison of recent determinations of the two quantities are, in fact, in very good agreement. Thus for tungsten  $V_0 = 4.52$  volts, and  $w_0 = 4.52$  volts; for sodium we have 1.8 and 1.9; and for platinum 6.27 and 6.30 volts respectively.

$V_0e$  is thus the energy used up by the electron in escaping, and thus the left-hand of Equation 56 is the total energy with which the electron starts its career. Thus if monochromatic light of frequency  $\nu$  falls upon any substance the initial energy,  $W$ , imparted to the electrons is given by the relation

$$W = h\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$



where  $h$  is a constant which is independent of the nature of the substance, and is equal to  $ke$ . Taking Millikan's value for  $k$  and converting volts to electrostatic units, we have

$$h = 4.80 \times 10^{-10} \times \frac{4.128 \times 10^{-15}}{300} = 6.6 \times 10^{-27} \text{ ergs sec.}$$

It will be seen that equation (57) is merely an expression of well-ascertained experimental facts.

Further, if  $\nu_0$  is the minimum frequency which will stimulate photoelectric emission in the metal (the *threshold frequency* as it is called), it is found experimentally that  $V_0 e = h \nu_0$ , where  $h$  is the same constant as before. The relation (56) can thus be written

$$Ve=h(\nu-\nu_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

68. **The quantum theory.** In order to explain the laws governing the thermal radiation from a perfect radiator Planck had been led to formulate the hypothesis that an accelerated electron did not radiate energy continuously, as demanded by the classical electromagnetic theory of light (see § 76), but only intermittently, each emission consisting of a definite burst or *quantum* of energy.<sup>(4)</sup> The energy,  $w$ , in one of these quanta was supposed to be directly proportional to the frequency of the radiation emitted. Thus if  $\nu$  is the frequency of the radiation the energy associated with each quantum of it is given by

$$w = h\nu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where  $h$  is a universal constant, known as Planck's constant. The value deduced for this constant from radiation data was

$$h=6.55 \times 10^{-27} \text{ ergs sec.}$$

It is therefore numerically identical with the  $h$  of the previous section.

We can illustrate Planck's law quite readily from phenomena which we have already considered. We have seen (§ 35) that an electron may be transferred from its normal position in the atom to some state of greater energy by the impact upon it of another electron, provided that the latter has a certain minimum energy,  $Ve$ , where  $V$  is the radiation potential for the atom. This minimum energy clearly represents the difference in energy of the atomic electron in its two conditions. Thus the potential energy of the electron in its abnormal condition will be  $Ve$  units greater than in its normal state, and  $Ve$  units of energy will be set free when the electron falls back into its stable condition.<sup>(5)</sup> This energy, in the case of the first radiation potential, is emitted as light of definite frequency, only one line

appearing in the spectrum. The frequency of this radiation, by Planck's law, should be given by the relation

$$Ve = w = h\nu, \text{ or } \nu = Ve/h.$$

If we take the case of mercury vapour the first radiation potential is 4.9 volts or  $1.63 \times 10^{-2}$  e.s.u. and the radiation emitted should thus be given by

$$\nu = \frac{1.63 \times 10^{-2} \times 4.80 \times 10^{-10}}{6.55 \times 10^{-27}} = 1.175 \times 10^{15}.$$

The frequency of the light actually emitted is  $1.18 \times 10^{15}$ . The agreement is perfect, considering the possible errors of measurement of resonance potentials.

In Planck's original form of the theory only the emission occurred in quanta, the atomic system being supposed to absorb radiation continuously until its energy reached that of the quantum for the radiation concerned. In 1905 Einstein <sup>(6)</sup> made the further assumption that the radiation actually travelled in minute bundles each possessing the quantum energy corresponding to its frequency, and that an atom absorbed either the whole of this quantum or none at all. This assumption leads immediately to our experimental relation (57). From monochromatic light of frequency  $\nu$  the atom will absorb a whole quantum of energy  $h\nu$ , and this energy when transferred to an electron will give it kinetic energy  $\frac{1}{2}mv^2 = h\nu$ . At the time the data for testing this equation did not exist, and the suggestion was not well received, as the idea of a definite quantum of radiation travelling through space as an entity appeared to be completely irreconcilable with the wave theory of light.<sup>(4)</sup>

So drastic an assumption is not absolutely necessitated by the results of § 67. It is possible to conceive that the atom absorbs energy from the radiation continuously until the quantum energy is reached and then ejects this superfluous energy in the form of a photo-electron. This mode of escape is, however, made impossible by the following considerations. However weak the incident light may be the emission of photo-electricity commences as soon as the light falls on the surface. The lag, if any, is certainly less than  $3 \times 10^{-9}$  sec. Now Rayleigh has shown that an electron can absorb energy from an area of the order of  $\lambda^2$  of an extended wave front of radiation of wavelength  $\lambda$ ; that is, taking  $\lambda$  as  $4 \times 10^{-5}$  cm. for the near ultra-violet, from an area of about  $16 \times 10^{-10}$  sq. cm. The photo-electric current from sodium can easily be detected in the light of a single candle at a distance of 10 metres, corresponding to an energy of about  $\frac{1}{100}$  erg per sq. cm. per sec. on

the surface of the metal, of which not more than one-third is of sufficiently short wavelength to excite the photo-electric effect. Modern photo-electric cells are sensitive to radiation as weak as  $10^{-6}$  erg per sq. cm. per sec. But the energy of the electron emitted is of the order of  $5 \times 10^{-12}$  erg. To accumulate this energy from an extended wave surface the electron would have to go on absorbing for 1 sec. in the case of the candle-light or, in the extreme case, for 3000 seconds, or 50 minutes. The suggestion of a gradual absorption from a uniform beam is therefore inconsistent with the facts, and we are driven back on Einstein's assumption.

Again, we have seen that the photo-electric emission ceases for a threshold frequency,  $\nu_0$ , given by  $h\nu_0 = V_0 e$ . No matter how intense the radiation may be an electron is only ejected if the value of  $h\nu_0$  is greater than the work necessary to eject an electron. Absorption, therefore, can only take place in complete quanta. If the energy in a single quantum is insufficient to provide the work necessary for the escape of an electron, no electron will escape.

The only possible conclusion from the facts outlined in this section is that when energy is interchanged between matter and radiation, the interchange is effected in definite units, the value of which is expressed by  $h\nu$ . Planck's relation (59), originally derived in a highly speculative manner, is thus found to express a highly important and well authenticated law in atomic physics; the exchange of energy between matter and radiation takes place in quanta. This is called the quantum theory.

The quantum theory in reality is not a theory: no theoretical basis has yet been found for it. It is a fundamental law governing the exchange of energy between radiation and matter, and in all cases in which it has been tested it is found to express the phenomena with great exactitude. We shall meet with further examples in the following chapter. The question as to how the quantum relation is to be reconciled with the electromagnetic theory of light is an interesting problem. The formal reconciliation of the two ideas is however not beyond the reach of mathematics, and is dealt with in what is known as Wave Mechanics.

**69. Distribution of velocities among the photo-electrons.** So far we have considered only the maximum velocity with which the electrons escape from the metal. If we maintain a constant difference of potential  $V$  between the radiator and its surroundings less than the maximum positive potential which would be acquired by the plate, the rate of loss of electricity from the plate, that is, the negative current

from it, will be proportional to the number of electrons which leave the plate with an initial energy greater than  $V_e$ . In this way the proportion of the electrons which emerge with various velocities less than the maximum can be measured.

We have seen that the maximum current is reached only when the illuminated plate is at the potential of its surroundings, that is when there is no retarding field. Electrons emerge with all velocities from zero up to the maximum, but it is very improbable that they actually commence their course with different velocities. Electrons will be projected at different depths in the metal (since light can penetrate small thicknesses of metal) and hence will have different thicknesses of metal to traverse before escaping from the surface. Now in passing through matter electrons lose velocity owing to collisions with the atoms. Thus those electrons which start at the surface, having no matter to traverse, will escape with the actual velocity given them by the radiation. Those starting deeper in the metal will lose some of their velocity before reaching the surface, until at last a layer is reached the electrons from which will be unable to penetrate to the surface at all.

**70. Magnitude of the photo-electric effect.** The magnitude of the photo-electric effect may be defined as the total current from unit area of the illuminated surface when its potential is favourable to the escape of negative electricity. Its value is only definite for surfaces not only kept but also prepared in the highest vacua obtainable and never exposed at any period to the action of a gas at a perceptible pressure.

An apparatus <sup>(7)</sup> suitable for the preparation and investigation of a metallic surface in vacuo is shown in Fig. 50. The thick lines represent the metallic part of the apparatus which extends far enough on all sides to screen the plate  $P$  from external electrostatic effects. When in the position shown the plate makes metallic contact with a Wilson electroscope  $E$ , while the two potentiometers  $V$ ,  $V$  can be adjusted so as to maintain a constant measured difference of potential between the radiator and its surroundings. To obtain a film on the radiator  $P$  it is lowered to the position  $P'$  opposite the mouth of a small quartz crucible  $F$ , which contains the metal under investigation and can be electrically heated by the wires  $\omega$ ,  $\omega$  passing through the walls of the tube. The metallic vapours condense on the cold plate forming a film which, as the whole apparatus is maintained at a high vacuum, has never been in contact with gas and so is free from the disturbing surface layer. If the outer case is earthed the maximum potential acquired by the plate  $P$  as measured by the electroscope

gives the maximum velocity of emission of the photo-electrons, as already explained. By applying a small accelerating potential sufficient to ensure the maximum emission, the total photo-electric effect can be measured.

With films prepared and kept in vacuo, the total photo-electric effect is found to be directly proportional to the intensity of the light for intensities varying in a ratio of as much as 1 to  $10^6$ . On account

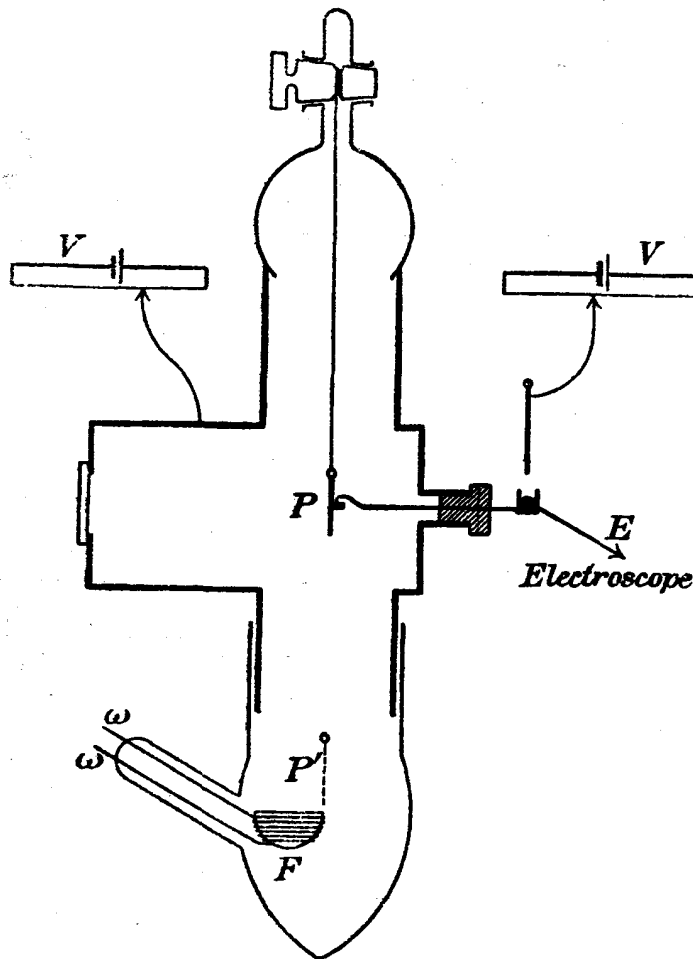


FIG. 50. Illustrating Hughes' apparatus for measuring the photo-electric effect

of its importance with respect to the quantum theory of radiation experiments with exceedingly small intensities of illumination have been made, but even when the energy was as small as  $10^{-7}$  ergs per sq. cm. per sec. no departure could be observed from the proportionality between the intensity and the current. The current is also independent of whether the light is continuous or is given out in a series of brief flashes. As long as the average energy remains constant the emission remains the same.

**71. Variation of the photo-electric effect with wavelength and state of polarization.** For the majority of metals the photo-electric effect increases as the wavelength of the exciting radiation diminishes. The alkali metals for a certain part of the spectrum and with oblique incidence of the light form an exception; the current rising to a fairly sharp maximum at a definite wavelength. In these metals the effect also depends on the plane of polarization of the incident light.

Let light be incident obliquely on a surface, say, of a liquid alloy of sodium and potassium. These liquid alloys are convenient since they give perfectly plane surfaces. The plane containing the ray and the normal to the surface is known as the plane of incidence. According to the electromagnetic theory of light, if the light is plane polarized in the

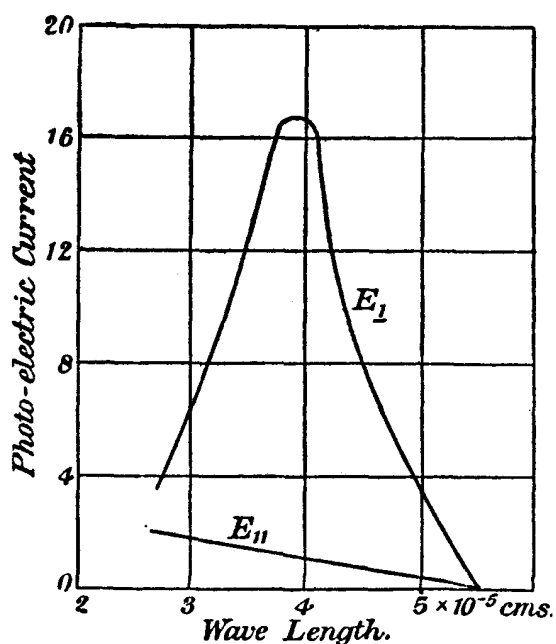


FIG. 51. Selective photo-electric emission from a sodium-potassium alloy

plane of incidence the electric field in the pulse will be perpendicular to this plane, that is, parallel to the surface of the radiator. We will denote this state as the  $E_{\perp}$  plane to indicate that the electric force is parallel to the radiating surface. On the other hand, if the light is plane polarized at right angles to the plane of incidence the electric field will be parallel to the plane of incidence and thus will have a component, if the ray is oblique, perpendicular to the surface. The more oblique the incidence the greater this component. We will denote this state of polarization by  $E_{\parallel}$ .

If now experimental curves are drawn between the photo-electric intensity and the wavelength of the incident light for the two states of plane polarization the results shown in Fig. 51 are obtained. It will be seen that when the electric field in the light waves is parallel to the surface of the radiator the emission decreases uniformly as the wavelength is increased. When, however, the light is polarized so that the electric field in the radiation has a component perpendicular to the radiator, the emission rises rapidly as the wavelength decreases, reaches a maximum for a certain definite wavelength and then decreases until it joins the other curve. The maximum intensity increases with the

obliquity of the rays, but the corresponding wavelength is constant. The ratio of the maximum to the normal effect differs widely for different surfaces even of the same alloy. It is normally as much as 15 to 1, but has been observed as high as 300 to 1, with an angle of incidence of  $60^\circ$ . The velocity of the electrons emitted is the same in the two cases.

Similar effects have been observed for other metal surfaces, when contaminated by absorbed gases. In this case there is usually a shift of the threshold wavelength towards the red end of the spectrum. In the diagram (Fig. 51) which represents the case of a sodium-potassium alloy the maximum effect is reached at a wavelength of  $3900 \times 10^{-8}$  cm.; for potassium the corresponding wavelength is 4490 and for rubidium about 5000, that is, in the yellow green of the visible spectrum. If the light is incident normally on the surface the effect, of course, disappears, since the light has then no component normal to the surface.

**72. Photo-electric cells.** The photo-electric effect is largely employed both for the accurate measurement of light intensities, and for the conversion of light energy into electrical energy. One form of photo-electric cell is shown in Fig. 52. The light enters through a quartz window *W* and passing through the ring-shaped anode *A* falls on the cathode *C*. The sensitive surface is deposited by evaporation in vacuo on the surface of the cathode. If the cell is required to respond to the visible part of the spectrum the metal employed is usually potassium or rubidium, and in this case the quartz window can be replaced by one of glass. If the ultra-violet only is to be measured, cadmium is often employed as the sensitive surface. The photo-electric emission can be largely increased by treating the surface in various ways, and notably by admitting hydrogen at low pressure to the cell and passing an electric discharge for a short time. This probably transforms the alkali metal into a hydride.

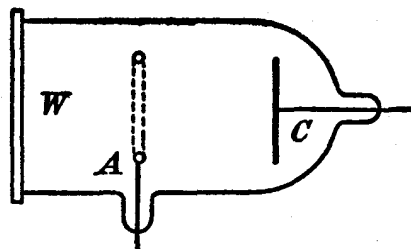


FIG. 52. Simple photo-cell.

If the cell is to be used for accurate photometry it is evacuated as completely as possible. The saturation current through the cell is then, for light of given wavelength, accurately proportional to the intensity of the light. Unless the light is intense, however, the current is usually too small to be measured by a galvanometer, and an electrometer method must be employed.

Much larger currents can be obtained by including an inert gas at low pressure in the cell, and applying potentials not much less than that necessary to produce an ordinary gas discharge through the cell. The photo-electrons are thus accelerated so that ionization by collision occurs, and the current can thus be multiplied many fold. Cells of this type are not suitable for accurate measurements as their sensitivity is very variable, and they also suffer from fatigue, owing to the condensation of the gas on the sensitive surface. They are used largely in sound film projection. Light is passed through the sound film on to the photo-electric cell. Variations in the density of the film produce corresponding variations in the intensity of the transmitted light, which in turn produces variations in the current through the cell. This current, which may be of the order of several microamperes can be amplified by a thermionic valve amplifier, and so transmitted to the loud speaker where the electrical fluctuations are converted into sound.

For fuller details the report on *Photo-electric cells and their Applications* <sup>(8)</sup> published by the Physical Society may be consulted.

**73. Photo-electric effect in non-metals and gases.** The photo-electric effect can also be detected in non-metals, but the threshold frequency is much greater than for metals. As there are normally no free electrons in a non-conductor, the energy of the absorbed quantum must be sufficient, not merely to overcome surface forces but also to extract the electron from its parent atom.

The photo-electric effect in the case of a gas would take the form of a simple ionization of the gas, the ejection of the photo-electron leaving the gaseous molecule with a positive charge. The minimum energy required is therefore  $Ve$ , where  $V$  is the ionization potential; thus  $\nu$ , the minimum frequency to produce ionization, should be given by

$$Ve = h\nu_0 = \frac{hc}{\lambda_0}$$

where  $c$  is the velocity of light. Taking 15 volts as the ionization potential of air, the threshold wavelength,  $\lambda_0$ , for ionization should thus be  $817 \times 10^{-8}$  cm.

Experiments in this part of the spectrum are difficult; the limit of transparency for quartz is  $1450 \times 10^{-8}$ , though some specimens of fluorspar transmit down to  $1000 \times 10^{-8}$  cm. Using one of these as a window, Hughes obtained ionization in air with light of estimated wavelength  $1250 \times 10^{-8}$  cm. It is difficult in such experiments to avoid spurious effects due to scattered light falling on the walls of the ionization chamber or on particles of dust in the gas. If the ionization



observed is genuine, it must be associated with the radiation potentials of the gas.

It is possible that if two molecules, each of which has passed into the excited state by the absorption of radiation, come into collision, their joint energies may be given to a single electron, which would then have sufficient energy to escape from the atom. Effects of this kind are known to occur among molecules stimulated by electron bombardment.

**74. Phosphorescence and the photo-electric effect.** No connection can be traced between photo-electricity and fluorescence, many fluorescent compounds being quite inactive. On the other hand, phosphorescent substances always show a well-marked photo-electric effect. Thus calcium sulphide with a trace of powdered bismuth and a flux of sodium borate yields a very actively phosphorescent substance. On testing this it was found to be exceedingly photo-electric also, giving a photo-electric effect nearly as large as brightly polished magnesium. The three substances taken separately and also in pairs were found to be neither phosphorescent nor markedly photo-electric. By using absorbing screens it could also be shown that light which did not produce phosphorescence did not produce a photo-electric effect. The two effects, therefore, seemed to run parallel to each other.

It has been shown that phosphorescence is due to certain active centres in the "phosphor." In all probability these centres under the action of light emit an electron which if the centre is sufficiently near the surface escapes as a photo-electron. If the centre is too deep for this the electron remains embedded in the surrounding substance, which is always a non-conductor. Eventually, however, under the action of the attraction of the now positively charged centre the electron finds its way back into the "centre," producing disturbances there which cause the emission of the phosphorescent light. The photo-electric effect is thus connected with the excitation of phosphorescence, and not with the phosphorescence itself. The photo-electric effect itself ceases as soon as the exciting radiation is cut off.

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- (5) For resonance phenomena see SOMMERFELD, *Atomic Structure*, p. 337.
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## CHAPTER X

### X-RAYS

**75. Production of X-rays.** Röntgen, in 1895, discovered that if a discharge tube at low pressure was worked in the neighbourhood of a cardboard screen covered with crystals of barium platinocyanide the crystals glowed with their characteristic yellowish green fluorescence. The effect was traced to radiation of some kind proceeding from those portions of the walls of the discharge tube on which the cathode stream impinged. The luminosity was not appreciably diminished if the tube was completely enclosed in black paper, and it was therefore not due to ultra-violet light. This new type of radiation was labelled X-radiation, pending further enquiry as to its nature. It is sometimes, but more rarely, called Röntgen radiation.

X-radiation can be detected by the luminosity it excites in certain fluorescent materials, by its action on a photographic plate or by the ionization it produces in any gas through which it passes. The latter effect is commonly employed to measure the intensity of the radiation.

X-rays have the property of penetrating many substances which are opaque to ordinary light. The absorbing power of a substance for X-rays depends partly on the density, and partly on the atomic weight of the elements contained in it. Light substances, and those containing only elements of low atomic weight are comparatively transparent, while substances of high atomic weight and great density are comparatively opaque. Thus if an experimenter places his hand between the source of X-rays and a fluorescent screen the shadows cast by the relatively opaque bones are clearly visible, surrounded by the much fainter shadow cast by the relatively transparent flesh. This property of the rays renders them an extremely valuable asset in medical diagnosis, and is also being widely applied in metallurgical and other technical investigations.

For medical and industrial purposes the original type of X-ray tube, in which the cathode rays were produced by means of a low-pressure gas discharge, has been replaced by a design originally due to Coolidge, one form of which is shown in Fig. 53. The cathode *C* consists of a spiral of tungsten wire which can be heated by passing a current through it either from an insulated battery, as shown in the diagram,

or, more conveniently, from a specially designed transformer. The tungsten wire, when raised to incandescence emits large numbers of thermions, which can be projected towards the anode *A* with any desired velocity by applying a suitable potential difference between the anode and the cathode. The cathode stream thus produced can be focused to a fine spot by surrounding the cathode with a molybdenum shield *M*.

The tube is exhausted as completely as possible, so that no discharge passes when the wire is cold. As the P.D. across the tube is always greatly in excess of the saturation potential, the current through the

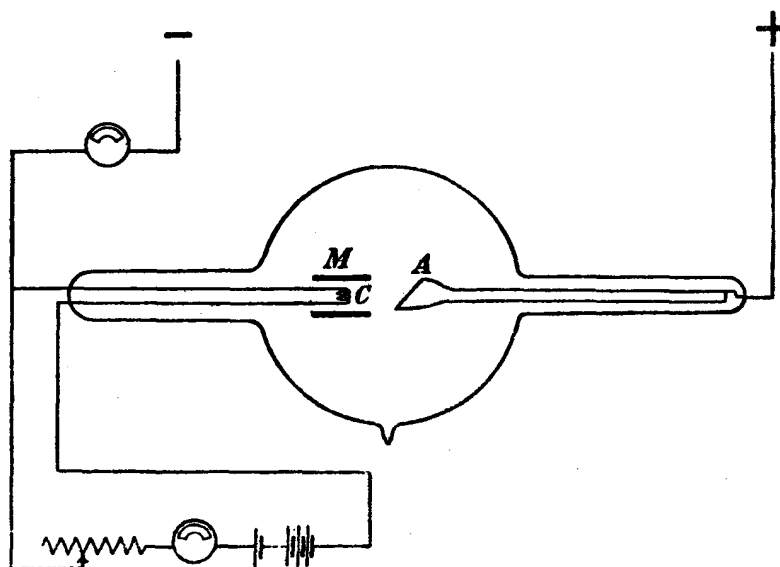


FIG. 53. Illustrating the principle of the Coolidge tube

tube depends only on the thermionic emission of the filament, which in turn is governed by the temperature of the filament. This can be controlled by a rheostat in the heating circuit. The current through the tube can thus be regulated independently of the P.D. across it.

The anode *A* is a massive block of copper, carrying a small piece of tungsten which serves as the actual target for the cathode rays. The efficiency of a target in converting the energy of the cathode rays into X-ray energy increases with the atomic number of the target, while the hardness and high melting-point of tungsten are also valuable qualities. In modern practice currents of 30 ma. at a P.D. of 250,000 volts are quite commonly employed in X-ray production. Only a minute fraction of this energy, usually about 0.2 per cent., is converted into X-radiation. The remainder is transformed into heat in the anode. As will be seen from the figures quoted above, this production of heat

may amount to 1800 calories per second, and in tubes designed for prolonged running special cooling devices have to be employed to remove it from the tube.

The high-potential supply for the tube is almost invariably obtained from a high-tension transformer. For small currents the tube itself acts as a rectifier, since current can only pass when  $C$  is negatively charged. When the current employed is large enough to raise the focal spot to incandescence, it must be rectified, by thermionic rectifiers, and is often smoothed by the use of condensers.

#### 76. Radiation emitted by the stoppage of a moving electrified particle.

**Classical theory.** The fact that the X-rays have their origin at the point struck by the cathode rays led Sir G. Stokes to suggest that they are electromagnetic waves set up by the sudden stoppage of the rapidly moving electrons which make up the cathode stream.

The effect is most readily understood by a consideration of the Faraday tubes of force. If a charged cathode particle is moving with a speed which is small compared with that of light, its electric field will be the same as that for a particle at rest, and the tubes will be therefore distributed uniformly round the particle. Assigning all the effects in the field round the charge to the action of the Faraday tubes it can be shown that the motion of a single Faraday tube of force produces a magnetic field in the medium which is perpendicular to the length of the tube and to its direction of motion, and equal to  $4\pi v$ , where  $v$  is the velocity of motion of the tube in a direction perpendicular to its length. (See J. J. Thomson, *Elements of Electricity and Magnetism*, Chap. XIII.)

Consider a single charged particle moving along a direction  $OX$  with a velocity  $v$ . This particle will carry with it a number of Faraday tubes of electric force, which when the motion has become steady will travel along as if rigidly attached to the particle.

Suppose the particle is suddenly stopped by striking a solid obstacle at the point  $A$  (Fig. 54 *a*), and let us suppose that disturbances are propagated along the Faraday tubes with a finite velocity  $c$ . It can be shown that this velocity is the velocity of light.

Let  $t$  be the time that has elapsed since the particle was stopped. Describe round  $A$  a sphere of radius  $ct$ . Then all the portions of the tubes inside this sphere will be radiating from the stationary position of the particle at  $A$ . Outside this sphere however the disturbance produced in the Faraday tubes by the stoppage of the particle will not have arrived. They will thus be still moving in the direction  $OX$  with their original velocity  $v$ , and will be radiating from the position which

the particle would have reached if it had not met the obstacle, that is, from a position  $B$  along  $OA$  produced where  $AB$  is equal to  $vt$ .

There will thus be a relative displacement between the two portions of the tube of force, and as we must regard the tube as continuous it is evident they must wrap themselves round the surface of the sphere.

This sphere is expanding with a velocity  $c$ , and we thus have a sheet of electric force travelling out with a velocity  $c$  from the position of the particle. But the motion of the Faraday tubes at right angles to their length produces a magnetic field perpendicular to the Faraday tubes and to the direction in which they are moving, that is to say, also in the surface of the sphere. Thus a sheet of electromagnetic disturbance

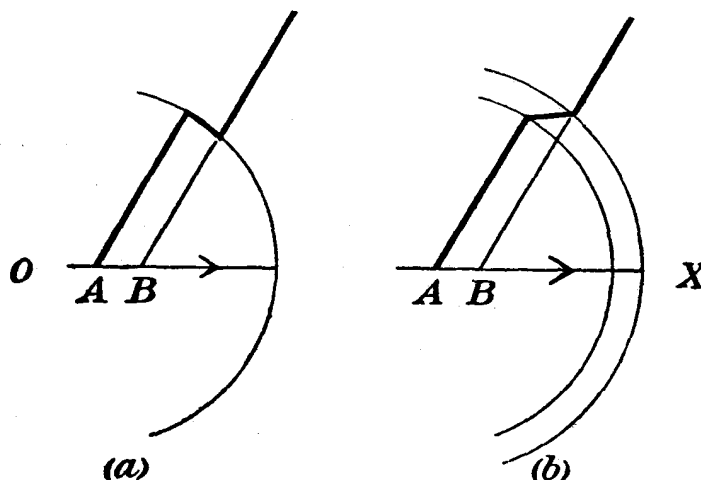


FIG. 54. Illustrating Stokes' theory of X-ray production

spreads out from the particle. This, on Stokes' theory, constitutes an X-ray pulse.

It will not in general be possible to stop the particle abruptly. If  $\tau$  is the time taken to reduce the particle to rest, the disturbance will be enclosed by two spheres of radii  $ct$  and  $c(t+\tau)$  (Fig. 54 *b*). The thickness of the disturbance or pulse will thus be  $c\tau$  and will be smaller the more quickly the particle is stopped.

Let Fig. 55 *b* represent this state of affairs. Then, since  $AB=vt$ ,

$$\frac{\text{tangential electric polarization}}{\text{normal electric polarization}} = \frac{vt \sin \theta}{\delta},$$

where  $\delta$  is the thickness of the pulse and  $\theta$  is the angle between the direction of the tube and  $AX$ . But the normal polarization is  $e/4\pi r^2$  and thus since  $r=ct$  the tangential electric polarization

$$= \frac{e}{4\pi r \delta} \frac{v \sin \theta}{c} \dots \dots \dots (60)$$

Now since this is moving at right angles to its direction with a velocity  $c$  it will produce a magnetic force  $H$ , equal to  $4\pi c$  times the polarization, that is

$$\frac{ev \sin \theta}{r\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Since these fall off inversely as the distance from the particle while the normal polarization falls off as the inverse square of the distance, the intensity in the pulse except for points near the particle will be great, compared with the intensity outside it. We shall thus get a pulse of electromagnetic disturbance travelling out from the particle, and behaving in many respects like ordinary light. The chief differences are that in the first place the thickness of the pulse is found to be small compared with the wavelength of visible light, while in the second place the X-rays lack that regular periodic character which occurs in a train of waves of constant wave length.

The energy in a pulse of this kind being equally divided between the electric and magnetic fields is equal to  $2 \times \mu H^2 / 8\pi$  per unit volume, i.e.

$$\frac{\mu e^2 v^2 \sin^2 \theta}{4\pi r^2 \delta^2}.$$

The energy radiated is thus zero along the direction of motion of the particle, and rises to a maximum at right angles to this direction. The volume of shell included between radii, making angles  $\theta$  and  $\theta + d\theta$  with the axis  $AB$ , is  $\delta \cdot 2\pi r \sin \theta \cdot r d\theta$ , and the total energy in the shell is therefore

$$\begin{aligned} 2 \int_0^{\pi/2} 2\pi r^2 \sin \theta \delta \frac{\mu e^2 v^2 \sin^2 \theta}{4\pi r^2 \delta^2} d\theta \\ = \frac{2}{3} \frac{\mu e^2 v^2}{\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (62) \end{aligned}$$

on substituting for  $H$  from (61) and evaluating the integral. The energy is thus inversely proportional to the thickness of the pulse. Thus the more suddenly the particle is stopped the greater the energy radiated.

The method can easily be extended to the case of the acceleration of a charged particle which is not actually brought to rest. If  $f$  is the acceleration then the time  $\tau$  which would be taken for the particle to be brought to rest is given by

$$v = f\tau.$$

Substituting this value for  $\tau$ , and the value  $c\tau$  for  $\delta$  in Equations 60 and 61 we have

$$\text{tangential polarization } D = \frac{ef \sin \theta}{4\pi rc^2} \quad . \quad . \quad . \quad (63)$$

$$\therefore \text{ Magnetic field } H = \frac{ef \sin \theta}{rc} \quad . \quad . \quad . \quad (64)$$

The rate of flow of energy across unit area (by Poynting's Theorem) is  $c^2 DH$ . Hence

$$\text{intensity of the radiation} = \frac{e^2 f^2 \sin^2 \theta}{4\pi r^2 c} \quad . \quad . \quad . \quad (65)$$

Integrating over the area of the sphere of radius  $r$  we have

$$\text{total energy of the radiation} = \frac{2}{3} \frac{e^2 f^2}{c} \quad . \quad . \quad . \quad (66)$$

**77. The scattering of X-rays by matter.** Suppose the radiation from one electron falls upon another. The second electron while the pulse is passing over it will be subject to an electric field  $X$  and will thus have an acceleration

$$f_1 = X \frac{e}{m} = \frac{4\pi e D}{m} = \frac{e^2 f \sin \theta}{mrc^2} \quad . \quad . \quad . \quad . \quad (67)$$

substituting for  $D$  from (63). This electron will, therefore, give out radiation which, since  $f_1$  is proportional to  $f$ , will have the same wave form and thus the same quality as the radiation exciting it.

Thus if a beam of primary X-rays falls on a material substance, since all matter contains electrons, secondary X-rays will be given out in all directions which, since they are found to be of the same quality as the rays exciting them, have been called "*scattered radiation*," in order to distinguish them from another type of secondary radiation which may also be emitted, the quality of which depends not on the primary radiation but on the chemical nature of the radiator. This latter type is often called "*characteristic*" radiation, since it is characteristic of the radiator used for the experiment.

These effects can easily be observed by allowing a narrow pencil of X-rays to fall on a thin sheet of aluminium, or paper. An electroscope placed near the radiator, but out of the direct line of the primary pencil, will lose its charge, showing that ionizing radiations are being given out by the radiator. If the radiator is removed the rate of loss of charge becomes very small. If, however, a sensitive electroscope is employed it is possible to observe a residual effect due to the action of the primary pencil on the molecules of the air.



The scattered radiation is not uniformly distributed about the radiator. The simple theory already outlined leads to the conclusion that  $I_\theta$ , the intensity of the radiation emerging from the radiator at an angle  $\theta$  with the primary rays should be expressed in the form

$$I_\theta = I_{\pi/2}(1 + \cos^2 \theta) \quad . . . . . (68)$$

where  $I_{\pi/2}$  is the intensity of the scattered radiation at right angles to the primary beam. The scattered radiation is thus a minimum in this direction, and approaches twice this value as the direction of the primary beam is approached.

Careful experiments have shown that this relation is not strictly true. In every case the intensity of the radiation on the side of the radiator from which the primary rays emerge is greater than that given out at a similar angle from the face by which the rays enter.<sup>(1)</sup> The distribution is thus unsymmetrical about a plane through the radiator at right angles to the primary beam. This is shown by the thick line curve in Fig. 55, which represents the author's results for the actual distribution of the scattered radiation around a thin aluminium radiator placed at  $R$ . The dotted line gives the theoretical distribution as determined by (68).

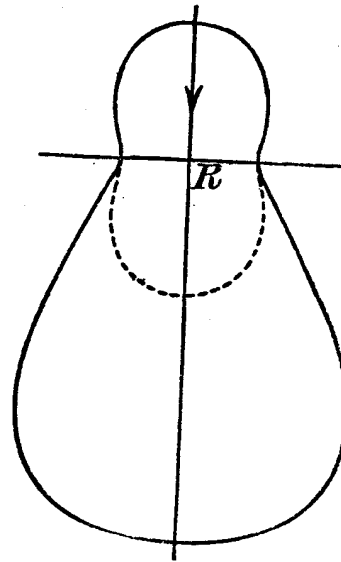


FIG. 55. The distribution of scattered X-radiation

**78. Polarization of the scattered radiation.** Consider a beam of cathode rays falling on an anti-cathode at  $O$  in the direction  $DO$  (Fig. 56). If the first impact of the electrons produced an X-ray the acceleration of the electrons would be in the direction  $OD$ , and the electric force in the resultant pulse at any point along the ray  $OP$  would be parallel to the plane of the paper. Actually the fact that the distribution of the primary rays round the anti-cathode is almost uniform shows that many of the cathode particles must experience considerable deviations owing to collisions with the electrons in the anti-cathode before producing an X-ray pulse, but at any rate we may expect a preponderance of direct collisions and therefore a maximum electric force parallel to the plane of the paper. Since there is an excess of electric field in the pulses in a definite direction the emerging beam may be said to be partly polarized.

Suppose now that this partially polarized radiation falls on a radiator

at  $P$ . The electric force will cause the electrons in the molecules to have an acceleration (§ 77), which if the beam is completely polarized will be in the direction  $PQ$  since the field must be in the plane of the paper and also perpendicular to the direction of the ray. Thus the electrons will be set vibrating in the direction  $PQ$ , and will emit radiation which, as we have seen, will be zero in the direction of motion of the electron, and a maximum at right angles to it. Thus if the primary rays were completely polarized there would be no scattered radiation in the direction  $PQ$ , while it would reach a maximum in a direction  $PR$  through  $P$  perpendicular to the plane containing  $OD$  and  $OP$ . In other words if we allow the rays from an X-ray tube to fall on a radiator we should expect that the scattered radiation would be a minimum in a line through the radiator parallel to the incident cathode rays, and a

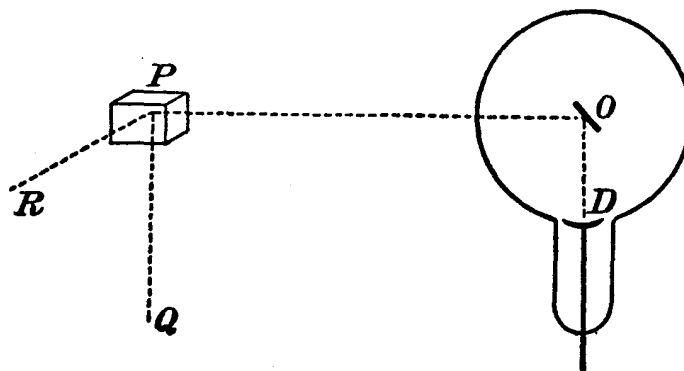


FIG. 56. Illustrating Barkla's method of demonstrating the polarization of X-rays

maximum in a direction at right angles to this. The experiments were carried out by Barkla <sup>(2)</sup> in 1904 and showed a very marked effect, the polarization often reaching as much as 20 per cent. The actual amount depends on the hardness of the rays and the nature of the anti-cathode.

The matter can, however, be pursued further. The electrons in the radiator are all moving in the plane  $RPQ$ . Since the component of motion parallel to  $PR$  produces no radiation in the direction  $PR$  the scattered radiation proceeding along  $PR$  will clearly be completely polarized, the electric displacement being parallel to the direction  $PQ$ . Thus if a second radiator of low atomic weight is placed at  $R$ , no radiation should be scattered from it in a direction parallel to  $PQ$ , the maximum scattering being in a direction at right angles to this. The relation will only hold exactly if the different beams are infinitely narrow, so that they can be regarded as lines. Compton, using very narrow slits to limit the beams, was, however, able to show that the

radiation scattered from  $R$  in a direction parallel to  $PQ$  was less than 1 per cent. of that scattered at right angles to this direction. The complete polarization of the beam scattered along  $PR$  has thus been demonstrated.

**79. Energy of the scattered radiation.** The total energy scattered by a single electron when a primary X-ray pulse passes over it can easily be calculated. Let the electron be situated at a distance  $r$  from the point of origin of the X-ray pulse and in a direction making an angle  $\theta$  with the direction of the acceleration of the primary electron producing the pulse. The acceleration of the scattering electron is then given (67) by

$$f_1 = \frac{e^2 f \sin \theta}{mrc^2},$$

and the total energy  $S$  radiated by it is, by (66), given by

$$S = \frac{2}{3} \frac{e^2}{c} \int f_1^2 dt = \frac{2}{3} \frac{e^6 \sin^2 \theta}{m^2 r^2 c^5} \int f^2 \cdot dt,$$

where the integration extends over the whole time that the primary pulse is passing over the electron. But, by (65),  $I$ , the energy per unit area in the primary pulse at the scattering electron, is given by

$$I = \frac{e^2 \sin^2 \theta}{4\pi r^2 c} \int f^2 \cdot dt,$$

from which we have immediately

$$\frac{S}{I} = \frac{8\pi}{3} \frac{e^4}{m^2 c^4}, \quad \dots \dots \dots (69)$$

where  $e$  is measured in electrostatic units.

If there are  $N$  electrons in unit mass of the radiator, and if we can assume that they scatter independently, so that there is no constant phase relation between the secondary wavelets from the different electrons, the total energy of the scattered radiation will be the sum of the intensities from the individual electrons. Thus if  $S$  is the total intensity of the radiation scattered from unit mass of a radiator when in a primary beam of intensity  $I$

$$\frac{S}{I} = \frac{8\pi}{3} \frac{Ne^4}{m^2 c^4} \quad \dots \dots \dots (70)$$

This result is historically important as it forms the basis of by far the earliest method of estimating the number of electrons in the atom. Since the primary and scattered radiations have the same pulse form

it is legitimate to assume that their energies are proportional to the ionization produced by them in unit volume of air. The ratio  $S/I$  can thus be determined by making suitable observations with an ionization chamber. Experiments showed that, for elements of low atomic weight and for radiation of medium wavelengths, the mass coefficient of scattering was approximately constant at a value of about 0.2.

If  $Z$  is the number of electrons per atom and  $W$  the atomic weight of the radiator, the number of atoms per gm. of the radiator is  $1/Wm_0$  and the number of electrons is  $Z/Wm_0$ , where  $m_0$  is the mass of an atom of unit atomic weight. Hence substituting in (70) we have

$$0.20 = \frac{8\pi}{3} \frac{Z}{Wm_0} \frac{e^4}{m^2c^4} = 0.40 \frac{Z}{W}$$

on evaluating. The number of electrons per atom is thus approximately equal to half the atomic weight.

We can express this result in a more fundamental manner. The existence of isotopes shows that the chemical atomic weight of an element is not a fundamental property as different atoms of the same element differ in mass. If we arrange the elements in the order of ascending atomic weights, and number them consecutively, beginning with hydrogen=1, the number assigned to any element is called its *atomic number*. All isotopes of the same element have the same atomic number, though they differ in mass. Except in the case of hydrogen the atomic number is not far removed from half its atomic weight. Hence the result we have just arrived at may be stated in the form that the number of electrons in an atom is equal to its atomic number. This suggestion is amply confirmed by phenomena with which we shall deal later.

For primary radiation of long wavelength, and for elements of high atomic number the mass coefficient of scattering increases above its classical value of 0.2. The effect may be ascribed to the interaction of the scattered waves from the different electrons. Suppose that the pulse thickness, or wavelength, of the primary pulse is large compared with the diameter of an atom. The different electrons in the atom will as the pulse passes over them be all moving simultaneously in practically the same phase, and the radiations they give out will also be in phase. In this case ordinary optical theory shows that the amplitude of the resulting wave is the sum of the separate amplitudes, the intensity being the square of the resultant amplitude. The intensity of the radiation scattered by the atom will thus vary as  $Z^2$ , not as  $Z$ . We may call this "coherent" scattering, and we should expect it to set in

when the wavelength of the radiation became comparable with the average distance between the electrons in the atom. Thus for long wavelength radiation and for elements of high atomic number, where the number of electrons per atom is large, the scattering is greater than that given by the simple theory.

It is also found that for very short wavelength radiation the scattering per electron falls gradually far below the classical value. This phenomenon is not explicable on the classical theory.

**80. The Compton effect.** The scattered radiation is as a whole found to be somewhat less penetrating than the primary radiation which produces it. In a brilliant series of experimental researches Compton succeeded in analysing spectroscopically the scattered radiation from a radiator illuminated by a beam of monochromatic X-rays. The experiments were difficult, partly owing to the very small intensity of the scattered radiation and partly owing to the smallness of the difference in wavelength to be detected. He found that the scattered radiation consisted of two components, the one of precisely the same wavelength as that of the primary beam; the second, known as the modified radiation, of somewhat greater wavelength. The difference  $\Delta\lambda$  in wavelength between the two radiations was found to vary with the direction in which the scattered radiation was viewed; in fact  $\Delta\lambda$  was proportional to  $\sin^2 \frac{1}{2} \theta$ , where  $\theta$  is the angle between the directions of the primary and secondary beams. The actual value of  $\Delta\lambda$  when measured in a direction at right angles to the primary beam was  $0.024 \times 10^{-8}$  cm. (0.024 Å.). The value was independent of the nature of the radiator and of the wavelength of the primary beam.

The distribution of energy between the two wavelengths, however, varies with the nature of the radiator. For elements of low atomic number the scattered radiation is mainly of the modified type. For elements of high atomic number it is nearly all unmodified radiation.

To explain these effects, Compton suggested that the unmodified line represented the scattering by electrons which were firmly bound in the atom; the modified line being due to the scattering by "free" electrons. An electron may be regarded as free if the energy binding it in the atom is much smaller than that in the primary X-ray pulse. On the average, electrons are much more firmly bound in elements of high atomic number, and it is thus the elements of low atomic number which would be expected to give the maximum proportion of the modified radiation.

**81. Compton's theory of scattering.** To account for the observed change in wavelength Compton made a somewhat daring, but very

successful application of the quantum hypothesis. He assumed that the quanta of energy in the primary X-ray beam were so highly localized that they might be regarded as particles of negligible size, and further, that the collision between one of these quanta and an electron might be treated by the ordinary laws of dynamics, in exactly the same way as the collision between two perfectly elastic particles.

From the fact that radiation exerts a pressure it can easily be shown that it possesses momentum equal to its energy divided by the velocity of light. The momentum of the *photon*, as we may call the localized

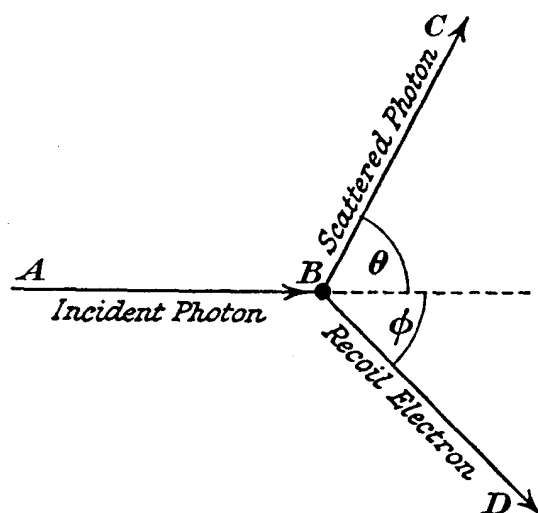


FIG. 57. Illustrating Compton's theory of X-ray scattering

quantum, is therefore  $h\nu/c$ . Consider a photon travelling in the direction  $AB$  (Fig. 57) and meeting an electron at  $B$ . After the collision the photon is scattered in some direction  $BC$ , the electron itself moving in a direction  $BD$ . It is assumed that the electron is free to move, and is not rigidly attached to an atomic structure.

Since the electron has received energy the photon must have lost an equal amount. Its energy is no longer  $h\nu$ , but

$h\nu_1$ , where  $\nu_1$  is less than  $\nu$ , since  $h$  is a universal constant. In other words the frequency of the quantum is diminished by collision with the electron, that is its wavelength is increased. This is, in essence, Compton's theory of the Compton effect.

We can easily put this in mathematical form. Let  $\theta$  (Fig. 57) be the angle between the original and deflected directions of the photon, and  $\phi$  the angle between the original direction of the photon and that of the electron;  $v$  the velocity of the electron and  $c$  the velocity of light. We shall see later (§ 115) that the mass of an electron varies with its velocity; and we must modify the usual expressions for kinetic energy and momentum to take this variation into account. The kinetic energy of the electron thus becomes  $m_0 c^2 \left( \frac{1}{\sqrt{1-\beta^2}} - 1 \right)$ , and its momentum  $\frac{m_0 \beta c}{\sqrt{1-\beta^2}}$ , where  $\beta = v/c$  and  $m_0$  is the mass of an electron

at rest. By the principle of the conservation of energy we have

$$h\nu = h\nu_1 + m_0c^2 \left( \frac{1}{\sqrt{1-\beta^2}} - 1 \right) \quad . \quad . \quad . \quad (71)$$

From the conservation of momentum we have further

$$\begin{aligned} \frac{h\nu}{c} &= \frac{h\nu_1}{c} \cos \theta + \frac{m_0\beta c}{\sqrt{1-\beta^2}} \cos \phi, \\ 0 &= \frac{h\nu_1}{c} \sin \theta - \frac{m_0\beta c}{\sqrt{1-\beta^2}} \sin \phi, \end{aligned}$$

from which we can readily deduce

$$\left( \frac{m_0\beta c}{\sqrt{1-\beta^2}} \right)^2 = \left( \frac{h\nu}{c} \right)^2 + \left( \frac{h\nu_1}{c} \right)^2 - 2 \frac{h\nu}{c} \cdot \frac{h\nu_1}{c} \cos \theta \quad . \quad (72)$$

Combining (71) with (72) and eliminating  $\beta$  we have

$$\nu_1 = \nu \left/ \left( 1 + \frac{2h\nu}{m_0c^2} \sin^2 \frac{1}{2} \theta \right) \right.$$

Since  $\nu = c/\lambda$ , and  $\nu_1 = c/\lambda_1$ , where  $\lambda$  is the wavelength of the primary radiation and  $\lambda_1$  that of the radiation scattered in the direction  $\theta$ , the equation, in terms of wavelengths instead of frequencies, becomes

$$\lambda_1 - \lambda = \frac{2h}{m_0c} \sin^2 \frac{1}{2} \theta \quad . \quad . \quad . \quad (73)$$

The change in wavelength thus depends on the angle  $\theta$ , and a factor involving only universal constants. Substituting the accepted values of  $h$ ,  $m$  and  $c$ ,

$$\lambda_1 - \lambda = \Delta\lambda = 0.0484 \sin^2 \frac{1}{2} \theta,$$

which, for radiation scattered at right angles to the primary beam, gives us

$$\Delta\lambda = 0.0242 \text{ \AA.},$$

a value identical with that found by direct experiment.

If, instead of being free to move, the electron with which the photon collides is rigidly attached to an atom,  $m$  becomes the mass of the whole system, i.e. the atom, and  $\Delta\lambda$  thus becomes vanishingly small. The photon is then scattered without loss of energy, and without change in wavelength. On Compton's theory this accounts for the unmodified radiation.

In the Compton scattering a part  $(h\nu - h\nu_1)$  of the energy of the photon is transferred to the scattering electron, which is therefore projected with a finite velocity. The short tadpole-shaped tracks seen along

the line of the X-ray track in Fig. 58 (Plate III) are those of the Compton recoil electrons. The longer tracks are those of the photoelectrons ejected when a photon is completely absorbed in the atom.

**82. Diffraction phenomena with X-rays.** The identification of X-rays with light of short wavelength naturally led to many attempts to detect diffraction or interference effects. Haga and Windt<sup>(3)</sup> in 1901 made experiments with a V-shaped slit, a few thousandths of a mm. wide at its narrowest end, and obtained certain variations of intensity in the resultant photograph which if really due to diffraction would indicate a wavelength of the order of  $1.3 \times 10^{-8}$  cm. The experiments were difficult at the time and were not generally accepted as decisive, although it is only just to say that the value obtained by these experiments has been confirmed by the method of crystal reflection which we are about to describe.

The difficulty in demonstrating diffraction for X-rays lies in the shortness of their wavelengths. With normal incidence a beam of sodium light of wavelength  $5.89 \times 10^{-5}$  cm. is deflected through an angle of some  $19^\circ$  by a transmission grating with 5500 lines to the cm. To produce a similar deviation in X-rays would require each of these spacings to be divided into ten thousand spaces, a process mechanically

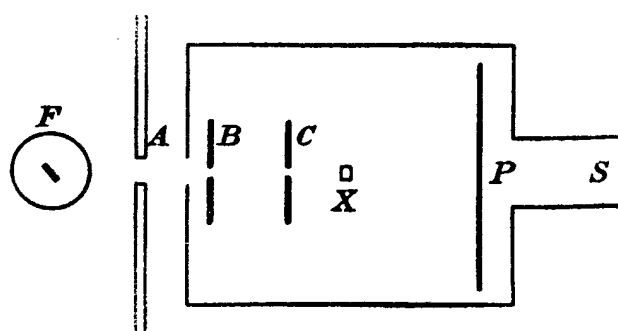


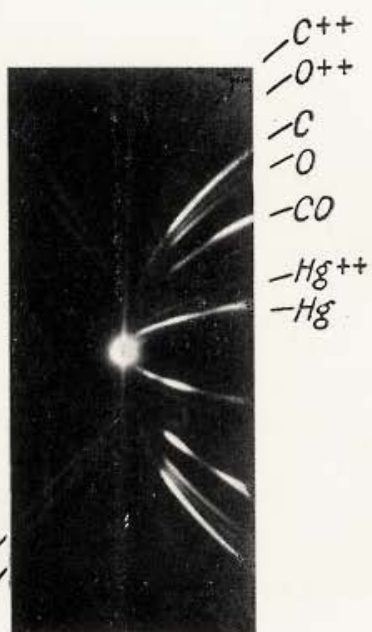
FIG. 59. Illustrating the Laue method of demonstrating X-ray diffraction

and physically inconceivable. It was not until 1912 that Prof. Laue<sup>(4)</sup> conceived the idea that the regular grouping of the atoms in a crystal, as affirmed by modern crystallography, might provide a natural grating of suitable spacing for the experiment. The problem is

of course more complicated than that of the ordinary grating in which the spacings are all parallel and all in one plane. Since the regularity of the crystal grating is in three dimensions instead of only one Laue was able to show that if a narrow pencil of X-rays was made to pass symmetrically through a crystal diffracted rays should emerge in various directions from the crystal and should form a symmetrical pattern of spots on a photographic plate placed perpendicular to the beam.

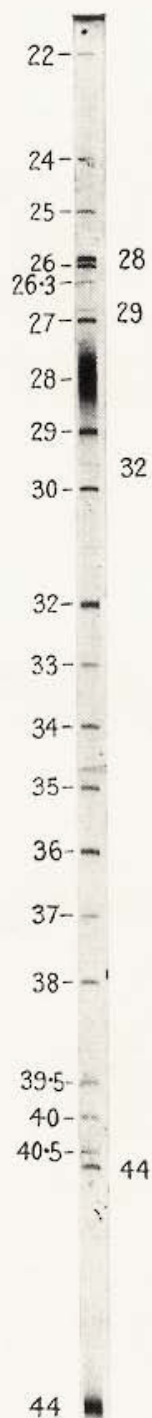
The theory was put to the test by Friedrich and Knipping in 1913. Their apparatus is indicated in Fig. 59. *F* is an X-ray tube, the rays





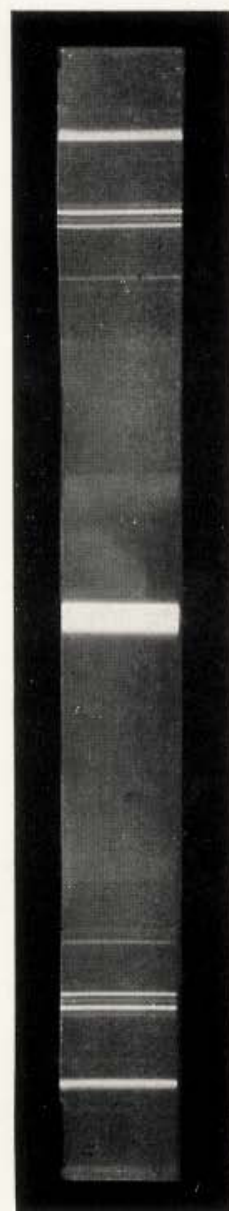
*Sir J. J. Thomson.*

FIG. 40. POSITIVE  
RAY PARABOLAS.



*F. W. Aston.*

FIG. 42.  
MASS SPECTRUM  
OF BROMINE  
WITH CARBON  
DIOXIDE.



*G. Shearer.*

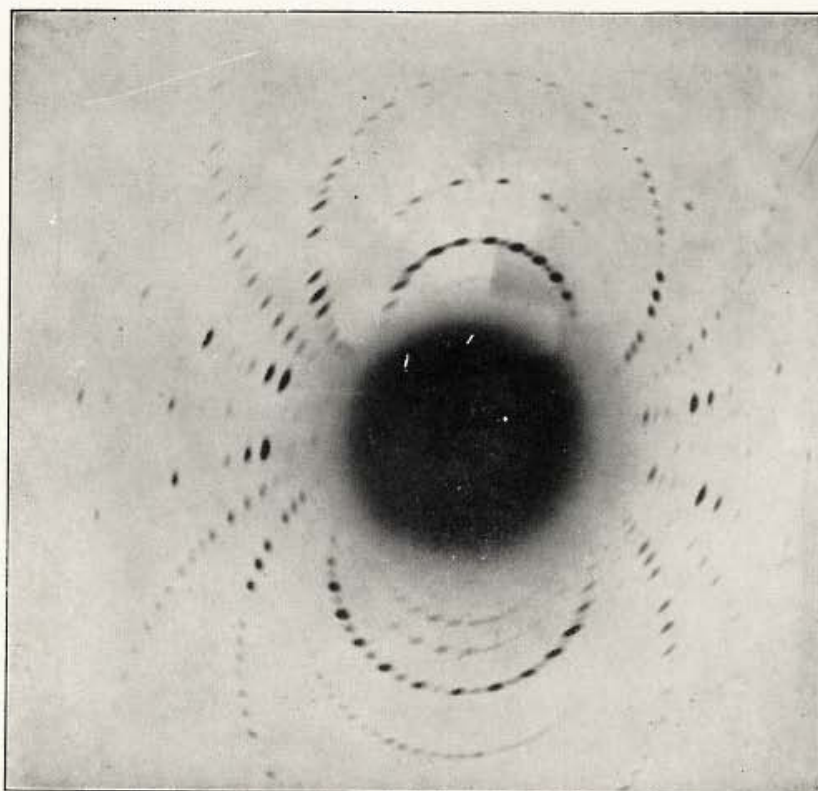
FIG. 67.  
L SPECTRUM OF  
TUNGSTEN.

PLATE III.



*C. T. R. Wilson.*

FIG. 58. X-RAYS SHOWING SHORT TRACKS OF RECOIL ELECTRONS.



*Sir William Bragg.*

FIG. 60. LAUE PATTERN OF NICKEL SULPHATE.

from which are limited to a narrow pencil by passing through a series of slits in the lead screens, *A*, *B*, *C*. They then pass along one of its axes of symmetry through a small crystal *X*. A sighting screen *S* enables the adjustment to be made. A photographic plate is then inserted at *P* and an exposure made lasting for some hours. On developing the plate it was found that the very black central patch made by the undeflected beam of rays was surrounded by a symmetrical pattern of spots as indicated by the theory, some of the spots being deviated by nearly  $40^\circ$  from the direct line of the primary beam. A Laue pattern for nickel sulphate is reproduced in Fig. 60 (Plate III).

**83. Reflection of X-rays by crystal planes.** Modern crystallographers, following Bravais, have been led to regard the atoms or molecules in crystals as arranged with a definite spacial symmetry. All the atoms of the same sort are regarded as forming a regular system of points in space, the different systems belonging to the different atoms of course interpenetrating. This net-like arrangement is known as a space lattice.

In order that a set of points shall form a space lattice it must satisfy certain geometrical conditions deducible from the general laws to which all regular patterns in space must conform.

It has been shown that the only method of dividing up space which will satisfy the conditions is as follows. A series of three sets of parallel planes is taken intersecting each other. All the planes in each set are parallel and equally spaced, but the spacings of the different sets need not be the same, and the planes may intersect at any angle. The space is thus divided up into a series of parallelepipeds (Fig. 61) or *unit cells* the corners of which form the space lattice. The three directions parallel to the edges of the unit cell form the axes of the crystal and the ratio of the lengths of these edges is the "axial ratio" of the crystal.

Looking again at Fig. 61 it will be noticed that along certain planes drawn in the crystal, the construction planes, for example, the particles are thickly studded, that is to say, these planes pass through a considerable number of points. It is possible to draw other sets of

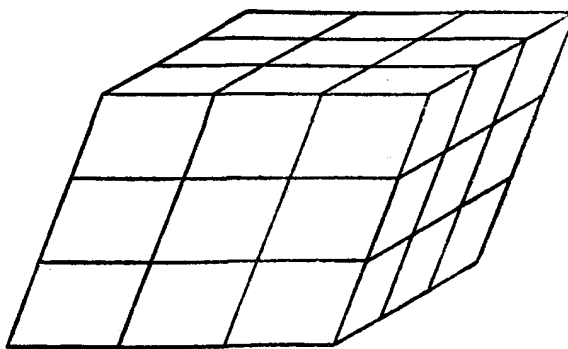


FIG. 61. The crystal space lattice. The atoms occupy the points of intersection of the planes

planes in the crystal, such for example as those drawn through opposite edges of the elementary parallelepipeds, which contain somewhat smaller but still considerable numbers of particles. The two-dimensional array of points figured in Fig. 64 may help to make the matter clearer. Here, in addition to the construction planes, two other such sets are indicated by firm lines; the planes labelled (1) passing from one corner to the opposite corner of the "unit cell," those marked (2) passing from a corner of one cell to the opposite corner of the next cell but one. These planes are closely related to the surfaces of the natural crystal. They represent lines along which the crystal may cleave or split with comparative ease, and all are parallel to possible faces on the crystal.

Consider now a parallel beam of X-rays falling on one of these thickly studded planes represented in section by the dots in Fig. 62.

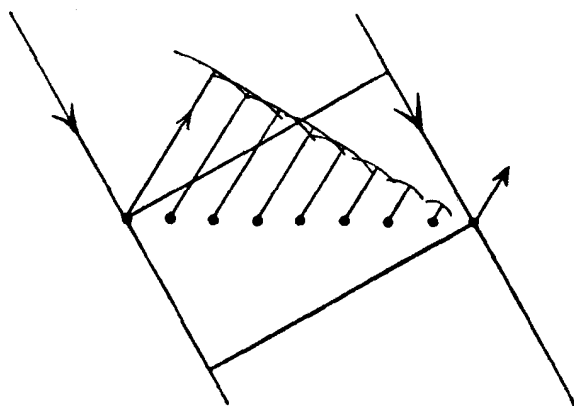


FIG. 62. Reflection of X-rays by a single crystal plane

Each particle, in turn, as the wave front passes over it, scatters a small fraction of the radiation and thus becomes the source of a secondary pulse similar in character to the primary radiation. These secondary pulses will combine to form a plane wave front inclined to the reflecting plane at an angle equal to that of the incident wave. We have, in

fact, a reproduction of Huygens' construction for the reflection of a plane wave at a plane surface, except that the reflecting "elements" are discontinuous.

There is, however, an important distinction between the specular reflection of light, and the so-called "reflection" of X-rays by crystal planes. The fraction of the incident energy which is reflected from a single plane is infinitesimally small. Most of the radiation is transmitted, a further minute fraction being reflected at each of the series of parallel planes through which it passes while penetrating into the crystal. If the waves reflected from these successive planes of atoms are in phase with each other when they reach some point outside the crystal they will co-operate to form a measurable reflected beam. If they are out of phase they will, by the ordinary laws of interference, simply cancel each other and there will be no reflected beam. The

effect, when it occurs, is thus a volume not a surface reflection. It depends on the internal structure of the crystal, not on its surface, and will occur even if the crystal exhibits no actual facet parallel to the planes we are considering.

Let  $pp, qq, rr, \dots$  (Fig. 63) be a series of parallel planes of atoms and  $PP'$  be a wave front advancing on the crystal in the direction  $PQ$ , making an angle  $\theta$  with the surface  $pp$ . The reflected ray must travel in a direction  $QR$ , making the same angle with the surface  $pp$  as the incident ray  $PQ$ . The waves reflected from the surfaces  $pp, qq$ , will be in phase at a point  $R$  if their path difference,  $P'Q'R - PQR$ , is an

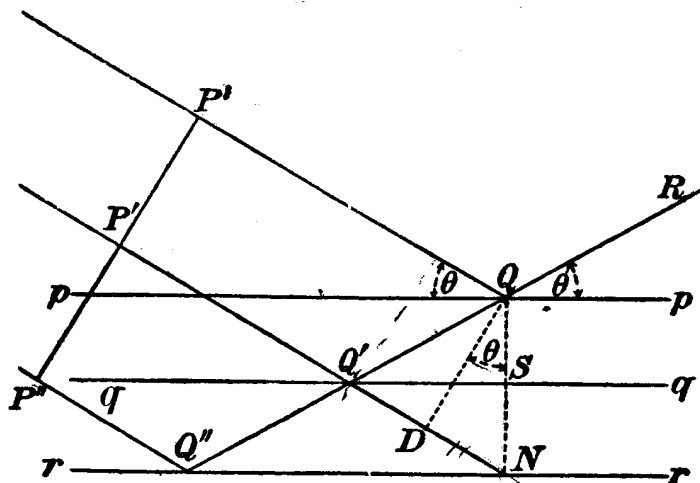


FIG. 63. Illustrating Bragg's law for crystal reflection of X-rays

integral number of wavelengths of the radiation. Draw  $QSN$  perpendicular to  $pp$  and  $QD$  perpendicular to  $P'N$ . Then since  $PQ$  and  $QR$  make equal angles with the line  $pp$  and  $QN$  is perpendicular to  $pp$ ,  $Q'Q$  is equal to  $Q'N$  and the path difference between the two rays under consideration is  $P'Q' + QQ' - PQ = P'Q' + Q'N - P'D$

$$\begin{aligned} &= DN = QN \sin \theta \\ &= 2d \sin \theta \quad \dots \dots \dots (74) \end{aligned}$$

where  $d$  is the perpendicular distance between the planes. For maximum brightness the path difference must be an integral multiple of the wavelength  $\lambda$ , of the radiation. Hence for maximum reflection by a given set of crystal planes

$$2d \sin \theta = n\lambda \quad \dots \dots \dots (75)$$

This important relation was first pointed out by W. L. Bragg and is known as Bragg's law.

It will be noticed that the problem is exactly analogous to that of the diffraction grating, the various parallel planes acting as lines in the

grating. Applying the usual optical principles it can be shown that a very small discrepancy from exact agreement in phase between the separate elements, such as would be introduced by a very slight change in the angle  $\theta$ , will result in the almost complete extinction of the radiation by interference. The angle at which reflection occurs is, therefore, very sharply defined.

84. **Theory of Laue's diffraction experiment.** Consider, now, a beam of X-rays incident on the crystal represented by the system of points in Fig. 64. For clarity, only one of the series of exactly similar crystal

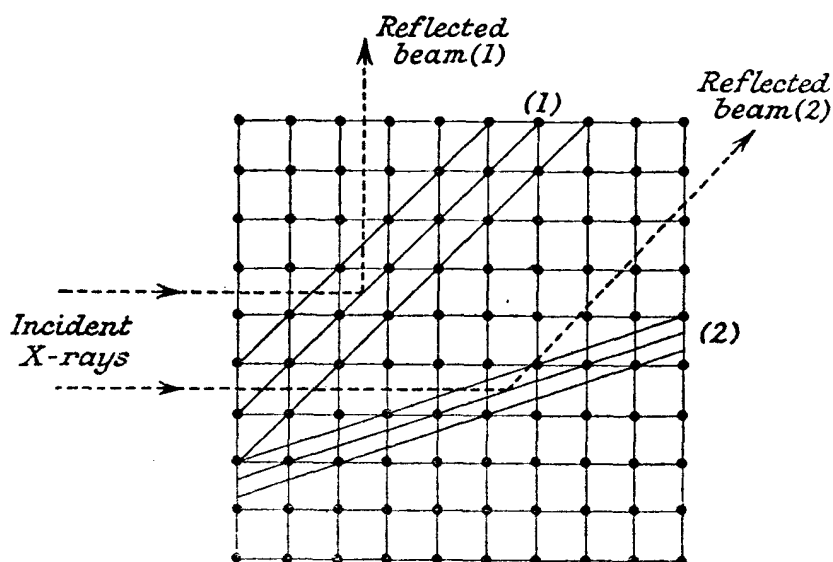


FIG. 64. Illustrating the formation of the Laue diffraction pattern

planes parallel to the plane of the paper is shown in the diagram. It will clearly be possible to pick out a considerable number of planes in the crystal, each of which passes through an appreciable number of the diffracting centres (atoms or molecules) represented by the dots in the diagram. Two such sets are indicated. Each of these sets is capable of producing a reflected beam providing that the Bragg condition  $n\lambda = 2d \sin \theta$  can be satisfied. For a given set of planes and a fixed direction of incident beam both  $d$  and  $\theta$  are fixed. If the incident rays are monochromatic, so that  $\lambda$  is also fixed it is improbable that any of the planes will satisfy the necessary conditions and there will be no reflected rays. If, however, the incident radiation forms a continuous spectrum, covering a suitable range of wavelengths, each set of planes will pick out, from the continuum, the particular wavelength which satisfies the Bragg condition. Thus, considering only the first order diffraction ( $n=1$ ), the wavelength of the radiation reflected from

the planes marked (1) will be given by  $\lambda_1 = 2d_1 \sin \theta_1$  and that from the planes (2) by  $\lambda_2 = 2d_2 \sin \theta_2$ .

As we shall see shortly, the radiation from an ordinary X-ray tube, such as was used by Laue for his experiment, does in fact yield a continuous spectrum. Each of the Laue "spots" thus represents the reflection of some particular wavelength in a particular set of crystal planes. Since the deviation of the diffracted ray is twice the glancing angle,  $\theta$ , at which the incident radiation strikes the reflecting plane, it is obvious that the symmetry of the Laue pattern reflects the symmetry of the crystal itself. The detailed interpretation of the pattern is, however, difficult, and sometimes uncertain.

**85. Bragg's method of X-ray analysis.** The theory of the Laue effect outlined in the previous section is due to W. L. Bragg<sup>(5)</sup>: Laue's own treatment of the subject being more complicated and, on the whole, less satisfactory. The conception of the diffracted beams as being due to reflection by crystal planes immediately suggested a simpler and more informative way of investigating the phenomenon. Suppose that, selecting a crystal with well marked faces, we mount it so that the glancing angle of the primary beam on one of these faces can be gradually increased from zero to  $90^\circ$ . If the incident radiation is homogeneous, of wavelength  $\lambda$ , reflection will occur when the glancing angle attains any of the values given by making  $n$  successively equal to 1, 2, 3, . . . in the general equation  $2d \sin \theta = n\lambda$ , where  $d$  is the spacing of the planes parallel to the selected crystal face. These may be termed the first, second, etc., order reflections. The intensity decreases rapidly with increasing order. These reflections may be identified as being due to the same primary wavelength by the fact that  $\sin \theta_1 : \sin \theta_2 : \sin \theta_3$  as 1 : 2 : 3. Since the maximum value of  $\sin \theta$  is unity, the longest wavelength which can be reflected is given by  $\lambda = 2d$ .

If the primary beam contains a number of definite wavelengths,  $\lambda_a, \lambda_b, \dots$ , each will be reflected at its own appropriate angle, so that, for the same crystal face and for the same order reflection  $\lambda_a/\lambda_b = \sin \theta_a/\sin \theta_b$ . If the primary beam forms a continuous spectrum, reflection will take place at all angles, within appropriate limits. This can be compared with the continuous spectrum formed by a grating illuminated with white light, and the radiation may be conveniently referred to as "white" X-radiation. The intensity of the reflected beam will be proportional to the intensity of the corresponding radiation in the primary radiation.

If, using the same monochromatic radiation, we remount the crystal

so that reflection takes place at a different set of planes, the relative value of the spacing,  $d$ , for the two sets of planes can be determined from the relation

$$n\lambda = 2d_1 \sin \theta_1 = 2d_2 \sin \theta_2. \quad . \quad . \quad . \quad (76)$$

This is the basis of the important method of investigating crystal structure known as X-ray analysis.

**86. The X-ray spectrometer.** The "single crystal" method. The reflected ray can be detected, and its deflection measured, either by a photographic or by an ionization method, and each has its advantages. The apparatus needed for the ionization method has been perfected by W. H. Bragg and is indicated in Fig. 65. The rays from the anti-

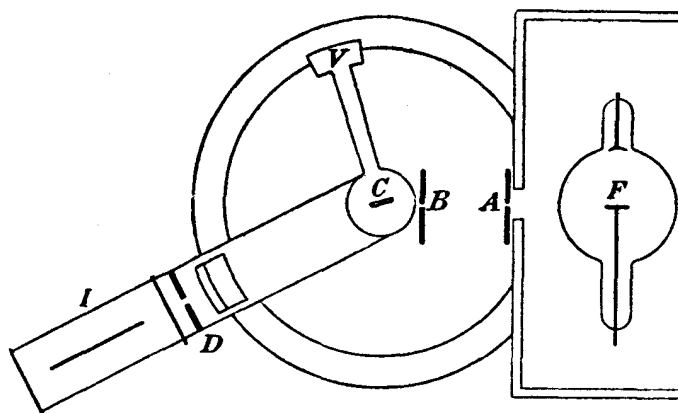


FIG. 65. W. H. Bragg's X-ray spectrometer

cathode of the X-ray tube are limited to a narrow pencil by two adjustable lead slits  $A$  and  $B$ . The arrangement may be compared to the collimator of a spectroscope. The crystal is mounted on the table of the spectrometer, as we may call it, at  $C$ , and its orientation may be read by the vernier  $V$  on the circular scale of the instrument. The ionization chamber  $I$  is carried by an arm also pivoted at the centre of the instrument, and the reflected beam can be limited by a third lead slit  $D$ . The current through the ionization chamber can be measured by an electroscope or electrometer in the usual way. As the intensity of the reflected ray is usually small the ionization chamber is generally filled with some heavy gas such as methyl iodide, in which the ionization produced by a given beam of X-rays is more intense than it is in air.

In using the apparatus the crystal and the chamber are moved together, the latter of course being always moved through twice the angle of the former. As the glancing angle between the crystal face and the incident beam is gradually increased a point is reached where



the electroscope shows a deflection, and the ionization current can be plotted against the glancing angle. If the rays are homogeneous the current will only flow at certain definite angles corresponding to the different orders of reflection for that particular wavelength, and the curve will be discontinuous.

This case is represented by the top curve of Fig. 66, which shows the current through the chamber against the glancing angle  $\theta$ . It will

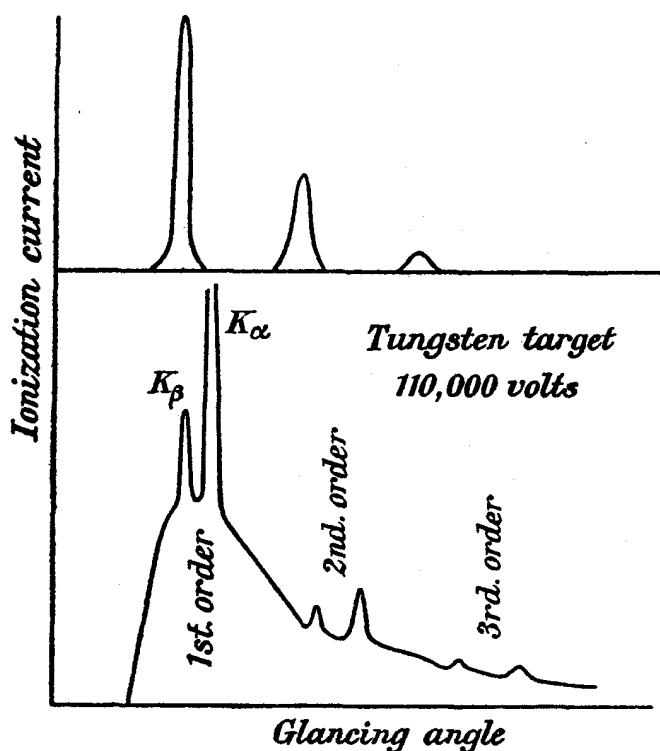


FIG. 66. Showing the relation between glancing angle and ionization for the single crystal spectrometer

be seen that the curve shows three peaks or "lines," and these three peaks can be identified as being due to the same waves by the fact that  $\sin \theta_1 : \sin \theta_2 : \sin \theta_3$  as 1 : 2 : 3. In general, however, if the radiation is that from an ordinary X-ray tube, it will be made up of certain rays of definite wavelengths together with more or less "white" radiation or radiation of continuous wavelength. The curve will then be a continuous curve rising to various peaks or maxima, the height of which affords a measure of the intensity of the corresponding wavelength in the primary beam. This is shown in Fig. 66, the lower curve of which gives the spectrum of the radiation from a tungsten anticathode. It will be seen that in this very important practical case we have a series of peaks on a background of "white" radiation. At larger angles peaks appear again in the second order reflections, and

again in the third order. It will be noticed that the intensity decreases with the order of the spectrum. These peaks are characteristic of the tungsten anti-cathode. They appear whatever the nature of the crystal used for the reflection: rock salt, calcite, and fluorspar all yielding the same result. The curves are, in fact, exactly similar in the three cases, only the scale of the curve depending, as might be expected from the theory, on the spacings of the crystal used.

If the relative intensities of the various reflections are not required, the ionization chamber can, with advantage be replaced by a photographic film, which is usually bent round a former into a circular arc concentric with the axis carrying the crystal. The crystal itself is slowly rotated, by a clockwork mechanism, throughout the exposure. When, in the course of rotation, the reflecting planes in the crystal make with the incident beam an angle which is the reflecting angle for one of the constituent wavelengths in the incident radiation, a reflected ray is momentarily produced which records itself on the photographic film. The rotation of the crystal is continued until the weak images so formed are sufficiently exposed.

A photograph of the  $L$  spectrum of tungsten, taken by Dr. G. Shearer, is reproduced in Fig. 67 (Plate II). The central bright band is due to radiation which has passed straight through the crystal. The characteristic lines of the tungsten spectrum show as bright lines standing out on a much fainter background due to the general radiation from the tube. Since the photograph was taken with the crystal describing complete revolutions, the spectrum appears symmetrically on each side of the central bright band. The photographic method is generally employed when measurements of wavelength of the highest order of accuracy are required.

**87. The powder crystal method.** The methods outlined in the previous section yield the maximum information of the highest accuracy. They are, however, tedious, and for many purposes can conveniently be replaced by a method devised independently by Debye and Scherrer, and by Hull. In this, the crystal is reduced to fine powder, and a sample of the powder is formed into a fine filament, either by coating it on to a hair or enclosing it in a thin-walled capillary tube. The filament is mounted along the axis  $C$  (Fig. 68) of a cylindrical chamber  $D$ , or "camera," round the circumference of which is stretched a photographic film. The incident X-radiation falls on the specimen through a pinhole  $P$ ; the radiation should preferably be monochromatic, or at any rate must contain one strong monochromatic line.

Since the millions of crystal fragments are orientated entirely at random, each set of crystal planes will intercept the primary beam at all possible angles. Those which do so at the critical Bragg angle will give rise to a reflected ray. Since even these planes will have all possible orientations about the primary ray, the reflected energy from a single

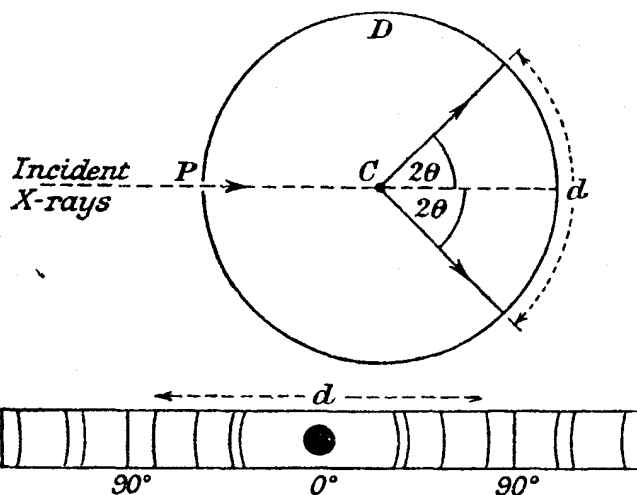


FIG. 68. The principle of the Powder spectrometer

set of planes will lie along the surface of a cone of semi-vertical angle  $2\theta$ . All the sets of planes are, of course, reflecting independently at the same time, so that on unrolling and developing the photographic film, a series of circular arcs is obtained, as indicated in the lower half of Fig. 68, marking the interception by the film of each of the reflection cones. Thus a single exposure suffices to give a complete picture of the crystal structure. If  $d$  is the distance measured along the film between the two reflections from the same crystal plane, and  $R$  is the radius of the cylinder, the glancing angle  $\theta$  is clearly equal to  $d/4R$ . In practice, however, the spectrometer is usually calibrated by using some substance whose constants are known.

**88. Determination of the wavelength of X-rays.** The theory so far developed only enables us to compare wavelengths. To determine them in absolute units we must know the distance apart of the planes in our crystal grating. The discussion involves some acquaintance with the elements of crystallography, but the argument is briefly as follows.

Taking the case of a simple cube (Fig. 69), the most important faces are (1) the face of the cube itself, (2) the face parallel to a plane through opposite edges of the cube, (3) the face parallel to a plane including one corner of the cube and passing through the diagonal of the opposite

**face.** The development of this face gives rise to regular octahedra and is exceedingly common on cubic crystals. The three sets of planes are denoted in crystallography as {100}, {110}, and {111}, respectively.

Now crystallographers recognize three classes of cubic symmetry:

(1) the simple cube arising from a simple cube lattice such as we have already described;

(2) a cube with a single particle at the centre, known as the cube centred lattice;

(3) a simple cube with a particle at the centre of each face: this is known as the face centred lattice.

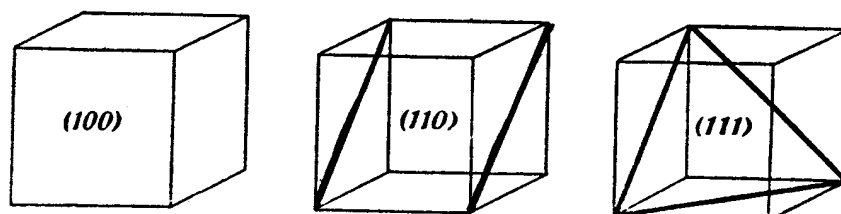


FIG. 69

Now the ratios of the distances between two successive planes in the three sets of planes corresponding to the systems {100}, {110}, {111} will not be the same for the three kinds of lattices. It can be shown by simple geometry that

$$\begin{aligned} \frac{1}{d_{100}} : \frac{1}{d_{110}} : \frac{1}{d_{111}} & \\ = 1 : \sqrt{2} : \sqrt{3} & \text{ for simple cube lattice .} \\ = 1 : \frac{1}{\sqrt{2}} : \sqrt{3} & \text{ for a cube centred lattice .} \\ = 1 : \sqrt{2} : \frac{\sqrt{3}}{2} & \text{ for a face centred lattice . . . (77) .} \end{aligned}$$

Now by using the same homogeneous X-radiation for the three faces {100}, {110}, {111} respectively we can by (75) find the ratios of  $d_{100} : d_{110} : d_{111}$ . We can thus identify for any cubic crystal, by applying the above results, the particular order of cubic symmetry to which it belongs.

Let us apply these results to the important sylvine group, comprising rock salt (NaCl), sylvine (KCl), and other similar halogen compounds of the alkali metals. These are chemically and crystallographically similar and may reasonably be expected to have the same structure. In the case of sylvine the measurements indicate that it has the simple

cubic structure. With rock salt, however, we meet a new phenomenon. The reflections from the  $\{100\}$  and the  $\{110\}$  faces are similar to those from sylvine. The reflections from the  $\{111\}$  faces give a weak first order spectrum, a strong second, a weak third, a strong fourth, and so on. Judging only by the strong spectra we should assign to rock salt the simple cube lattice already suggested for the crystallographically similar sylvine crystal. Judging by the weak first order spectrum it would be a face centred lattice.

To reconcile these apparently conflicting conclusions W. L. Bragg made the suggestion that the diffracting particles making up the lattice structure were not molecules of the salt but atoms of its constituent elements. Since the intensity of the radiation scattered is proportional to the atomic number, reflection from a plane of sodium atoms ( $\text{Na}=11$ ) would be appreciably less than that from a plane of chlorine atoms ( $\text{Cl}=17$ ). A set of planes alternately made up of all sodium atoms and all chlorine atoms would thus resemble a grating in which every odd ruling was somewhat wider than the even ones. Such a grating, as is well known, gives in addition to its normal spectra a set of weaker spectra with half the normal deviations. The experimental observations will thus be explained if the  $\{111\}$  planes are composed alternately of sodium atoms and chlorine atoms, while the  $\{100\}$  and  $\{110\}$  planes are each composed of equal numbers of the two kinds of atoms.

These conditions are satisfied if the atoms are arranged as in Fig. 70. Here the black dots indicate the metallic atoms, the circles the atoms of chlorine. It is easy to show that the same structure will serve for the sylvine crystal. Potassium has an atomic number 19, very close to that of chlorine. In fact, since the compound  $\text{KCl}$  is formed by the passage of an electron from the potassium atom to the chlorine atom, the number of electrons in each will be the same, and their radiating powers will be identical. The structure of Fig. 70 obviously reduces to a simple cubic lattice if the dots and the circles become identical. The pattern of Fig. 70 must, of course, be envisaged as repeating in every direction.

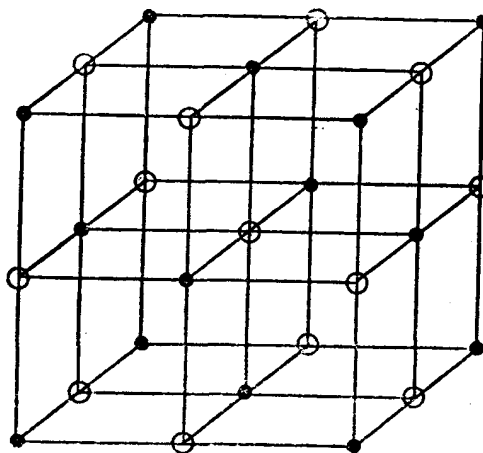


FIG. 70. Bragg's suggested structure for rock salt

Accepting this structure for the rock salt crystal we see (considering the adjacent cubes in all directions) that each sodium atom is associated with eight of the small cubes into which the figure can be divided, and assuming its mass to be equally divided among the cubes, one-eighth of the mass is included in each cube. There are four sodium atoms associated with each small cube and thus each small cube includes the mass of half an atom of sodium, and therefore half a molecule of the compound sodium chloride. Thus the whole structure of Fig. 70 represents four molecules.

The distance apart of the planes bounding the small cubes is obviously  $d_{100}$ , all the planes being exactly similar. Hence the volume of each small cube is  $(d_{100})^3$ . The mass associated with each small cube is one half that of the sodium chloride molecule, that is  $\frac{1}{2}(23+35.5) \times$  (the mass of an atom of unit atomic weight), i.e.  $1.65 \times 10^{-24}$  gm., assuming the value deduced for this constant from Millikan's oil-drop experiment.

Hence if  $\rho$  is the density of the rock salt crystal ( $=2.17$ ) the mass of the cube  $=\rho(d_{100})^3=29.3 \times (1.65 \times 10^{-24})$  and

$$d_{100}=2.814 \times 10^{-8} \text{ cm.}$$

For the most pronounced radiation from platinum the glancing angle for the first order spectrum using the  $d_{100}$  planes of rock salt is  $11.4^\circ$ . Hence substituting in the equation  $\lambda=2d \sin \theta$  the wavelength of this particular X-radiation is given by  $\lambda=1.10 \times 10^{-8}$  cm.

**89. The X-ray spectra. The continuous background.** We have seen that the X-ray spectrum emitted when cathode rays impinge on a target consists, to use optical phraseology, of a series of bright lines superimposed on a continuous luminous background. The lines are characteristic of the substance forming the target and constitute its X-ray spectrum. The background is independent of the nature of the target except for the fact that its intensity is proportional to the atomic number of the element of which the target is made. For a given target the intensity of the radiation is proportional to the square of the P.D. across the tube, and to the current passing through it. The energy in the radiation is, however, only a minute fraction (about  $\frac{1}{1000}$ ) of that of the incident cathode rays, the remaining portion of the energy appearing in the target in the form of heat.

The distribution of intensity in the spectrum is indicated in Fig. 71.<sup>(6)</sup> The maximum of the curve moves in the direction of shorter and shorter wavelengths as the P.D. across the tube is increased.

It will be noticed that the curve falls to zero at some definite wave-

length, which depends on the P.D. used to excite the tube. There is thus for each P.D. a minimum value for the wavelength of the radiation emitted, or in other words a maximum frequency, which cannot be exceeded. Very careful investigation of this end part of the spectrum by Duane and Hunt<sup>(7)</sup> and others has shown that this frequency  $\nu$

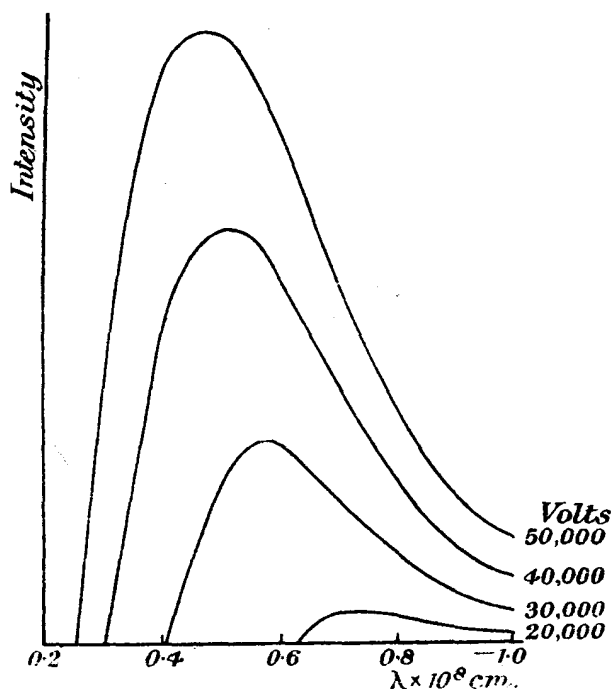


FIG. 71. Distribution of energy in the continuous X-ray spectrum

is directly proportional to the applied P.D. If  $V$  is the potential difference in volts it was found that over the whole range of potentials employed (from 24,000 to 100,000 volts) the relation could be expressed by the formula

$$V = 4.128 \times 10^{-15} \nu,$$

or converting volts to absolute e.s.u. and multiplying both sides by  $e$ , we have

$$\begin{aligned} Ve &= (1.376 \times 10^{-17}) \times (4.803 \times 10^{-10}) \nu \\ &= 6.61 \times 10^{-27} \nu \end{aligned} \quad (78)$$

where  $Ve$  is the maximum energy acquired by the electrons in the X-ray tube.

This equation is not only identical in form but has the same numerical constant as the equation (57) for the photo-electric effect. In the photo-electric effect we were considering the transference of energy from radiation to an electron; in the present case we are dealing with the transference of energy from an electron to radiation—the “inverse

photo-electric effect " as it may be called. Both are governed by the same experimental law. The inverse photo-electric effect is one of the most accurate ways of determining the value of the constant,  $h$  (Planck's constant), involved.

The fact that much of the radiation has a smaller frequency than the maximum is due to the rapid loss of energy of the electrons in penetrating the material of the target. Only those which make a radiative collision at the surface of the target will still possess the full energy  $Ve$  when they emit the radiation. The quantum relation  $T=h\nu$  is, of course, inconsistent with the classical theory of X-radiation, outlined in § 76; no satisfactory theory of the phenomenon has yet been given.

**90. The X-ray spectra. The characteristic radiations.** The characteristic spectrum of an element may be mapped by making it the target in an X-ray tube. Special demountable tubes, which can be rapidly opened up for changing the target and then re-evacuated, are generally employed for the purpose. The rotating crystal method described in § 86 gives the most accurate results. If the position of the line to be investigated is known approximately it is unnecessary to rotate the crystal through  $360^\circ$ . It is sufficient to set the crystal at approximately the correct angle, and to rock it backwards and forwards through a few degrees on each side of the expected value. In this way the relative wavelengths of the different lines can be compared to an accuracy approaching one part in a million.

Experiments made prior to the development of X-ray spectroscopy had indicated that the characteristic radiations from an X-ray tube fell into certain groups known as the  $K$  and  $L$  radiations. Elements of high atomic number gave indications of a still more absorbable group known as the  $M$  radiation. A study of the absorption of these radiations indicated that they were approximately homogeneous.

When examined by the spectrometer it is found that neither the  $K$  nor the  $L$  radiations are homogeneous. The  $K$  radiation consists of four principal lines; the  $L$  radiation shows ten or more. Thus the  $K$  spectrum of tungsten, the element most often employed in the target of an X-ray tube, shows four  $K$  lines

$$K_{\alpha_2}=0.21345; K_{\alpha_1}=0.20862; K_{\beta_1}=0.18422; K_{\beta_2}=0.17898;$$

all in Ångstrom units, and an  $L$  spectrum (Fig. 68) of thirteen distinct lines ranging from 1.6750 to 1.0963 Å. Some lines of an  $M$  series of wavelength between 6.969 and 6.076 Å. have also been observed.<sup>(11)</sup>

The outstanding feature of the X-ray spectra is their great similarity to each other. In fact, with some minor exceptions, the characteristic



spectrum of one element differs from that of another only in being displaced bodily in the spectral scale, the wavelength of the various lines becoming smaller as the atomic number of the element becomes greater. Moseley,<sup>(8)</sup> to whom the first extensive series of observations is due, found that if  $\nu$  is the frequency of some particular line in the  $K$  spectrum, say the  $K_{\alpha_1}$  line for example, then its value for any element can be expressed by the equation

$$\nu = a(Z - b)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

where  $Z$  is the atomic number of the element, and  $a$  and  $b$  are constants for that particular line, but vary slightly as we pass from one line to another, say from the  $K_{\alpha_1}$  to the  $K_{\alpha_2}$  line. A similar relation, but with quite different values of the constants, holds also for the  $L$  spectrum. For the  $K$  series  $b$  has a value nearly equal to unity; for the  $L$  series its value is about 7.4. The relation is not quite exact, but if  $\sqrt{\nu}$  is plotted against  $Z$  a perfectly smooth line is always obtained which, however, shows a slight curvature for large values of  $Z$ .

It is interesting to note that when plotting his original observations Moseley found that it was necessary (in order to avoid definite breaks in the curves) to postulate that four elements, of atomic number 43, 61, 72, and 75, remained to be discovered. These missing elements have since been discovered and identified by their X-ray spectra.

**91. The absorption of X-rays.** If a sheet of any substance is interposed in the path of a beam of homogeneous X-rays the intensity of the beam is diminished, and falls off exponentially with the thickness of the sheet. Thus if  $I_0$  is the initial intensity of the beam,  $I$  its intensity after passing through a thickness  $d$  of material,

$$I = I_0 e^{-\lambda d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

where  $\lambda$  is the linear coefficient of absorption of the radiation in the particular absorbing substance. The value of  $\lambda$  for radiation of any wavelength can be measured conveniently with Bragg's spectrometer. The instrument is adjusted so that radiation of the desired wavelength falls on the slit of the ionization chamber, and the absorbing substance, in the form of thin sheets, is placed in the path of the reflected ray, between the crystal and the ionization chamber.

It is found experimentally that the absorption depends only on the number of atoms of the substance in the thickness of the absorbing layer, and is independent of their state of aggregation. Thus  $\lambda/\rho$ , where  $\rho$  is the density, is constant for a given substance and independent of any changes in density due to its physical condition.  $\lambda/\rho$  is called the mass coefficient of absorption and is usually denoted by  $\mu$ .

The absorption can obviously be expressed in the form  $I = I_0 e^{-\mu m}$  where  $m$  is the mass of unit area of the absorbing sheet.

The coefficient of absorption, as defined above and obtained by direct experiment, covers two or, to be more precise, three different

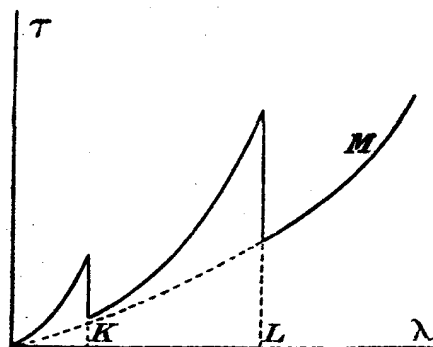


FIG. 72. Curve showing the relation between the absorption coefficient and the wavelength

processes. In the first place energy may be absorbed from the X-ray beam by an atom of the material. This, as we shall see later (§ 92), results in the ejection of an electron from the atom and hence, by analogy may be described as the photo-electric absorption. Again, the collision between a photon and a free electron may result in the projection of a Compton electron. Both these processes result in a true absorption of energy from the beam. In addition,

however, the X-radiation may suffer simple scattering in the absorbing sheet. This is not an absorption effect, but if we are dealing with a narrow limited pencil of radiation practically the whole of the scattered radiation is deflected outside the limits of the beam, and its intensity is thereby reduced. Thus the coefficient  $\mu$  as measured in this way is the sum of two distinct coefficients, the one  $\tau$  measuring the actual absorption, the other  $\sigma$  the scattering. Thus

$$\mu = \tau + \sigma$$

The coefficient  $\tau$  is itself complex, and includes both the photo-electric and the Compton absorption. The latter, however, is relatively unimportant, except for very short wavelength radiation.

For radiation of much shorter wavelength than the  $K$  radiation of the absorbing element, the true mass coefficient of absorption  $\tau$  increases rapidly with increasing wavelength,  $\lambda$ , approximately according to the relation

$$\tau = C\lambda^3$$

as shown by the initial portion of the curve in Fig. 72. On reaching a certain definite wavelength, known as the critical absorption wavelength there is a sharp and very marked discontinuity in the curve, so that the absorption coefficient for a wavelength slightly greater than the critical is only about one-seventh of the value just before the limit is reached. This discontinuity is known as the  $K$  absorption edge.

After passing the discontinuity, the absorption coefficient again increases roughly as the cube of the wavelength, until a second discontinuity—the  $L$  absorption edge—is reached, when the coefficient again suffers a large and sudden decrease in value. Increasing the wavelength still further we again get a rapid increase in the coefficient of absorption. For elements of high atomic number a third or  $M$  absorption limit can be found, towards the extreme long-wave end of the X-ray spectrum. The relation between the mass coefficient of absorption and the wavelength may thus be expressed by a formula of the type

$$\tau = (C_K + C_L + C_M)\lambda^3. \quad . \quad . \quad . \quad . \quad . \quad (81)$$

where  $C_K$ ,  $C_L$ , and  $C_M$  are constants for the given substance which come into operation when the wavelength of the incident radiation is less than the corresponding  $K$ ,  $L$ , and  $M$  critical absorption wavelengths.

Closer examination shows that the  $L$  discontinuity in reality consists of three discontinuities, as shown in Fig. 73, occurring quite close together on the wavelength scale. The  $M$  discontinuity consists of no fewer than five separate absorption edges. The critical absorption wavelengths are very sharply defined and can be determined with a precision at least as great as that with which an emission line can be measured.

For the same absorption edge (e.g.  $K$ ,  $L_1$ , . . .) the relation between the critical frequency  $\nu_c$ ; and the atomic number  $Z$  of the absorbing element is given by the Moseley relation

$$\nu_c = a(Z - b)^2 \quad . \quad . \quad . \quad . \quad . \quad (82)$$

It may be mentioned that in X-ray absorption the atoms act individually, so that an absorption curve drawn for, say, silver bromide will show all the absorption edges both of silver and of bromine. These particular edges can generally be noticed when an X-ray spectrum is photographed; the sudden decrease in absorption on passing through

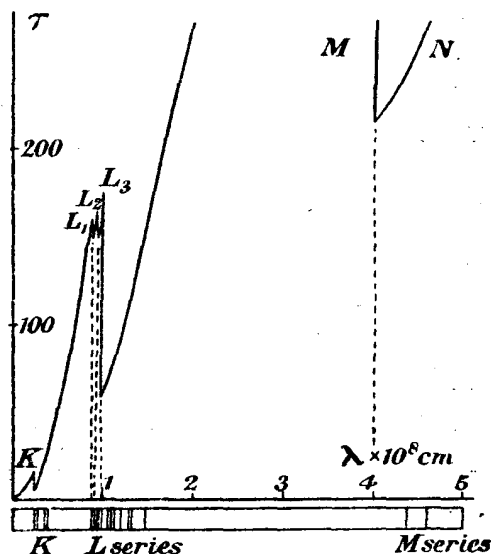


FIG. 73. Curve showing the relation between wavelength, absorption, and secondary spectrum

an edge being accompanied by a corresponding decrease in the chemical action on the silver bromide emulsion.

**92. Secondary radiations. The photo-electric emission.** The energy absorbed by the atom from the incident X-ray beam is re-emitted in two forms. The primary effect is the ejection from the atom of an electron which, by analogy with the similar effect for ultra-violet light, is generally referred to as a photo-electron. This is followed by the emission from the atom of one or more lines of its characteristic X-ray spectrum. The absorbing material thus becomes a source both of photo-electrons and of a secondary X-radiation which is characteristic of its material.

The existence of this characteristic secondary radiation had been recognized for some years before the development of X-ray spectroscopy enabled its wavelength and structure to be investigated. Barkla had found that the radiation from an absorbing sheet contained (in addition to the ordinary scattered radiation) two components, the absorption of which in aluminium was approximately exponential, and which were therefore presumably approximately homogeneous. The coefficients of absorption of the radiations, and therefore the radiations themselves, were characteristic of the material of the absorber. The more penetrating of the two he called the characteristic *K* radiation; the less penetrating the *L* radiation. This nomenclature, as we have seen, is still in use.

The production of the photo-electrons can be studied in detail by allowing the absorption to take place in the gas of a C. T. R. Wilson cloud chamber. The photo-electrons ionize the atoms through which they pass, and their tracks can thus be made visible by the deposition of water drops on the trail of ions which they leave behind. Two such photographs, both due to C. T. R. Wilson, are reproduced in Figs. 17 (Plate I) and 58 (Plate III). The narrow beam of X-rays, which is invisible since it does not ionize directly, passes across each photograph from left to right. The long tracks, starting from this line, are the photo-electrons; the much shorter tracks represent the Compton electrons.

It will be noticed that each of the photo-electron tracks starts off nearly normally to the direction of the primary beam, that is to say, in the direction of the electric vector in the X-ray pulse. This is what we should expect, on the classical theory, since the force exerted on the electrons will be in the direction of the electric field in the pulse. It can also be noticed that the majority of the tracks show an inclination towards the direction in which the X-rays are travelling, showing

that the momentum of the absorbed radiation may also be transmitted to the ejected electron.

The distribution of velocities among the photo-electrons has been investigated by de Broglie <sup>(15)</sup> and by Robinson <sup>(16)</sup> using a magnetic deflection method. Robinson's apparatus is indicated in Fig. 74. An incident beam of homogeneous X-rays enters a highly evacuated chamber through a thin aluminium window  $w$ , and falls on a thin radiator  $R$ , of the substance under investigation. The secondary electrons emerge from  $R$  in all directions, and a limited pencil of them pass through a slit  $S$  immediately above the radiator. The apparatus is placed in the uniform magnetic field produced

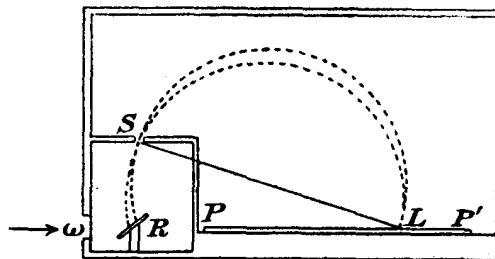


FIG. 74. Apparatus for determining the energies of X-ray photo-electrons

by a pair of Helmholtz coils, which deflects the rays in a semicircle in the plane of the diagram, so that they fall on a photographic plate  $PP'$ , placed film side upwards to receive them. Since the particles are deflected through a semicircle all those having the same velocity will be brought to a line focus at  $L$ , on the photographic plate. The diameter,  $SL$ , of the semicircle can thus be measured with precision, and hence the momentum of the electrons can be deduced from the relation  $\frac{1}{2}SL = mv/eH$ . The corresponding energy  $T$  of the photo-electrons can then be calculated.

Using homogeneous X-rays of frequency  $\nu$ , where  $\nu$  is greater than any of the critical absorption frequencies of the radiator, it is found that the photographic plate is crossed by a series of sharp lines, indicating the presence of groups of electrons of identical energy. Further, it is found that each of these lines can be represented, to a high degree of precision, by a relation

$$T = h\nu - h\nu_c \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

where  $\nu_c$  is one of the critical absorption frequencies of the radiator. If the incident radiation has a frequency less than, say, the critical  $K$  absorption frequency, the corresponding line,  $T = h\nu - h\nu_K$ , is absent.

Now  $h\nu$  is the energy of a photon of the incident radiation, and  $h\nu - T$  is thus the difference between the energy of the absorbed photon and the kinetic energy of the electron ejected. It thus represents the work done in extracting the electron from the atom. We see then

that the electrons in an atom are confined to a comparatively small number of very sharply defined "energy levels," which can be defined, and measured by the energy,  $h\nu_c$ , required to extract an electron from them. We shall find ample support for this view in the following sections.

**93. Conditions for the excitation of the characteristic radiation.** Experiment shows that the critical absorption wavelength is always somewhat shorter than that of the shortest line in the corresponding characteristic radiation. Thus the critical  $K$  absorption wavelength for tungsten is  $0.1785 \text{ \AA}$ ., while the  $K_{\beta_2}$  line of tungsten has a wavelength of  $0.1790 \text{ \AA}$ ., and the  $K_{\alpha_2}$  line  $0.2135 \text{ \AA}$ .. On the other hand, if tungsten is irradiated with homogeneous X-radiation of gradually decreasing wavelength, it is found that no lines of the  $K$  spectrum of tungsten are emitted until the wavelength has been decreased to  $0.1785 \text{ \AA}$ ., when all the lines in the  $K$  spectrum are emitted simultaneously. Thus the secondary  $K$  radiation is not excited unless the quantum energy of the incident radiation reaches that of the  $K$  absorption limit.

Exactly similar considerations apply when the characteristic spectrum is excited directly by the impact of electrons on a target of the material. The matter was investigated in detail by Webster.<sup>(14)</sup> Webster used an X-ray tube with a rhodium anti-cathode and, in order that the voltage applied to the tube should be constant and accurately measurable, constructed a battery of some 12,000 accumulators to provide the current for the tube. He found that the  $K$  radiation was not excited by a potential difference of 23.2 kilovolts, but that all the lines appeared simultaneously when the voltage was increased to 23.3 kilovolts. Thus the minimum energy which an electron must have to excite the  $K$  radiation of rhodium is 23,300 electron-volts.

Since one electron-volt is  $1.60 \times 10^{-12}$  erg, the frequency of the radiation which has a quantum energy equal to 1 eV is  $1.60 \times 10^{-12} / 6.61 \times 10^{-27}$  or  $0.242 \times 10^{15}$  per sec. and the corresponding wavelength,  $c/\nu$ , is  $12.4 \times 10^{-5}$  cm. It is often convenient to specify a homogeneous radiation by its quantum energy measured in electron-volts. Thus the familiar yellow sodium lines may be described as 2.1-volt radiation. To return to Webster's experiments, the wavelength corresponding to a quantum energy of 23.3 kilovolts is  $0.533 \times 10^{-8}$  cm. This is exactly the value found for the  $K$  absorption edge of rhodium. The wavelengths of the  $K$  lines of rhodium range from  $0.616$  to  $0.534 \text{ \AA}$ .. Thus a given line in the spectrum is not excited when the energy of the exciting electron becomes the quantum energy of the line itself, but only when it reaches that of the critical absorption wavelength.

**94. The theory of X-ray spectra.** The results outlined in the previous sections lead to a very definite picture of the electronic structure of the atom, and a convincing theory of the characteristic X-ray spectra. We have seen from experiments on the photo-electric emission that the energy required to extract an electron from the atom assumes a quite small number of perfectly definite values, measured, in each case by  $h\nu_c$ , where  $\nu_c$  is one of the critical absorption frequencies for the atom. These energies can be determined either by absorption measurements, or by measurement of the energies of the ejected photo-electrons; the former being usually employed for the high frequency end of the spectrum, the latter for the low frequencies, where the great absorbability of the radiation makes direct absorption measurements difficult. The system of energy levels for a given atom can thus be mapped out experimentally, without having recourse to any theory of atomic structure.

The absorption of energy, either from an X-ray photon or from an electron exceeding  $h\nu_K$ , where  $\nu_K$  is the critical  $K$  absorption frequency, will enable an electron to be expelled from the lowest, or  $K$ , level of the atom or, of course, from any one of the higher levels. Assuming the expulsion to take place from the  $K$  level this will leave a vacancy which, since the atom is stable in its normal state, will certainly be filled up by some other electron falling into the vacant space either from outside the atom or more probably, and hence more frequently, from one of the higher levels. Let us suppose, for example, that the electron falls into the vacant space from the  $L_1$  level. Energy will thus be liberated, in amount equal to the difference between the work necessary to extract an electron from the  $K$  level, and that necessary to extract an electron from the  $L_1$  level, that is to say, to  $w_K - w_{L_1}$ . On the quantum theory this will be radiated in the form of homogeneous radiation the frequency of which will be given by

$$h\nu = w_K - w_{L_1} = h\nu_K - h\nu_{L_1} \quad . . . . . (84)$$

$\nu$  should thus be the frequency of a line in the  $K$  series of the element. Now, as we have seen, all the quantities in this equation can be determined to an accuracy of at least one part in ten thousand. It provides, therefore, a very searching test of the validity of our reasoning.

The experimental data are most complete for tungsten, since this metal is the usual target in an X-ray tube. The wavelengths of the  $K$  and  $L_1$  discontinuities are respectively  $0.1785 \times 10^{-8}$  cm. and  $1.2136 \times 10^{-8}$  cm. We can put Equation 84 in the form  $1/\lambda = 1/\lambda_K - 1/\lambda_{L_1}$ , where  $\lambda$  is the wavelength of the radiation which should be emitted

TABLE VI

X-ray emission spectra and absorption edges  
The measurements are mainly due to Siegbahn <sup>(9)</sup>

Element.	K series $\times 10^8$ cm.			L series $\times 10^8$ cm.		
	$K_{\alpha_1}$	$K_{\beta_1}$	Ab- sorption limit	$L_{\alpha_1}$	$L_{\gamma_1}$	$L_3$ ab- sorption limit
11. Na	11.88	11.59	—	—	—	—
24. Cr	2.285	2.081	2.066	—	—	—
26. Fe	1.932	1.753	1.739	—	—	—
27. Co	1.785	1.617	1.604	—	—	—
28. Ni	1.654	1.497	1.484	—	—	—
29. Cu	1.537	1.389	1.377	13.31	—	—
30. Zn	1.432	1.293	1.280	12.23	—	—
42. Mo	0.7078	0.6310	0.6185	5.395	—	4.904
47. Ag	0.5583	0.4962	0.4845	4.146	3.515	3.691
50. Sn	0.4896	0.4343	0.4239	3.592	2.995	3.149
74. W	0.2086	0.1842	0.1782	1.473	1.096	1.213
78. Pt	0.1852	0.1637	0.1577	1.310	0.956	1.071
92. U	0.1264	0.1119	0.1066	0.909	0.614	0.721

TABLE VII

Absorption of X-Rays

Wave- length $\lambda \times 10^8$	Mass absorption coefficient $\tau$					
	O	Al	Cu	Mo	Ag	Pb
0.1	—	0.164	0.323	—	1.13	3.78
0.2	0.183	0.269	1.53	4.02	5.75	4.62
0.3	0.240	0.531	4.47	12.7	18.0	13.9
0.4	0.338	1.05	10.1	26.7	38.4	32.7
0.5	0.498	1.91	18.8	48.6	11.0	59.3
0.6	0.746	3.18	31.6	80.7	18.7	91
0.7	1.10	5.00	49.2	18.8	25.6	133
0.8	1.55	7.50	—	27.2	—	—
0.9	2.12	10.3	97	37.5	57	140
1.0	2.87	13.8	133	51	75	77



The value of  $1/\lambda_K - 1/\lambda_{L_1}$  is  $4.792 \times 10^8$ , and the value of  $\lambda$  should thus be the reciprocal of this, that is  $0.2086 \times 10^{-8}$  cm. The wavelength of the  $K_{\alpha_1}$  line is found, by direct measurement, to be

$$0.2086 \times 10^{-8} \text{ cm.}$$

Equally satisfactory agreement is obtained for the other lines of the  $K$  spectrum. Fig. 75 indicates the origin of the different  $K$  lines, the levels being indicated diagrammatically by horizontal lines. This theory, which is due to Bohr, has been applied with equal success to the more complicated  $L$  radiations. In the case of the  $L$  radiation we have three possible final levels corresponding to the three  $L$  discontinuities.

The relations we have been discussing are very vividly illustrated by the "cloud" photograph reproduced in Fig. 17A (Plate I), and due to C. T. R. Wilson. A very faint beam of X-rays is directed against the thin copper plate which is to be seen in the centre of the picture. The beam is so weak that during the exposure only a single quantum has been absorbed by the copper. The track of the ejected photo-electron is seen emerging from the copper plate. An electron falling back into the vacant place in the  $K$  ring of the atom causes the emission of a quantum of copper  $K$  radiation.

This after travelling outwards for a short distance is completely absorbed in a molecule of oxygen, and the resultant photo-electron forms the short track which is visible towards the lower left-hand corner.

**95. Refraction of X-rays. Total reflection.** On the classical theory the refractive index  $\mu$  of a substance for radiation of frequency  $\nu$  is given by

$$\mu^2 = 1 + \frac{e^2}{\pi m} \sum_1^N \frac{n_s}{(\nu_s^2 - \nu^2)}. \quad (85)$$

where  $n_s$  is the number of electrons per unit volume with a natural frequency  $\nu_s$ , the summation extending to all the electrons present.

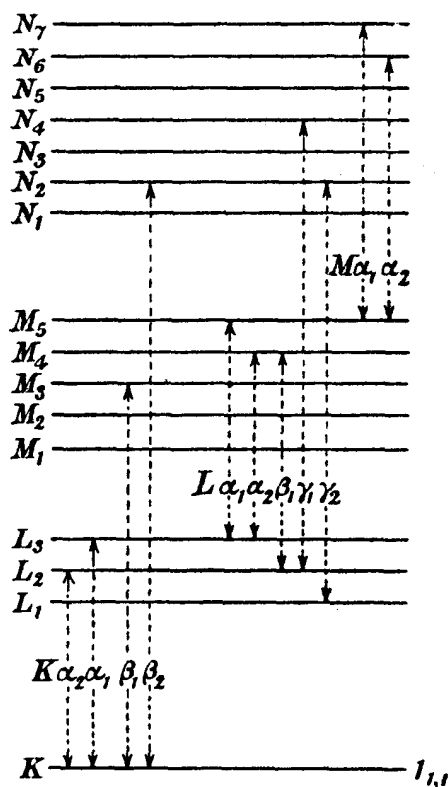


FIG. 75. Diagram showing atomic energy levels, and the production of characteristic X-ray lines

The frequency of X-radiation is usually greater than the natural frequency of the electrons in the atom. The second term is, therefore, negative and the refractive index of any substance for X-rays is less than unity.

A pencil of X-rays falling on a polished surface is therefore passing into an optically less dense medium, and should be totally reflected if the angle which it makes with the surface is sufficiently small. This conclusion was verified by Compton, who found that at glancing angles of the order of  $10'$  or less as much as 90 per cent. of the incident X-radiation was reflected from a well polished glass surface. The value of  $\mu$  for the glass could be determined by measuring the critical angle. The value of  $(1-\mu)$  was found to be  $8.12 \times 10^{-6}$  for radiation of wavelength  $1.54 \text{ \AA}$ . The value calculated from the formula is  $8.14 \times 10^{-6}$ . The value of  $(1-\mu)$  for silver is  $21.5 \times 10^{-6}$  for a wavelength of  $1.28 \text{ \AA}$ . In general  $(1-\mu)$  decreases with decreasing wavelength.

By directing a very fine pencil of X-rays at nearly grazing incidence at the edge of a large angle prism Siegbahn has obtained excellent dispersion spectra corresponding exactly to the optical spectra obtained with an ordinary prism spectrometer. Using the radiation from a tube with an iron target, the four  $K$  lines of the iron spectrum were quite clearly separated.

**96. Determination of X-ray wavelength by diffraction grating.** It is well known that diffraction spectra can readily be seen if ordinary light, from a narrow slit, is reflected, at nearly grazing incidence, from the graduated edge of a polished steel millimetre scale. The demonstration of the fact that X-rays can be optically reflected from a polished surface at nearly grazing incidence suggested to Compton <sup>(17)</sup> that true

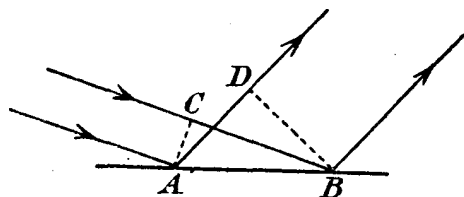


FIG. 76. Diffraction by a reflection grating

X-ray diffraction spectra might be producible by an analogous experiment, using an ordinary reflection grating.

Let  $A$  and  $B$  (Fig. 76) be two adjacent reflecting spaces on an optical ruled grating and  $AC$  an incident wave front. A diffracted ray will exist in the direction  $AD$  if  $CB-AD=n\lambda$ , where  $n$  is an integer. Calling the angle  $CBA$   $\phi$  and the angle  $DAB$   $\phi+\alpha$  (where  $\alpha$  is the angle between the diffracted and the ordinary reflected ray) we have

$$n\lambda = AB\{\cos \phi - \cos (\phi + \alpha)\} = AB\left(\phi\alpha + \frac{\alpha^2}{2}\right),$$

if all the angles are small. Thus using a grating of only 500 lines per cm. and an angle of incidence of  $10'$ , then for  $\text{Mo}_{K\alpha}$  radiation of wavelength  $0.70 \times 10^{-8}$  cm. the value of  $\alpha$  for the first order spectrum ( $n=1$ ) should be  $3.5'$ . This, though small, is quite measurable.

The experimental arrangement is indicated in Fig. 77. The radiation from the target  $X$  of an X-ray tube after passing through the slit  $S_1$  is rendered approximately monochromatic by reflection at the crystal  $C$ , the crystal being set so as to pick out one of the strong emission lines of the target. It then passes through a second slit  $S_2$  on to a speculum grating  $G$  about 5 mm. wide and having 500 lines to the centimetre, which it strikes at a glancing angle of a few minutes only. The directly reflected radiation, for which the angles of incidence and reflection are equal, makes an intense band at  $O$  on the photo-

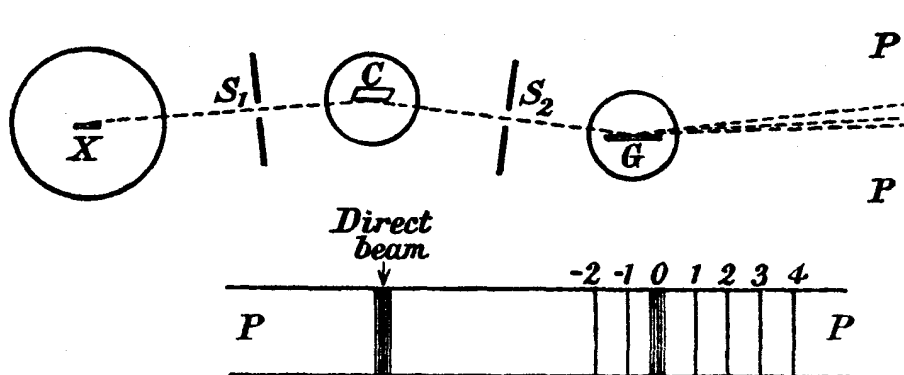


FIG. 77. Determination of X-ray wavelengths by diffraction grating

graphic plate  $PP$ , and is accompanied by diffracted rays corresponding both to positive and negative values of  $n$  as indicated in the lower part of Fig. 77. The angles can be determined from the geometry of the apparatus, and hence the wavelength of the radiation.

In spite of the smallness of the angles to be measured this method of determining X-ray wavelengths has been developed by Bearden (18) and others to give an accuracy of not less than 1 part in 10,000. The results were consistently some 0.25 per cent. higher than those given by the crystal reflection method based on Bragg's calculation of the lattice constant of rock salt. The discrepancy was traced, as we have already seen (§ 88) to an error in the value of the electronic charge  $e$  used in that calculation. It is evident that, if we adopt the value of  $\lambda$  given by the grating method, the calculation of § 88 can be worked in reverse to give a value for  $e$ . This is, in fact, the most accurate method of determining that constant and the value so obtained,  $4.804 \times 10^{-10}$  e.s.u., is the one now generally adopted.

**97. Ionization by X-rays.** The ionization produced by a beam of X-rays in its passage through a gas is the property generally employed to measure the intensity of the beam. X-rays, however, ionize, not directly, but by means of the corpuscular radiation excited in the gas. This is shown very clearly by the photographs reproduced in Figs. 17 and 58. The ions formed along the tracks of the photo-electrons are clearly shown, but there are no ions marking the path of the radiation itself. Ionization is produced both by photo-electrons and by Compton recoil electrons, the former being much the more effective except for short wavelength radiation.

In either case the number of ions produced is directly proportional to the energy absorbed in the gas from the radiation. It thus depends on the coefficient of absorption, i.e. on the wavelength, as well as on the energy of the radiation.

The international unit of quantity of X-radiation is defined as the quantity of X-radiation such that the associated electronic emission per 0.001293 gm. of air produces, in dry air, ions carrying 1 electrostatic unit of electricity of either sign. The unit is known as the *röntgen*. The actual energy corresponding to 1 röntgen will be much less for radiation of long wavelength than for short-wave radiation. It is only when the beams are of the same quality that the ionization currents are a measure of the energy. Experiments seem to show, however, that if the ionization chamber could be made so long that the beam of X-rays was completely absorbed the total ionization produced by the beam would be independent of the quality. Unfortunately this is usually impracticable.

Gases containing elements of high atomic number, and hence of large absorptive power, are much more intensely ionized than air, and can be substituted for the latter in the ionization chamber when radiation of small intensity has to be measured. Methyl iodide is commonly employed for this purpose.

The measurement of the ionization produced in a gas by a beam of X-rays demands certain experimental precautions if the results are to be of any value. The number of photo-electrons produced in a gas is very small compared with the number emitted from a metallic surface under the same conditions. It is very important therefore that the beam should pass through the gas in the ionization chamber without touching the electrodes. The windows through which the beam enters and leaves the chamber should be so far from the electrodes that the electrons emitted by them do not reach the gas between the electrodes. The form of chamber shown in Fig. 5 is suitable for comparative measurements.

For absolute measurements it is necessary to surround the insulated electrode with a guard ring so that the field across the gas may be uniform, and the volume of gas from which the ions are collected well defined. To reduce errors arising from the radiation scattered in the gas which may fall on the electrodes and give rise to secondary emission, the electrodes are made of an element of low atomic number, usually aluminium, though paper covered with graphite is sometimes used. The distance between the beam and the electrodes must be sufficiently long to allow of the complete absorption of the photo-electrons in the gas itself. Two or three centimetres is usually sufficient. The voltage applied must be sufficient to ensure saturation.

Under these conditions very accurate measurements can be obtained both of weak and of intense beams.

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## CHAPTER XI

### THE $\alpha$ -RAYS

**98. Properties and nature of the  $\alpha$ -rays.** The  $\alpha$ -rays from radioactive substances are distinguishable from the other radiations by their absorbability, being completely stopped by less than 10 cm. of air or  $\frac{1}{10}$  mm. of aluminium. They can be detected by their action on a photographic plate, which is, however, very weak, by the ionization they produce in the gases through which they pass, which is very intense, or by the fluorescence they produce on a fluorescent screen. The fluorescence observed through a low-power microscope is found to consist of a succession of scintillations produced by the successive impact of the individual particles in the rays. It has been shown that each particle produces a separate flash on colliding with the screen.

The tracks of  $\alpha$ -particles can readily be observed in a Wilson cloud chamber. Owing to the intense ionization produced the track of an  $\alpha$ -particle appears as a thick white streak, the separate drops being usually indistinguishable. Such a track is shown in Fig. 15 (Plate I).

By applying electric and magnetic fields it can be shown that the  $\alpha$ -rays consist of particles carrying a positive charge. The velocity of the particles and the value of the ratio  $e/m$  can be deduced from the deflections in the usual ways. The value of  $e/m$  is the same for all the  $\alpha$ -particles no matter what their source and is equal to 4823. The initial velocity depends on the radio-active substance from which they are ejected. These velocities range from  $1.45 \times 10^9$  to  $2.2 \times 10^9$  cm. per sec.

**99. Nature of the  $\alpha$ -particle.** The ratio of charge to mass for a hydrogen atom when carrying a single electronic charge is 9649 e.m.u. per gm. The value for an  $\alpha$ -particle is 4823 or half that of the hydrogen atom. If the charges were the same the particle would thus be twice the mass of the hydrogen atom, and hence probably a hydrogen molecule. It will be shown later that the particle carries double the charge carried by an electron; its real mass is therefore four times that of a hydrogen atom, and thus corresponds very closely with that of the helium atom (atomic weight 4.0).

The proof of the identity of the particles with helium has been rendered complete by an experiment due to Rutherford. A very thin-

walled glass tube *A* (Fig. 78) was sealed into an outer tube *B* which was highly exhausted and connected to a small discharge tube *C*. To prove that there was no connection between *A* and *B* the former was filled with helium under pressure and left for some hours. No trace of the helium spectrum was obtained in the discharge tube *C*. The helium was carefully removed and radium emanation was passed into *A*, and allowed to stand. The glass walls of *A* were sufficiently thin to allow the  $\alpha$ -particles from the emanation to pass into *B* where they were stopped by the outer walls. Under these circumstances the helium spectrum became visible in *C* in a few hours, and became brighter as the experiment was continued. As the  $\alpha$ -rays were the only particles entering *B* during the experiment it is clear that the  $\alpha$ -particles must be atoms of helium.

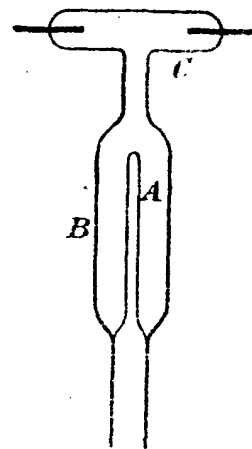


FIG. 78. Rutherford's apparatus for demonstrating the nature of  $\alpha$ -particles

**100. The counting of  $\alpha$ -particles.** Owing to its high speed and comparatively large mass, a single  $\alpha$ -particle possesses appreciable kinetic energy. The kinetic energy of the  $\alpha$ -particle from radium *C'* for example is  $1.2 \times 10^{-5}$  erg or 7.7 million electron volts. This is comparable with the energy in a faint flash of light or a weak sound, and it is thus possible to detect the advent of a single particle.

The simplest method is to allow the particles to fall on a fluorescent screen: each  $\alpha$ -particle producing a scintillation at its point of impact. The most suitable material is zinc sulphide in which about 0.01 per cent. of copper has been incorporated. The screen, covered thinly with the powdered sulphide, is viewed by a low-power microscope of large aperture. Observations must be made in complete darkness, as the scintillations are very faint. Under good conditions a trained observer can count scintillations at the rate of about 20 per minute, and will record about 95 per cent. of the scintillations which actually occur. Most of the earlier experiments on  $\alpha$ -particles were carried out in this way.

The method is slow, and very fatiguing to the observer. It has largely been superseded by electrical methods, which, by the use of electron valve circuits, can be made completely automatic, so that the final count is recorded by a mechanical counter. A single  $\alpha$ -particle from radium *C'* produces rather more than 200,000 pairs of ions in air, and thus permits the passage across an ionization chamber of about

$10^{-4}$  e.s.u. of charge. This is measurable, directly, with a sensitive electrometer, but the period of such an instrument is necessarily long, and the rate of counting unduly slow.

Counting can be speeded up by replacing the electrometer by an electrometer valve (§ 8). Assuming that the capacity of the ionization chamber and grid is 50 e.s.u. the production of 200,000 pairs of ions in the ionization chamber will raise the potential of the grid by  $6 \times 10^{-4}$  volt. Thus each  $\alpha$ -particle entering the chamber produces a signal strength of the order of 0.6 millivolts, which can be amplified in the usual way.

After amplification, the output can be used to work an electromagnetic counter of the type used for recording telephone calls. The entry of a single  $\alpha$ -particle into the ionization chamber is thus recorded automatically on a dial. Owing to the inertia of its moving parts, the mechanical counter takes an appreciable time (about  $\frac{1}{20}$  second) to reset itself. Thus two  $\alpha$ -particles entering the chamber within  $\frac{1}{20}$  second would be recorded as one only. The counting can be speeded up by the use of a special type of valve relay, known as a thyratron, which allows counting to proceed at the rate of 400 particles per second.<sup>(1)</sup>

An alternative method is to pass the output impulses through an oscillograph of very short period. Each  $\alpha$ -particle then produces a deflection of the oscillograph, which can be recorded on a moving photographic film. The advantage of this method is that the deflections are roughly proportional to the ionization produced by the  $\alpha$ -particle in the ionization chamber, and it is thus possible to discriminate between particles of varying energies.

As early as 1908, Rutherford and Geiger had counted  $\alpha$ -particles electrically, using the phenomenon of ionization by collision (§ 29) to multiply the small primary ionization produced by the  $\alpha$ -particle. The ionization chamber consisted of a charged metal cylinder *L* (Fig. 79), with an insulated axial wire electrode *M* connected to one plate of an electrometer. The gas pressure in the chamber, and the applied potential difference were adjusted so that ionization by collision occurred, when a supply of ions was produced by the entry of an  $\alpha$ -particle. The original current could thus be increased many fold, and a relatively insensitive, but short period, electrometer could be employed. To prevent the accumulation of charge on the needle, the latter is earthed through a high resistance, so that after each rush of current the charge leaks to earth.

The method has been developed by Geiger, and others, and in the



form of the "Geiger counter" provides the most sensitive device available for detecting ionizing particles. The central wire  $M$  is replaced by a short pointed platinum needle, which is maintained at a voltage just below the critical discharging potential. The formation of a few ions in the sensitive volume of gas near the point initiates a discharge, the magnitude of which seems to be independent of the number of the ions which "trigger" it off. The discharge is quenched by interposing a high resistance between the chamber and the source of potential.

The sensitivity of the Geiger counter depends on the sharpness of the point, and on the nature and pressure of the gas. With a sufficiently fine point, the instrument is capable of detecting the passage of a single  $\beta$ -particle. If it is wished to make it responsive only to  $\alpha$ -particles, the tip of the wire is fused into a small sphere.

**101. Determination of the charge on an  $\alpha$ -particle.** If the number of  $\alpha$ -particles emitted per second from a known mass of radioactive substance is counted, and the total positive charge emitted by the substance in the same time is measured, the charge on a single  $\alpha$ -particle can be determined. The experiments were carried out by Rutherford and Geiger using the apparatus sketched in Fig. 79. A trace of the

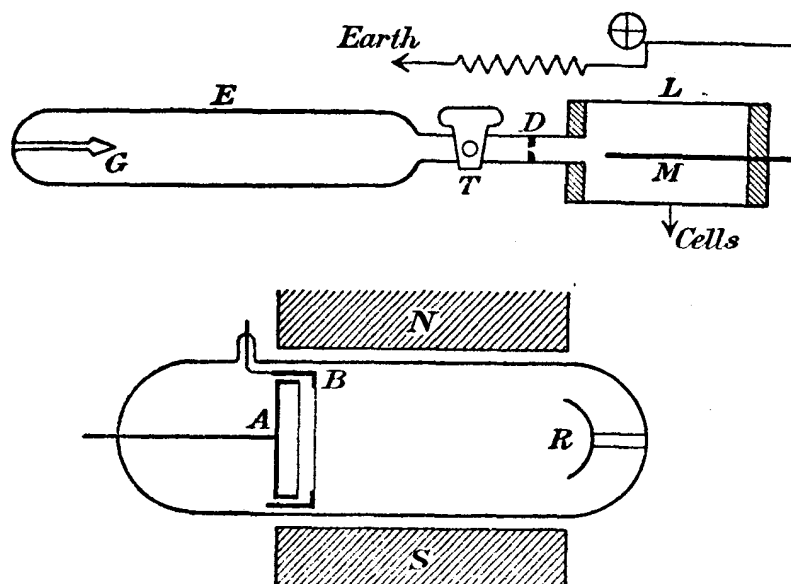


FIG. 79. Rutherford's apparatus for determining the charge on an  $\alpha$ -particle

radio-active substance is deposited on the metal cone  $G$ , and a beam of  $\alpha$ -particles passes, by way of a large bore tap,  $T$ , through a stop  $D$ , which subtends a known solid angle  $\Omega$  at the source. They then enter the ionization chamber  $L$ , the inner electrode  $M$  of which is connected

to an electrometer. The entrance of each  $\alpha$ -particle into the chamber is marked by a kick of the electrometer needle. If  $n$  is the number of kicks recorded per second, the total number of  $\alpha$ -particles emitted per second by the source is  $4\pi n/\Omega$ . A gram of radium, free from its dissociation products, emits  $3.72 \times 10^{10}$   $\alpha$ -particles per second.

In the second part of the experiment the  $\alpha$ -particles from a source  $R$  are collected in a Faraday cylinder. To avoid loss of charge through ionization currents, the chamber  $A$  (Fig. 79) is enclosed in a high vacuum. It is shielded from electrostatic disturbances by an earthed aluminium leaf  $B$ . Surfaces exposed to  $\alpha$ -radiation give off large numbers of slow negative electrons, known as  $\delta$ -rays. If these were allowed to escape from the Faraday cylinder, they would leave it with a spurious positive charge. The apparatus is placed between the poles of a strong magnet,  $NS$ , which, by coiling the  $\delta$ -ray tracks into small spirals, prevents them from leaving their place of origin.

The strengths of the sources  $G$  and  $R$  are compared by measuring the ionizations they produce with an electroscope. If the angle subtended by  $A$  at the source  $R$  is measured, the number of  $\alpha$ -particles entering  $A$  is known. The charge on each particle can thus be deduced. Rutherford obtained the value  $9.3 \times 10^{-10}$  e.s.u.

Regener, using the scintillation method to determine the number of  $\alpha$ -particles, obtained a value for the charge on a single  $\alpha$ -particle of  $9.58 \times 10^{-10}$  e.s.u. It will be seen that the values obtained both by Regener and by Rutherford are each very close to twice the electronic charge. The agreement of the two values was taken as evidence that every  $\alpha$ -particle produces a scintillation when it strikes a fluorescent screen.

**102. Passage of the  $\alpha$ -particles through matter.** If a narrow pencil of  $\alpha$ -particles is formed in air by placing a small diaphragm in front of a thin layer of some radio-active material—so thin that the absorbing effect of the layer itself on the  $\alpha$ -particles it emits is negligible—and the particles are viewed by a fluorescent screen, which is gradually withdrawn farther and farther from the source, it is found that the number of  $\alpha$ -particles falling per second on the screen remains constant until a certain critical distance is reached. If this distance is exceeded, the number of scintillations falls off with such rapidity that they entirely disappear within the next few millimetres.

Similarly, if the screen is kept fixed close to the diaphragm and very thin sheets of aluminium, mica or other substance are placed successively in front of the diaphragm, the number of  $\alpha$ -particles reaching the screen remains constant until some definite thickness of material has

been placed in the path of the particles. This critical thickness is called the *range* of the  $\alpha$ -particle in the absorbing material.

The range of an  $\alpha$ -particle depends on the particular radio-active substance from which it is emitted, all  $\alpha$ -particles from the same substance having the same range. It also depends on the nature of the absorbing material. For the same material the range depends only on the mass of material traversed by the rays. Thus for a given gas the range is inversely proportional to the density of the gas. The range of an  $\alpha$ -particle is usually specified as the distance the particle will penetrate through air at a pressure of 760 mm. and a temperature of  $15^{\circ}$  C. The range of the  $\alpha$ -particles emitted by radium is 3.4 cm. and of those from radium C' 6.9 cm. in air under standard conditions.

These results can be illustrated by experiments with a cloud chamber. If a speck of radio-active material is enclosed in the chamber, the tracks of the particles are clearly visible after exposure and can be photographed and measured. Fig. 80 (Plate IV) shows the result obtained using a mixture of thorium C and thorium C'. The existence of two definite ranges and the practical equality of the lengths of track of particles of the same kind are very clearly brought out.

Since the number of  $\alpha$ -particles in a beam remains constant up to the end of the range, the absorption of the rays must be due to a gradual absorption of the energy of the particles, which ultimately leaves them with insufficient energy to produce either scintillations or ionization. This conclusion can be verified by measuring the velocity of the particles after passing through various absorbing sheets of matter. The rays from a wire  $R$  coated with a very thin layer of radio-active material pass down a long air-tight glass tube  $T$  (Fig. 81) and are

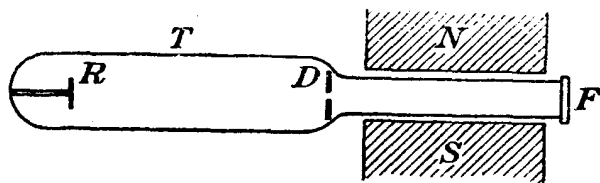


FIG. 81. Determination of absorption of velocity of the  $\alpha$ -particles

formed into a narrow pencil by the lead diaphragm  $D$ . This beam then continues between the poles of an electromagnet  $NS$ , and falls on a fluorescent screen  $F$ . From the geometry of the apparatus and a measurement of the shift produced in the fluorescent spot when the magnetic field is applied, the radius of curvature of the rays in the magnetic field can be calculated, and using the equation  $\rho = vm/He$  (29)  $v$  can be calculated since the ratio of  $e/m$  for the  $\alpha$ -particles is known.



admit the passage of the  $\alpha$ -rays is connected to a battery. The saturation current between the electrodes measures the mean intensity of ionization produced by the beam in the portion  $AB$  of its path. By altering the distance between  $AB$  and  $R$  the intensity at different parts of the path can be determined.

A more accurate and convenient design is shown in Fig. 83. The source  $R$  is placed at one end of a tube, some 10 cm. in length, the

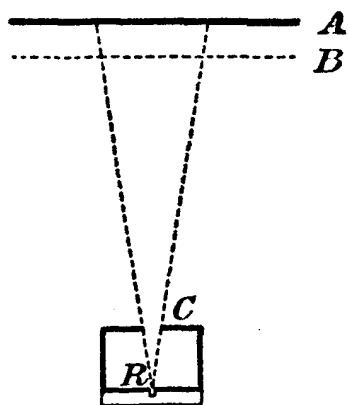


FIG. 82. Bragg's method of determining the specific ionization of  $\alpha$ -particles

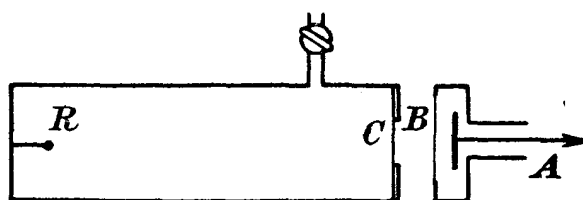


FIG. 83. Accurate method of determining specific ionization

other end of which is closed with a stop  $C$ , covered with thin metal leaf.  $AB$  is a shallow ionization chamber. The tube is air-tight and can either be completely evacuated or filled with gas at a known pressure. If the tube contains air at a pressure  $p$  mm., the effective distance that the particles have traversed before reaching the stop  $C$  is  $RC \times p/760$  cm. and can thus be adjusted without moving the apparatus. The effect of the foils at  $C$  and  $B$  can be measured and allowed for.

**104. The straggling of the  $\alpha$ -particles.** The experimental curves for the homogeneous rays from polonium and from radium  $C'$  are shown in Fig. 84. Their general form agrees quite closely with the curve given by (87) except that the maximum is not so sharp, and the fall to zero not so precipitous as the simple formula would lead us to expect. This is due to what may be called the straggling of the  $\alpha$ -particles. The absorption of energy from the particle is due mainly to the ionization it produces. This will depend on the number of molecules it strikes along its path, and to the particular way in which it strikes them. Since the molecules in a gas are widely separated, some  $\alpha$ -particles will strike rather more, others rather fewer than the average number, in passing through a centimetre of air, and thus the actual distance from the source at which their energy is completely

expended will be somewhat different for different particles. This naturally causes a certain rounding off of the maximum in the experimental curves, which, of course, give the mean effect for a large number of particles.

Owing to straggling the range of an  $\alpha$ -particle is not quite definite. To avoid this difficulty an *extrapolated range* is usually employed. This is obtained by producing to zero the straight descending portion of the curve indicated by the dotted line in Fig. 84.

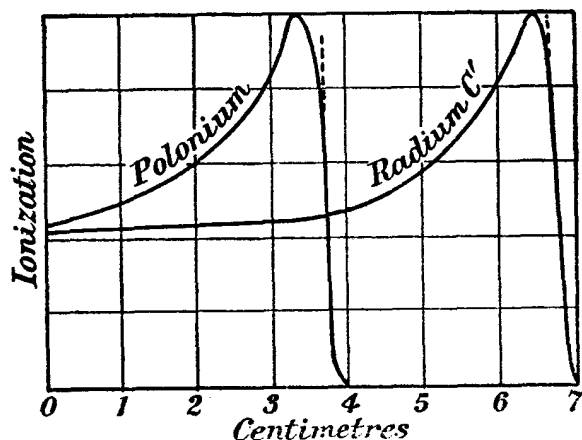


FIG. 84. Variation with range of the specific ionization of  $\alpha$ -particles

The ionization curve for a short-range  $\alpha$ -particle is almost identical with the corresponding portion of the curve for a long-range particle. Thus the curve for polonium (range 4 cm.) is practically identical with the last 4 cm. of the curve for the radium C' particles (range 7 cm.). The range of a beam of homogeneous  $\alpha$ -particles thus depends only on  $V$ , the velocity of projection. Putting  $x=0$  in (86) we have

$$V^3 = aR \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89)$$

This is Geiger's law, and while not quite exact, is sufficiently so for many purposes.

**105. The stopping power of substances.** The range of the  $\alpha$ -particles in any gas can be determined accurately by the method of Fig. 83. The stopping power of solid materials can be determined with the same apparatus, the absorbing foils being inserted between the stop  $C$  and the window  $B$  of the ionization chamber. Thus if the insertion of a given sheet of mica is found to reduce the range in air of the  $\alpha$ -particles from radium C' from 7 cm. to 5 cm., the sheet would be said to have a stopping power of 2 cm.

The ratio of the mass of a column of air of unit cross-section to the mass per unit area of the substance which produces the same diminu-

tion in the speed of the  $\alpha$ -particle is called the *stopping power* of the substance.

It was found better to express the stopping power in terms of the number of atoms traversed rather than in terms of the mass. The transformation can easily be made. Thus the stopping power of silver is 0.37; that is to say, in passing through a given mass of silver the velocity of the  $\alpha$ -particle is only reduced by 0.37 of the decrease produced in passing through an equal mass of air. But for equal masses of air and silver the former contains  $108/14.4$  times the number of atoms, hence atom for atom silver stops  $0.37 \times 108/14.4 = 2.8$  times as much as air. This is called *the stopping power of the atom*. Experiment has shown that except in the case of very light elements *the stopping power per atom is proportional to the square root of the atomic weight*; the mean value of the stopping power over the root of the atomic weight being about 0.26.

The stopping power of an atom is independent of its state of chemical combination, being the same in compounds as in the free state. Thus the stopping power of a compound can be calculated from that of its constituents.

**106. Alpha-ray spectra.** The statement that the  $\alpha$ -particles emitted by a particular radio-active substance are projected with identical energy requires some modification. In addition to the normal  $\alpha$ -particles, Ra C', Th C', and Ac C' also emit occasional  $\alpha$ -particles of range appreciably longer than the normal. The proportion of such particles is very small, and does not exceed one long-range particle to 50,000 normal particles.

To determine the exact energy of emission of these long-range  $\alpha$ -particles Rutherford<sup>(2)</sup> constructed a powerful electromagnet capable of bending the path of the  $\alpha$ -rays into a semicircle, and thus made it possible to use the focusing method already described in § 55. The source of  $\alpha$ -rays is placed at one end of a semicircular channel, cut in the magnetic poles in a plane at right angles to the magnetic field, and a small counting chamber is placed diametrically opposite to it, at the other end of the channel. The channel is completely exhausted of air, so that the particles lose no velocity in passing round the channel.

If  $\rho$  is the radius of the semicircular canal, a particle will only be able to traverse the canal if the radius of curvature of its path in the magnetic field  $H$  is identical with that of the canal, that is if the particle has a velocity  $V$  equal to  $H\rho E/m$ , where  $E/m$  is the ratio of the charge to the mass of an  $\alpha$ -particle. By gradually varying the magnetic

field, successive groups of  $\alpha$ -particles can be brought in turn into the counter. The corresponding value of  $H$  enables the velocity, and hence the kinetic energy, of the particles to be determined, the accuracy being of the order of one in ten thousand.

By a similar method Rosenblum had previously shown that the normal  $\alpha$ -rays from certain other substances were not strictly homogeneous. Thus radium emits two groups of particles of energy 4.87 and 4.68 million volts respectively, while the  $\alpha$ -rays from Th C are divisible into six definite groups, with a maximum difference of energy of about 0.6 million volts. The  $\alpha$ -rays from such substances are said to show *fine structure*. These results, as we shall see later, furnish important clues as to the structure of atomic nuclei.

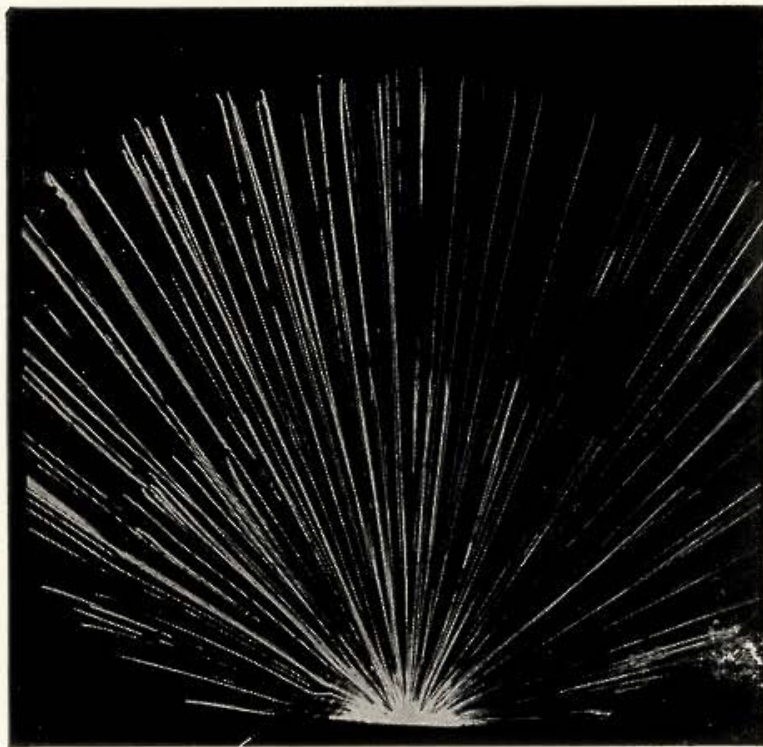
**107. The scattering of the  $\alpha$ -particles.** As is well shown in Fig. 80 (Plate IV), the track of the average  $\alpha$ -particle is approximately a straight line from beginning to end. A few  $\alpha$ -particles, however, suffer appreciable deflections. For example, the track shown in Fig. 15 shows two such deflections, the first about two-thirds of the way along, and the second near the end of its track. The deflections are generally more numerous near the end of the track, where the velocity is small.

The scattering of  $\alpha$ -particles can be investigated by allowing a narrow pencil of the rays to fall on a thin sheet of some material, and investigating the number of  $\alpha$ -particles emerging at various angles to the incident pencil by means of a fluorescent screen. Thus Geiger found that using a thin sheet of gold leaf of stopping power 3.68 cm. about one particle in about 8000 suffered a deflection greater than  $90^\circ$  and re-emerged on the side on which it entered. The *average* deflection was only of the order of  $7^\circ$ , and increased proportionally with the thickness of the scattering foil.

If we assume that the atomic radius is of the order of  $10^{-8}$  cm. it is clear that an  $\alpha$ -particle in traversing several centimetres of air must pass through some thousands of atoms of the gas. Only a minute proportion of such encounters, however, produce any appreciable deflection of the  $\alpha$ -particle. It is difficult to avoid the conclusion that the greater part of the atomic volume is occupied only by electrons, which are too light to deflect the comparatively massive  $\alpha$ -particle, and that the main mass of the atom must be concentrated in a particle, or nucleus, much smaller than the atomic structure as a whole. The nuclear theory of the atom was first propounded by Rutherford <sup>(3)</sup> in 1911, and has revolutionized our ideas of atomic structure.

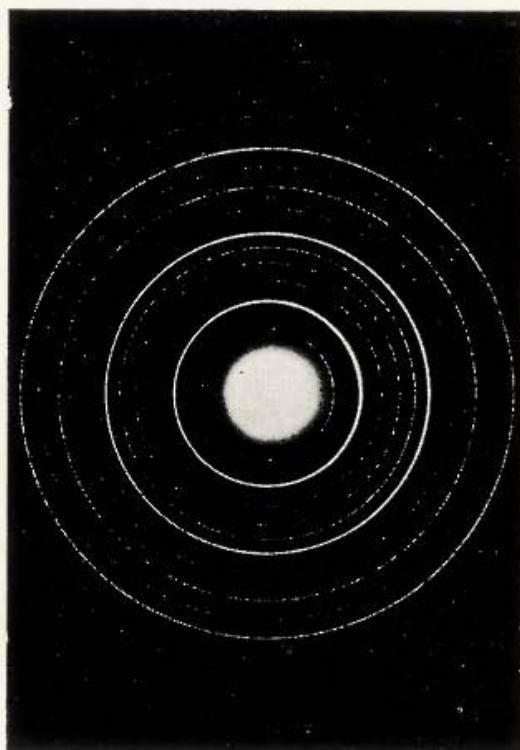
**108. Rutherford's theory of scattering.** Let us assume that the mass of the atom is concentrated on a nucleus, the dimensions of





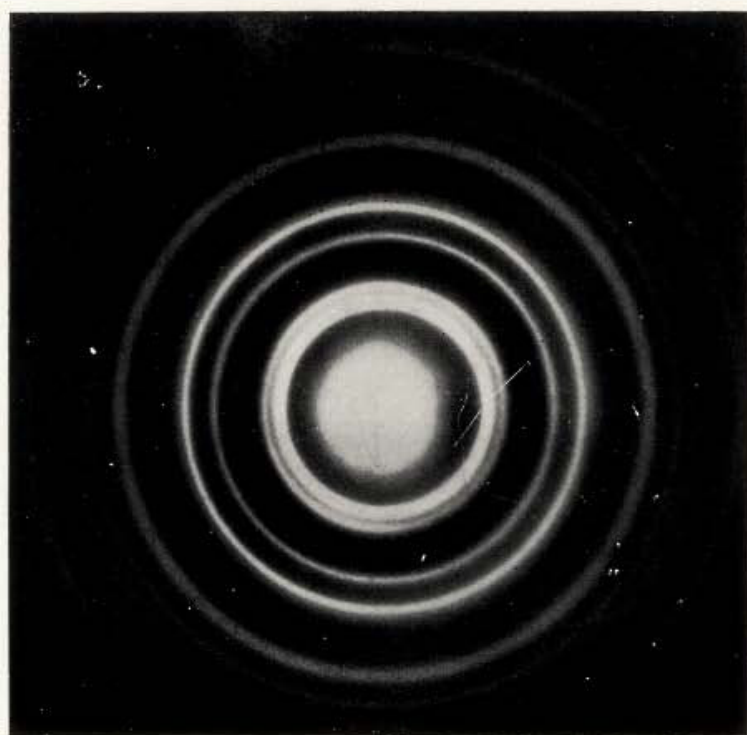
*Lord Rutherford, J. Chadwick and C. D. Ellis.*

FIG. 80. TRACKS OF  $\alpha$ -PARTICLES FROM THORIUM C AND THORIUM C'.



*G. I. Finch.*

FIG. 92A. ELECTRON DIFFRACTION  
PATTERN OF AMMONIUM CHLORIDE.



*P. Thomson.*

FIG. 92B. DIFFRACTION OF ELECTRONS BY A METAL FILM.

which are negligible in comparison with the radius of the atom. We will assume that the positive charge in the atom also resides in this nucleus, and is equal numerically to  $Ze$ . Since the atom is neutral as a whole, there must be  $Z$  negative electrons outside the nucleus, arranged in some manner to be discussed later.  $Z$  is thus the atomic number of the atom. The atomic system thus resembles in many respects a planetary system, the nucleus playing the part of the central sun. The  $\alpha$ -particle, since it has lost its two planetary electrons, will consist of a nucleus only of atomic mass 4, and charge  $2e$ .

The electrons, on account of their small mass, cannot appreciably affect the motion of the  $\alpha$ -particle. It also turns out that during a collision the colliding nuclei come within such a minute distance of each other that most of the deflection occurs while the  $\alpha$ -particle is well within the innermost ring of electrons. The electrical screening of the nuclear charge by the planetary electrons is therefore also negligible.

The  $\alpha$ -particle and the nucleus, being both positively charged, repel each other with a force which, at distances which are large compared with the radius of the nucleus, will vary inversely as the square of the distance. It is

assumed that the observed deflections are due to this force. The problem thus resembles that of two gravitating bodies, except that the force involved is one of repulsion, not attraction. If the mass of the deflecting nucleus is large compared with that of the  $\alpha$ -particle, so that the motion communicated to the nucleus is negligible, the path of the particle will be a hyperbola, with the nucleus at one focus.

Let  $S$  (Fig. 85) be the nucleus, supposed fixed,  $PO$  and  $OP'$  the initial and final directions of the  $\alpha$ -particle,  $V$  its initial velocity, and  $v$  its velocity in passing through the apex  $A$  of its trajectory. Let  $p$  be the length of the perpendicular  $SN$  let fall from  $S$  on the initial direction of the  $\alpha$ -particle.

By the principle of conservation of angular momentum

$$pV = SA \cdot v \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

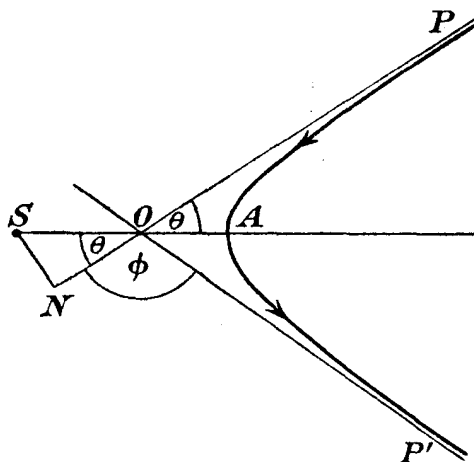


FIG. 85. Illustrating Rutherford's theory of  $\alpha$ -ray scattering

and by the conservation of energy

$$\frac{1}{2}MV^2 = \frac{1}{2}Mv^2 + \frac{Ze \cdot E}{SA},$$

$M$  being the mass and  $E$  the charge of the  $\alpha$ -particle.

Hence

$$v^2 = V^2 \left( 1 - \frac{b}{SA} \right),$$

where

$$b = \frac{2Ze \cdot E}{MV^2} = \frac{Ze \cdot E}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

where  $T$  is the initial kinetic energy of the  $\alpha$ -particle. Combining this equation with (90) we have

$$p^2 = SA(SA - b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

If we write  $S\hat{O}N = P\hat{O}A = \theta$ , the eccentricity of the hyperbola is  $\sec \theta$ , and by the geometry of the conic,  $SO = OA \sec \theta$ . Thus

$$\begin{aligned} SA &= SO + OA = p \operatorname{cosec} \theta (1 + \cos \theta) \\ &= p \cot \theta / 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (93) \end{aligned}$$

Substituting in (92) we have

$$b = 2p \cot \theta.$$

The resultant deflection  $\phi$  of the  $\alpha$ -particle is the angle  $NOP'$  between the two asymptotes, or  $\pi - 2\theta$ , whence finally

$$\cot \frac{\phi}{2} = \frac{2p}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad (94)$$

Thus, to be deflected through an angle greater than  $\phi$ , the initial direction of the  $\alpha$ -particle must pass within a distance  $p$  of the nucleus, given by (94).

Let  $t$  be the thickness of the scattering foil, which we will suppose so small that the probability of the same  $\alpha$ -particle suffering more than one deflection is negligibly small. Let  $n$  be the number of atoms per unit volume of the material of the foil. The number of atoms contained in a cylinder of length  $t$  and base  $\pi p^2$  is  $\pi p^2 nt$ . This measures the probability that the  $\alpha$ -particle will pass within a distance  $p$  of a nucleus, and thus suffer a deflection greater than the corresponding value of  $\phi$ . The probability  $\rho$  of a deflection exceeding  $\phi$  is thus given by

$$\rho = \frac{1}{2} \pi n t b^2 \cot^2 \frac{\phi}{2}$$

If we are dealing with a sufficiently large number of particles  $\rho$  is obviously also the fraction of the whole number which suffers displacements greater than  $\phi$ .

Similarly the fraction  $d\rho$  of particles whose deflections lie between  $\phi$  and  $\phi + d\phi$  is given by

$$\begin{aligned} d\rho &= 2\pi p n t \cdot d\rho \\ &= \frac{1}{4} \pi n t b^2 \cot \frac{\phi}{2} \operatorname{cosec}^2 \frac{\phi}{2} \cdot d\phi. \end{aligned}$$

Hence the number  $q$  of  $\alpha$ -particles which will reach a screen subtending a small solid angle  $\omega$  at the scattering foil, in a direction making an angle  $\phi$  with the direction of the incident  $\alpha$ -rays, can be shown to be given by

$$q = \frac{1}{16} Q n t b^2 \omega \operatorname{cosec}^4 \frac{1}{2} \phi \quad . \quad . \quad . \quad . \quad . \quad (95)$$

where  $Q$  is the number of incident  $\alpha$ -particles.

**109. Experimental verification of the theory of scattering.** A very elegant method of verifying the theory was devised by Chadwick.<sup>(4)</sup> The source of the  $\alpha$ -particles was placed at  $A$  on the axis of a brass plate in which had been cut a narrow circular zone  $PP$ . The scattering substance (very thin leaves of copper, silver, or gold) covered this clear zone, while the observing screen was placed, also on the axis of the plate, at  $S$ . The apparatus can be visualized from the section in Fig. 86 by imagining the diagram to be rotated about the axis  $AS$ .

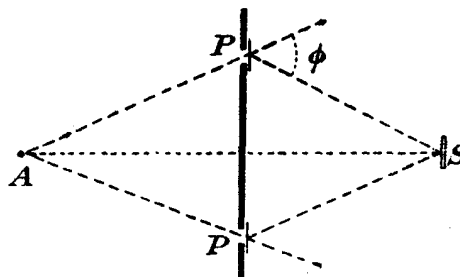


FIG. 86. Chadwick's method of measuring  $\alpha$ -ray scattering

Thus all the particles which reach  $S$  must have been scattered through the same mean angle  $\phi$ . The angle can be varied by varying the distance  $AS$ . The variation of scattering with the kinetic energy of the particles can be investigated by placing absorbing screens of known stopping power in front of the source  $A$ . The apparatus is enclosed in a highly evacuated box to eliminate absorption in the air.

Knowing the dimensions of the screen  $S$  and of the other parts of the apparatus, it is easy to calculate, from (95), what fraction of the particles incident on the foil should be scattered to the screen. The predictions of the theory were fully borne out by the experimental results.

If the number  $Q$  of  $\alpha$ -particles falling on the scattering annulus is found, the only unknown in Equations (91), (95) is the factor  $Z$ , the number of charges on the scattering nucleus.  $Q$  can obviously be calculated if the number of particles per unit solid angle emitted by  $A$  is known. This can be measured by removing  $PP$ , and so allowing the incident beam to fall directly on the screen  $S$ . In this way Chadwick found for copper, silver, and gold values of  $Z$  equal respectively to 29.3, 46.3, and 77.4. The atomic numbers of these elements are 29, 47, and 78. Moseley's suggestion that the charge on the nucleus of an element is equal to its atomic number is thus confirmed.

The closest distance of approach between an  $\alpha$ -particle and a nucleus for a head-on collision is obviously given by  $b$ , or  $Ze \cdot E/T$ . It is thus directly proportional to the atomic number of the scattering element, and inversely proportional to the energy of the incident particle. For oblique collisions it is given by (93). A swift  $\alpha$ -particle will thus penetrate more closely to a nucleus than a slow one, and more closely to a nucleus of low than to one of high atomic number. The closest distance of approach in Chadwick's experiment was of the order of  $10^{-12}$  cm. for the copper foil. The accuracy with which the experimental results agreed with the theoretical predictions shows that at distances equal to or greater than  $10^{-12}$  cm. the nucleus behaves as a point charge and the law of force is that of the inverse square.

**110. Scattering by light elements. Departure from the inverse square law.** For elements of small atomic mass it is necessary to take into account the momentum transmitted by the collision to the deflecting nucleus itself. The necessary modification in the theory has been made by Darwin.<sup>(5)</sup> The correction is of the order of  $2(M/m)^2$ , where  $M$  is the mass of the  $\alpha$ -particle and  $m$  that of the nucleus. It amounts to about 4 per cent. for aluminium, and is greater for elements of still lower atomic number.

Using the annular ring method just described, Bieler<sup>(6)</sup> found that for aluminium and magnesium the number of  $\alpha$ -particles scattered at any given angle was less than that predicted by the theory, the effect being most marked for the faster particles and for large angles of scattering where the particles have approached more closely to the nucleus. The closest distance of approach of an  $\alpha$ -particle from radium  $C'$  to an aluminium nucleus is of the order of  $5 \times 10^{-13}$  cm.

At distances of this order of magnitude the electrical force between the particles is, therefore, less than that calculated from the inverse square law. Bieler's experiments show that an attractive force (varying as the inverse fourth or fifth power of the distance) is super-

posed upon the ordinary Coulomb law of repulsion. Since the attractive force increases more rapidly with diminishing distance than the Coulomb repulsion, at sufficiently small distances there would actually be attraction between the particles. It is these attractive forces, which have been christened *exchange forces*, which presumably retain within the atomic nucleus the positively charged particles of which it is largely composed.

**111. Scattering of  $\alpha$ -particles in hydrogen and helium.** In the case of hydrogen we are dealing with the collision of an  $\alpha$ -particle with a nucleus less massive than itself. The effect of the collision on the path of the  $\alpha$ -particle is comparatively small; the maximum deflection is, for example, only  $14^\circ$ . On the other hand the hydrogen nucleus is projected forward by the impact with considerable velocity. Assuming the ordinary laws of elastic collision the velocity  $u$  with which a hydrogen particle of mass  $m$  will be projected by the impact of an  $\alpha$ -particle of mass  $M$  and velocity  $V$  is given by

$$u = 2V \frac{M}{M+m} \cos \theta = \frac{8}{5}V \cos \theta \quad . \quad . \quad . \quad (96)$$

where  $\theta$  is the angle between the direction of the hydrogen atom and the initial direction of the  $\alpha$ -particle. The velocity of projection of the hydrogen particles may thus exceed that of an  $\alpha$ -particle, and they acquire sufficient energy to ionize the gas through which they pass, and to produce scintillations on a fluorescent screen. They may be observed by passing a beam of  $\alpha$ -particles down a tube containing hydrogen and having a fluorescent screen at its further end. If absorbing sheets of known stopping power are interposed in front of the screen it is found that after sufficient material has been used to cut off all the  $\alpha$ -rays, occasional scintillations are observable until the total stopping power of the absorbing sheets is approximately four times that required to stop the  $\alpha$ -particles themselves. The nature of the particles producing these scintillations can be determined by measuring the ratio of the charge to the mass, in the usual way. The value obtained,  $10^4$  e.m.u. per gm., identifies them as hydrogen atoms carrying a single positive charge. They thus consist simply of hydrogen nuclei, or *protons*.

It is known that the range of a proton is practically the same as that of an  $\alpha$ -particle of equal velocity. Hence, assuming Geiger's relation (89) the range  $R_p$  of the proton projected by an  $\alpha$ -particle of range  $R_\alpha$  should be given by

$$R_p = R_\alpha (8/5)^3 \cos^3 \theta.$$

The maximum range of the proton should thus be  $(1.6)^3$  or 4.1 times that of the  $\alpha$ -particle. This is in agreement with the observations just described.

Experimentally it is more convenient to study the distribution of the protons than to measure the comparatively small deflections of the  $\alpha$ -particles themselves. Since the forces binding the hydrogen atoms in a chemical compound are negligible in comparison with those generated in a collision, the hydrogen may be used in the form of paraffin wax, and experiments on the distribution of the protons can be carried out by the method of Chadwick and Bieler. The theoretical distribution can be calculated from the general theory of scattering.

For very short range  $\alpha$ -particles the proton distribution is that given by the theory. For more energetic particles, however, there is a large excess of protons making small angles with the direction of the incident  $\alpha$ -rays.<sup>(7)</sup> An analysis of the results indicates that for head-on collisions the inverse square law breaks down when the distance between the colliding particles is less than  $4 \times 10^{-13}$  cm. For very oblique collisions the law ceases to hold at an even greater distance, about  $8 \times 10^{-13}$  cm. The distribution of the particles is inconsistent with any central law of force.

Very similar results are reached when the collisions of  $\alpha$ -particles in helium are studied, and the inverse square law breaks down for even greater separations between the particles.

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## CHAPTER XII

### THE $\beta$ - AND $\gamma$ -RADIATIONS

**112. Nature of the  $\beta$ -rays.** The  $\beta$ -rays can be distinguished from the  $\alpha$ -rays by their greater penetrating power, as they are able to produce measurable effects after passing through absorbing sheets of 100 times the thickness required to stop the  $\alpha$ -particles. Their nature was first investigated by Becquerel, who showed that they consisted of charged particles by the following simple experiment. A small quantity of uranium oxide was placed in a small lead dish on the back of a photographic plate, the film side being downward. The whole was then placed between the poles of an electromagnet in a dark room. After some hours the plate was developed, and was found to be blackened immediately below the lead dish, the paths of the rays having been bent into circles by the action of the field.

A modification of the same experiment (Fig. 87) showed that the rays were heterogeneous. The plate was placed film side upwards between the poles of the magnet and exposed to the rays for some time. On developing the plate a diffuse patch was found, showing that some of the rays had been bent into smaller circles than others. By placing different thicknesses of aluminium foil on the plate it was found that the part of the patch nearest the source disappeared sooner than that further away. The more deviable rays were thus more absorbable than the others. Since

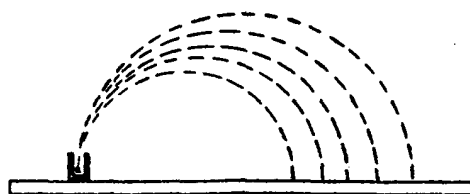


FIG. 87. Magnetic deflection of  $\beta$ -rays

$$\rho = mv/He$$

these rays are the slower ones, assuming that the ratio of  $e/m$  is constant.

The similarity of the  $\beta$ -rays to cathode rays was further established by Curie, who showed that they carried a negative charge. In performing this experiment it is necessary to avoid the presence of air, as the latter becomes conducting under the action of the rays and the charge is thus unable to accumulate. In modern experiments this

difficulty is overcome by working in a high vacuum. Curie, however, surrounded his plate by a solid dielectric instead. His apparatus is shown in Fig. 88. The  $\beta$ -rays from the radio-active material in  $R$  penetrate the thin aluminium leaf  $A$  and the thin layer of wax, but are stopped by the lead plate  $B$ , giving up their charge to it. On performing the experiment it was

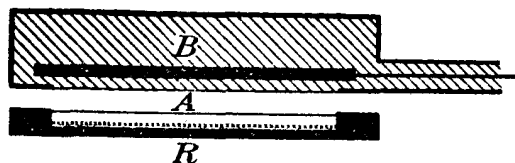


FIG. 88. Curie's demonstration of the charge on  $\beta$ -rays

found that the electrometer connected to  $B$  showed a gradual but steadily increasing deflection when the source of radio-active substance was placed below  $A$ . The sign of the charge was negative.

**113. Determination of  $e/m$  and  $v$  for the  $\beta$ -rays.** Since the  $\beta$ -rays are rapidly moving negatively charged particles we can apply to them the methods of measuring the ratio of the mass to the charge, and the velocity, which were employed for the cathode rays. The first accurate experiments were made by Kaufmann <sup>(1)</sup>; they have been repeated by numerous observers.

The method adopted was the one where the electric and magnetic fields are parallel, and the two deflections thus at right angles to each other, the theory of which has already been considered (§ 50) in connection with the positive rays. The apparatus used is shown in Fig. 89 (a).

The source of the rays was a small speck of radium compound placed at  $R$ , while the rays were limited to a narrow pencil by a slit  $A$  in a thick lead plate. Before reaching the hole  $A$  the rays passed between two metal plates, which could be adjusted by levelling screws to be perfectly parallel. The plates could be raised to different potentials by means of wires which were fused through the walls of the containing glass vessel. The photographic plate  $P$  was mounted at the top of the chamber, which was carefully exhausted to a very high vacuum. The whole vessel was placed between the poles of a strong electromagnet  $N, S$ .

On developing the plate the trace of the rays was a single continuous curved line (Fig. 89 (b)). If the rays had differed only in velocity this curve would by (41) have been a parabola. The line, however, was not parabolic, showing that the value of  $e/m$  was not constant, but varied with the velocity. The fact that the line was single and unbroken showed that this important ratio was a continuous function of the velocity of the rays. In other words for every value of the velocity

there was a single, definite value for the ratio  $e/m$  which depended on the velocity alone.

The calculations necessary to evaluate the results are hardly so simple as those for Thomson's experiments on the positive particles. Allowance has to be made for the fact that the two fields are not coterminous. Very careful experiments and elaborate calculations were made to determine the end-corrections for the two fields.

Since the curve showed no signs of breaks it was very improbable that the rays consisted of sets of different particles. It is equally improbable

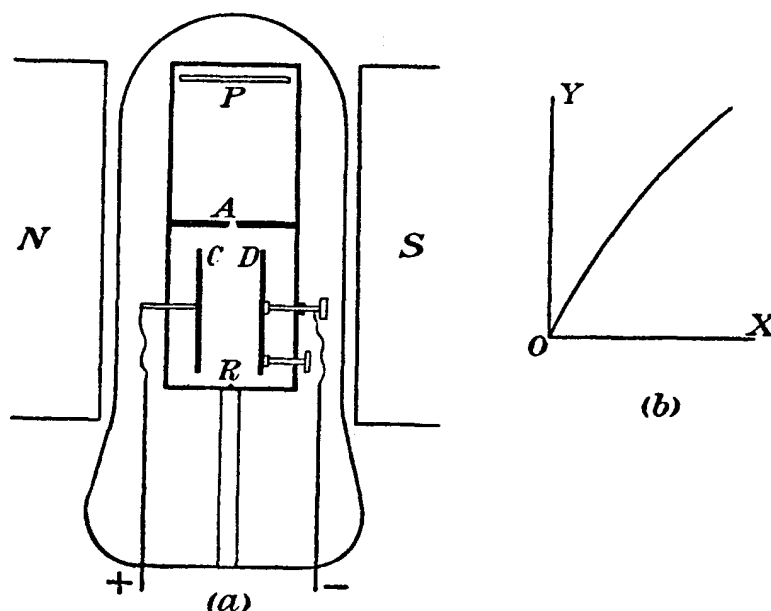


FIG. 89. Kaufmann's determination of  $e/m$  for  $\beta$ -rays

that the charge was gradually changing, since the great accumulation of evidence has shown that the charge  $e$  is atomic. We are thus led to the conclusion that the mass of a  $\beta$ -particle is a function of the velocity with which it is moving. The value of  $e/m$  for the more slowly moving  $\beta$ -particles is identical with the value obtained for the cathode rays. The  $\beta$ -particles are thus simply swift electrons.

**114. Electromagnetic mass.** It can easily be shown that a moving charge will act as if it possessed mass from the mere fact that it carries a charge. Consider a point charge moving with a velocity  $v$ . This will be equivalent to a current element coinciding with the path of the particle and equal to  $ev$ , where  $e$  is the charge and  $v$  the velocity. The magnetic field due to the moving charge at a distance  $r$  from it in a direction making an angle  $\theta$  with the direction of motion will thus be  $ev \sin \theta / r^2$ .

The energy in a magnetic field of strength  $H$  is  $\mu H^2/8\pi$  per unit volume. Hence if  $du$  is a small element of volume at the point considered the magnetic energy in that element of volume will be  $\frac{\mu}{8\pi} \left( \frac{ev \sin \theta}{r^2} \right)^2 du$ .

The whole magnetic energy in the space round the particle will be the integral of this from the surface of the particle to infinity. To evaluate this, with the electron as centre describe two spheres of radii  $r$  and  $r+dr$  and draw two radii making angles  $\theta$  and  $\theta+d\theta$  with the direction of motion. If these are supposed to rotate about the direction of motion of the electron they will cut out from the spherical shell an annulus the volume of which is

$$2\pi r \sin \theta \cdot r d\theta \cdot dr.$$

But the magnetic field is obviously constant throughout the space so obtained and hence the energy in the annulus is

$$\frac{\mu}{8\pi} \left( \frac{e^2 v^2 \sin^2 \theta}{r^4} \right) 2\pi r^2 \sin \theta d\theta dr = \frac{\mu e^2 v^2 \sin^3 \theta d\theta dr}{4r^2}.$$

The energy in the spherical shell is thus

$$\begin{aligned} 2 \int_0^\pi \frac{\mu e^2 v^2 \sin^3 \theta dr}{4r^2} d\theta &= \frac{\mu e^2 v^2 dr}{2r^2} \int_0^\pi \sin^3 \theta d\theta \\ &= \frac{1}{3} \frac{\mu e^2 v^2}{r^2} dr. \end{aligned}$$

If the charge is carried by a small sphere of radius  $a$  the total magnetic energy in the space is  $\int_a^\infty \frac{\mu e^2 v^2}{3 r^2} dr$

$$= \frac{\mu e^2 v^2}{3 a} \quad \dots \dots \dots (97)$$

This energy must be given to the particle when it is set in motion. Comparing this expression with that for kinetic energy ( $\frac{1}{2}mv^2$ ) we see that the charge behaves as if it had a mass of  $\frac{2\mu e^2}{3 a}$  due to its charge  $e$ ; this is called its *electromagnetic mass*. If we assume that the whole mass of an electron is electrical in origin the radius  $a$  of the electron can be deduced from this equation. The value so obtained,  $1.9 \times 10^{-13}$  cm., is of an order of magnitude not inconsistent with our knowledge of the dimensions of other fundamental particles.

115. Variation of the mass of an electron with velocity. Bucherer's

**experiments.** It can be shown on the electromagnetic theory that the above analysis is only true if the velocity of the particle is small compared with that of light (practically if it is less than one-tenth that of light). If the velocity of the particle approximates to that of light the distribution of the electric field round the moving charge is altered in such a way as to increase the electromagnetic energy of the field, and thus the electromagnetic mass of the particle.

Lorentz has shown that if  $m_0$  is the electromagnetic mass of a particle when moving with a velocity small compared with that of light (the rest-mass, as it is called) its mass  $m_v$  when moving with a velocity  $v$  is given by

$$m_v = m_0(1 - \beta^2)^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

where  $\beta$  is the ratio of the velocity of the particle to that of light.

On the Principle of Relativity all mass, whether electrical in its origin or not, should vary with velocity in accordance with (98). Since  $\beta$ -particles move with velocities ranging up to within about 2 per cent. of the velocity of light, they afford a very suitable means of testing the accuracy of the formula. An experimental test was carried out by Bucherer<sup>(2)</sup> using a very ingenious method. The source of the  $\beta$ -radiation was a small speck of radium fluoride  $R$  placed at the centre of two parallel plates  $A$  and  $B$  (Fig. 90) which were very close together.

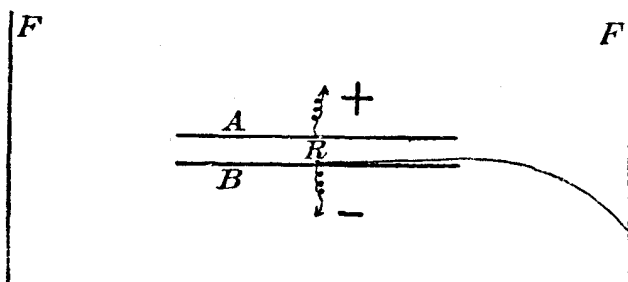


FIG. 90. Bucherer's method of investigating the variation of mass with velocity

The plates were maintained at a considerable difference of potential, and the apparatus was placed in a uniform magnetic field at right angles to the plane of the paper. The  $\beta$ -particles from the radium will obviously only be able to escape from between the plates if the electric and magnetic forces upon them are exactly equal and opposite. Otherwise the particles will be deflected by whichever of the two fields is the stronger, and will strike one or other of the plates. The particles are projected at all angles to the magnetic field. Those which emerge

from between the plates at an angle  $\theta$  with the magnetic field will have a velocity given by

$$Xe = Hev \sin \theta$$

$$\therefore v = \frac{X}{H \sin \theta}.$$

Thus for any given value of the angle  $\theta$  the rays which succeed in escaping from between the plates have a definite and calculable velocity.

On emerging from the plates the electrons are acted upon only by the magnetic field, and their deviation is proportional to  $e/m$ , where  $m$  is the mass corresponding to the velocity  $v$ . To measure the deviation the rays are allowed to fall upon a photographic film  $F, F$ , which is bent into a cylinder coaxial with the plates. The field is reversed during the experiment and the distance between the two traces on the film gives twice the magnetic deviation corresponding to that direction, from which  $e/m$  can be calculated. Hence  $v$  and  $e/m$  are known for each point on the film.

The values of  $m_v/m_0$  given by the Lorentz formula are contained in Table VIII. It will be noticed that the increase in mass is inappreciable until  $v$  is at least one-tenth the velocity of light. The discrepancy between the Lorentz formula and the experimental results of Bucherer was less than 1 per cent.

TABLE VIII

$\beta$	$m_v/m_0$
·01	1·000
·10	1·005
·30	1·048
·50	1·115
·60	1·250
·70	1·400
·80	1·667
·90	2·294
·95	3·203
·98	5·025

**116. The kinetic energy of an electron.** The variation in the mass of an electron with its velocity necessitates a modification of the usual expression for the kinetic energy of a moving particle. Let us suppose that an electron, initially at rest, is acted upon by a constant force  $F$ .

By Newton's second law of motion, this force is equal to the rate of change of momentum produced; that is  $F = d(mv)/dt$ . In classical mechanics we regard the mass  $m$  of the particle as a constant, and in fact the law is often stated in the form  $F = m dv/dt$ . For the electron, however (and according to the Principle of Relativity for all moving bodies), the mass depends on the velocity, and hence both  $m$  and  $v$  are functions of the time.

Writing  $m_0$  for the *rest mass* of the particle and putting  $v = \beta c$ , where  $c$  is the velocity of light, we have, by (98),

$$\text{Momentum} = mv = \frac{m_0}{(1-\beta^2)^{\frac{1}{2}}} \beta c \quad . . . . (99)$$

and the force on the particle  $F = d(mv)/dt$ . The kinetic energy,  $T$ , of the particle when its velocity has reached the value  $\beta c$  is thus given by

$$\begin{aligned} T &= \int_0^\beta F ds = \int_0^\beta \frac{d}{dt}(mv) ds = \int_0^\beta \frac{ds}{dt} d(mv) \\ &= \int_0^\beta \beta c d\left(\frac{m_0 \beta c}{(1-\beta^2)^{\frac{1}{2}}}\right) \\ &= m_0 c^2 \int_0^\beta \frac{\beta}{(1-\beta^2)^{\frac{3}{2}}} d\beta = m_0 c^2 \left[ \frac{1}{(1-\beta^2)^{\frac{1}{2}}} \right]_0^\beta \\ &= m_0 c^2 \left( \frac{1}{(1-\beta^2)^{\frac{1}{2}}} - 1 \right) \quad . . . . . (100) \end{aligned}$$

It can easily be seen, by expanding the first term within the bracket by the binomial theorem, that the expression reduces to the form  $T = \frac{1}{2}mv^2$ , if  $\beta$  is small.

**117. Equivalence of mass and energy. Einstein's relation.** Equation (100) can be written in the form

$$T = c^2 \left( \frac{m_0}{(1-\beta^2)^{\frac{1}{2}}} - m_0 \right) = c^2(m_v - m_0) = c^2 \cdot \Delta m$$

where  $\Delta m$  is the difference between the mass of the particle when in motion and at rest. In this form the work done upon the particle in setting it in motion is represented by an increase in mass of the particle, just as work done in rubbing one body against another is represented by the heat evolved. There is thus a relation between mass and energy, equivalent to that between heat and energy as expressed by Joule's law.

It is tempting, from the form of the relation, to regard  $c^2 m$  as the total energy of the particle in motion and  $c^2 m_0$  as its energy when at





As can be clearly seen from the Wilson photographs, the specific ionization produced by a  $\beta$ -particle is much smaller than that of an  $\alpha$ -particle. The  $\beta$ -particles expend their energy much more slowly than the  $\alpha$ -particles, and their range is consequently much greater. A 0.5 MeV  $\beta$ -particle has a range in air, at atmospheric pressure and temperature, of about 1 metre. As with the  $\alpha$ -particles, the specific ionization increases rapidly towards the end of the track.

Owing to the large number of deflections experienced by the particle its actual path in an absorbing sheet of material may be considerably greater than the actual thickness of the sheet and will differ for different particles according to the laws of probability. This "straggling" is so pronounced that it is not possible to get much information as to the actual range of the  $\beta$ -particles by the simple methods employed for  $\alpha$ -rays. For the same reason a homogeneous beam of  $\beta$ -particles falling on an absorbing sheet will emerge on the further side with a considerable range of velocities, owing to the differences in the actual lengths of their tracks in the sheet.

Probably the most informative experiments on the subject are those made by White and Millington,<sup>(3)</sup> using the focusing method described in § 93. The radiator  $R$  of Fig. 74 is replaced by a wire coated with radium B and C. These substances emit certain strong homogeneous groups of  $\beta$ -rays, each group being characterized by a definite velocity of emission. Thus when the apparatus is placed in a strong magnetic field several strongly marked sharp lines appear on the photographic plate. If the source is covered with a thin sheet of mica each line is displaced towards the source, showing that the particles have been retarded by passing through the absorbing screen, and at the same time the line is very much broadened, owing to the large straggling of the particles in the sheet. By placing the mica so that it covers only part of the source, both sets of lines can be recorded simultaneously on the same plate and the displacement can thus be accurately measured. Since the magnetic field is constant this displacement is, by (29), proportional to the change in velocity in passing through the sheet.

The relative intensity of the  $\beta$ -particles at a given point in the broadened line can be estimated by measuring the density of the photographic trace by a microphotometer. One set of such results is shown in Fig. 91, the curves I-III referring to three different thicknesses of mica. It will be seen that the actual retardation suffered by a  $\beta$ -particle in passing through a given sheet of matter varies within fairly wide limits, according to the number and kind of collisions it has chanced to make in its passage. If we take the highest point of each

peak as a measure of the most probable retardation, it is found that the most probable retardation  $\delta$  is roughly proportional to the thickness of the absorbing screen. The value of  $\delta/\sigma$ , where  $\sigma$  is the thickness of the sheet expressed as its mass per unit area, is known as the stopping power of the material. It is found that the stopping power  $S$  of a

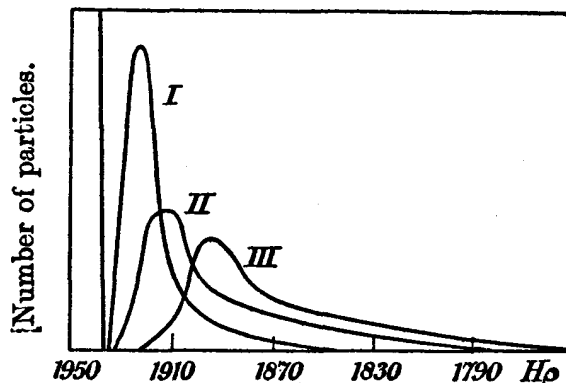


FIG. 91. Illustrating the straggling in velocities of  $\beta$ -particles in their passage through matter

given substance is roughly, but not accurately, inversely proportional to the cube of the velocity of the  $\beta$ -particles. Thus approximately

$$Sv^3 = \text{constant} \quad \text{or} \quad T_0^2 - T^2 = k\sigma \quad . \quad . \quad . \quad (103)$$

where  $T_0$  is the initial and  $T$  the final kinetic energy of the particle after passing through a thickness  $\sigma$ .

The stopping power does not vary much with the nature of the absorbing foil. It appears to be somewhat greater for elements of low atomic number.

**119. Diffraction of electrons by crystals.** We have seen (§ 83) that if a pencil of X-rays is directed obliquely at the surface of a crystal, reflection takes place for certain values of the glancing angle, the crystal acting as a diffraction grating for the radiation. Davisson and Germer<sup>(4)</sup> in 1922 showed that exactly similar results can be obtained if a pencil of fast cathode rays is substituted for X-rays in the experiment. Thus a pencil of electrons can be diffracted by a crystal grating. The critical glancing angle depends on the velocity of the particles, the angle becoming smaller as the velocity increases. The beam of electrons thus behaves exactly in the same way as a beam of light, the "wavelength" of the beam becoming smaller as the velocity increases.

The results of Davisson and Germer were confirmed by a somewhat different method by G. P. Thomson.<sup>(5)</sup> Thomson allowed a narrow pencil of fast cathode rays to fall normally on very thin metallic films,

the emergent rays falling on a photographic plate placed some distance on the further side of the film. It was found that the central spot marking the position of the undeflected beam was surrounded by a series of well-marked diffraction haloes, exactly similar to those which would have been obtained with a beam of homogeneous X-rays, but of smaller diameter, showing that the wavelength concerned was rather shorter than that of X-rays (see Fig. 92 (b), Plate V). If a magnetic field is applied between the film and the plate the ring system is deflected as a whole, showing that the rings are formed by electrons and not by any X- or  $\gamma$ -radiation which might conceivably be present in the tube.

X-ray measurements show that a metal foil consists of a large number of irregularly orientated crystals of the metal, and Thomson's arrangement thus corresponds to the "powder" method used in X-ray analysis (§ 87). Thus by measuring the radii of the rings the corresponding wavelength  $\lambda$  can be deduced since the lattice spacing of the crystals is known from X-ray measurements. For cathode rays of velocity  $10^{10}$  cm. per sec. the value of  $\lambda$  was  $7.8 \times 10^{-10}$  cm., corresponding to that of a hard  $\gamma$ -ray. He was able to show that the wavelength  $\lambda$  was related to the velocity  $u$  of the electron by a relation of the form

$$\lambda = \frac{h}{mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where  $m$  is the mass of the electron and  $h$  is Planck's constant.

Electron diffraction is now being extensively used as a means of studying surface conditions, as owing to their small penetrating power electrons are diffracted mainly in the surface layers. The perfection to which the method has been brought is shown by Fig. 92 (a) (Plate V), due to Finch.

**120. The wave theory of matter.** Before the publication of Davisson and Germer's results de Broglie<sup>(6)</sup> had already advanced the suggestion that an electron might be regarded as a group of waves, or "wave packet" as it is now called. The original suggestion seems to have been inspired by the idea that, since radiation was known, under certain circumstances, to exhibit the characteristics of particles, so particles under certain circumstances might be expected to exhibit some of the characteristics of wave motion. The argument was supported by certain formal analogies between the generalized laws governing the motion of particles and those relating to the transmission of light.

Accepting the suggestion the appropriate wavelength  $\lambda$  to be

assigned to the particle can easily be deduced. Suppose that  $n$  quanta per second, each of energy  $h\nu$ , are falling normally every second on a square centimetre of some absorbing surface. As is well known, this will produce a pressure on the surface equal to the energy density in the radiation, that is to  $n \cdot h\nu/c$ , where  $c$  is the velocity of light. But pressure is rate of change of momentum and therefore each quantum must be supposed to have momentum equal to  $h\nu/c$ , a result we have already employed in § 81. On the other hand, the momentum of a particle is given by  $mu$ , where  $m$  is its mass and  $u$  its velocity. Equating these two expressions we have

$$mu = h\nu/c = h/\lambda; \quad \text{or} \quad \lambda = \frac{h}{mu}$$

where  $\lambda$  is the wavelength of the radiation constituting the particle.

The velocity of a group of waves is not necessarily that of the waves themselves: the two are, in fact, only identical in a non-dispersive medium. Observation of a group of waves formed by dropping a pebble into a pond will show that the group is travelling more slowly than the waves themselves. It can be seen that waves form at the back of the group, travel through it, and finally disappear as they reach the group front. It is, therefore, not necessary that the electrons should move with the velocity of light; any velocity can be obtained for them by assigning appropriate (though rather unusual) dispersive properties to the medium. De Broglie's theory has been developed by Schrödinger and others with great success. The quantum relationships emerge from it quite naturally, and it leads to formulae identical with those of Bohr for the structure of the hydrogen atom. The relation between the velocity and the wavelength is of the form of (104) which has been verified by Thomson.

The theory is not confined to electrons but is taken to apply to all small particles whether charged or uncharged, the wavelength being given by (104) in all cases. Thus a hydrogen molecule at ordinary temperatures has, owing to its velocity of thermal agitation, a wavelength of about  $10^{-8}$  cm. and Stern and Esterman <sup>(7)</sup> have succeeded in demonstrating the diffraction of a stream of hydrogen molecules by a crystal surface. An  $\alpha$ -particle from radium C' of mass  $6.60 \times 10^{-24}$  gm. and velocity  $1.92 \times 10^9$  cm. per sec. must be regarded as a wave packet of wavelength almost exactly  $5 \times 10^{-13}$  cm. It is not without significance that the ordinary laws of scattering begin to break down at a distance from the nucleus of this order of magnitude.

It is probably impossible to form any mental image of an entity

which has some of the properties of a particle and some of the properties of a wave. The matter can, however, be dealt with quite rigidly by mathematical reasoning, and wave mechanics has now a very large and growing literature of its own. It is entirely beyond the scope of this work, though we shall have occasion to refer to some of its conclusions.

**121. Nature and properties of the  $\gamma$ -rays.** The  $\gamma$ -rays are distinguished from the  $\alpha$ -rays and  $\beta$ -rays by their much greater penetrating power. The  $\gamma$ -rays from some radio-active substances can be detected through 30 cm. of iron. They are not deviated by a magnetic field and, therefore, carry no charge. Their behaviour corresponds with the assumption that they are electromagnetic disturbances of short wavelength. If in sufficient intensity they produce luminosity on a fluorescent screen, and affect a photographic plate. They also produce ionization in gases, and are generally detected by this action.

The  $\gamma$ -rays differ from X-rays of equivalent wavelength merely in their mode of origin. The wavelengths of the  $\gamma$ -rays are characteristic of the radioactive substance which emits them and range from about 3.9 Å. for radioactinium up to 0.047 Å. for thorium C". The corresponding quantum energies are 0.032 and 2.65 MeV respectively. For many years owing to technical difficulties it was not found possible to generate X-rays at more than about 100,000 volts, and the  $\gamma$ -rays provided the only means of extending our knowledge of the properties of the short-wavelength end of the electromagnetic spectrum. More recently the technical difficulties of the production and application of high voltages have been gradually overcome and X-ray sets, more or less of the orthodox pattern, are now on the market capable of producing 2-million-volt X-rays. An electron accelerator of an entirely new type, known as a "Betatron" (§ 147), has been built which produces radiation with a quantum energy up to 100 MeV. There seems no doubt that these high energy radiations will be spoken of as X-rays; the term  $\gamma$ -ray being confined to those high-frequency radiations emitted spontaneously in radioactive and other natural processes.

**122. Determination of the wavelength of  $\gamma$ -rays.** Direct determinations of the wavelengths of some of the less penetrating  $\gamma$ -radiations have been made by Rutherford.<sup>(8)</sup> The method adopted was an interesting modification of that of the X-ray spectrometer already described. As the glancing angle decreases with the wavelength it is very small for  $\gamma$ -rays, and hence could not be measured with any precision. To overcome the difficulty a crystal *S* of rock salt was mounted with cleavage planes parallel to the line *RO* (Fig. 93), the source of  $\gamma$ -radiation being placed at *R* and a photographic plate at *O*. The

$\gamma$ -rays strike the crystal at all angles, but only those which fall upon it in such a way that the angle between them and the reflecting planes is the glancing angle for the particular wavelength will be reflected. The rays are thus left as it were to pick out their own reflecting planes, and beams of reflected rays  $AB$  and  $A'B'$  will emerge from the crystal and fall on the photographic plate.

Neglecting the absorption of the crystal, which will be very small, the energy transmitted plus the energy reflected must be equal to the energy in the primary beam. Hence a ray such as  $RA$  which is selectively reflected will suffer a much greater loss of energy in going through the crystal than rays which pass through without any loss by

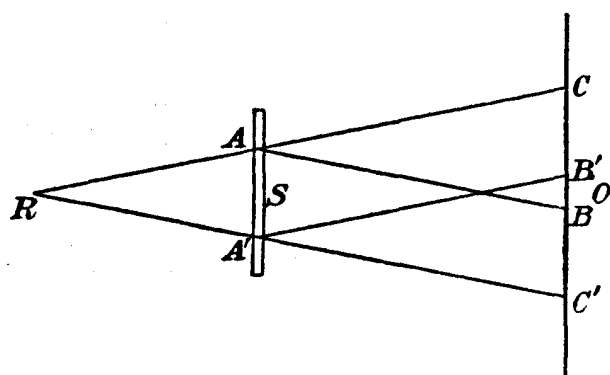


FIG. 93. Rutherford's method of determining  $\gamma$ -ray wavelengths

reflection. Thus in the direction  $RA$  produced the rays will be relatively enfeebled and the effect on the photographic plate will be small. The reflecting plane thus casts a shadow on the photographic plate at a point  $C$  in the direction  $RA$  produced. Similarly the plane  $A'$  casts its shadow at  $C'$ . It is

evident that the angle  $CRC'$  subtended by the distance between two corresponding bright lines at the source  $R$  is equal to twice the glancing angle for the rays. In this way by measuring the distance  $CC'$  on the plate and the distance of the plate from the source the angle of selective reflection for the rays used can be accurately determined.

The values of the glancing angle for the  $\gamma$ -rays from the mixture of radium B and radium C used for the experiment ranged from  $44''$  to  $4^\circ$ . The corresponding wavelengths ranged from  $0.71 \times 10^{-9}$  to  $4 \times 10^{-9}$  cm.

**123. The passage of  $\gamma$ -rays through matter.** A narrow pencil of  $\gamma$ -radiation becomes enfeebled in its passage through matter, the intensity  $I$  after passing through a thickness  $d$  of absorbing material being given for homogeneous radiation by  $I = I_0 e^{-\mu d}$ . The thickness  $d$  is usually expressed as mass per unit area, and  $\mu$  is then the mass coefficient of absorption.

As in the case of X-rays (§ 91) the coefficient of absorption  $\mu$  may thus be written in the form

$$\mu = \tau + \sigma_a + \sigma_s,$$

where  $\tau$  is the coefficient of photo-electric absorption,  $\sigma_a$  the Compton absorption coefficient, and  $\sigma_s$  the coefficient of scattering.

As we have already noted in the case of X-radiation, the probability of photo-electric absorption decreases rapidly with decreasing wavelength, and is almost negligible for  $\gamma$ -rays, except for elements of high atomic number. Thus the Compton absorption and Compton scattering are the principal cause of absorption for these high-frequency radiations.

Klein and Nishina have obtained, on quantum mechanical considerations, a formula for  $\sigma$ , the absorption per electron due to scattering. They find that

$$\sigma = \frac{\pi e^4}{m^2 c^4} 2 \left\{ \frac{1+\alpha}{\alpha^2} \left[ \frac{2(1+\alpha)}{1+2\alpha} - \frac{1}{\alpha} \log(1+2\alpha) \right] + \frac{1}{2\alpha} \log(1+2\alpha) - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}, \quad \dots \quad (105)$$

where  $\alpha = h\nu/mc^2$ ,  $h\nu$  being the energy of the  $\gamma$ -ray and  $m$  the mass of the electron. The experimental evidence is in agreement with this formula for photons of energy less than 1 MeV. For photons of still higher energy, a new absorption process comes into operation, and increases rapidly with increasing photon energy, so that penetrating power of the radiation actually reaches a maximum for photons of energy 5 MeV (see Fig. 99). This process, which results in the production of pairs of positive and negative electrons, is discussed more fully in § 133.

**124. Characteristic  $\gamma$ -ray spectra. Ellis's experiments.** Although  $\gamma$ -rays can be produced by the impact of  $\alpha$ - or  $\beta$ -rays on matter, our main source of  $\gamma$ -radiation is that emitted by the nuclei of radioactive elements during their disintegration. This emission is found to consist of a number, often quite large, of homogeneous groups of waves, the wavelengths of which are characteristic of the particular element from which they are emitted.

Although it is possible, as we have seen, to determine the wavelength of  $\gamma$ -radiation by crystal reflection, the method is incapable of great accuracy owing to the very small glancing angles involved. The most accurate method is to measure the energy of the photo-electrons ejected when the  $\gamma$ -ray is absorbed in some suitable material. In the analogous case of X-rays we have seen that the absorption of radiation of frequency  $\nu$  in an electron level of energy  $w_K$  gives rise to a photo-electron the kinetic energy  $E$  of which is given by  $E = h\nu - w_K$ . In the X-ray part of the spectrum the relation is used to determine the value

of  $w$ , since the direct measurement of  $\nu$  presents no difficulties. It is obvious, however, that if the electron energy levels are known from X-ray measurements, it is possible to use the relation to determine the frequency  $\nu$  of the radiation employed.

The experimental arrangements <sup>(9)</sup> are essentially the same as those adopted for the X-ray measurements. The source of  $\gamma$ -radiation is usually a fine wire on which the appropriate radio-active substance is deposited. This is wrapped round with a sufficient thickness (about 3 mm.) of the absorbing material to cut off completely any  $\beta$ -radiation due to the source itself. The  $\beta$ -rays emerging are, therefore, all produced by the transformation of the  $\gamma$ -radiation in the absorbing material.

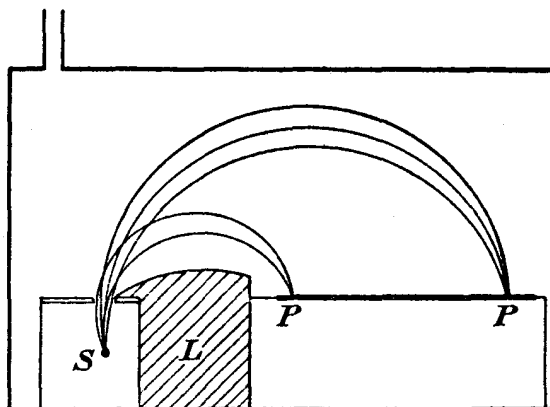


FIG. 94. Ellis's apparatus for measuring the  $\beta$ -ray spectrum

The source, in its wrapping, is placed at  $S$  (Fig. 94) immediately below the slit. The whole apparatus is then highly evacuated to prevent scattering or absorption of the  $\beta$ -particles in their path to the photographic plate  $PP$ .  $L$  is a block of lead used to screen the photographic plate  $PP$  from the direct action of the  $\gamma$ -rays from  $S$ , which would tend to produce a general fogging of the plate.

Since the energies of the photo-electrons range between about 300,000 and 3,000,000 volts, strong magnetic fields are required to produce the necessary deflection, and the apparatus is placed between the poles of a strong electromagnet. For absolute measurements the distribution of the magnetic field along the path of the particle must be determined, as it is usually not quite uniform. It is also necessary to take into account the variation of the mass of the  $\beta$ -particle with velocity. The radius of curvature  $\rho$  and the corresponding energy  $E$  of the particle are thus given by the equations

$$H\rho = \frac{m_0 c}{e} \frac{\beta}{\sqrt{1-\beta^2}}, \quad \dots \dots \dots (106)$$

$$E = m_0 c^2 \left( \frac{1}{\sqrt{1-\beta^2}} - 1 \right), \quad \dots \dots \dots (107)$$

where  $m_0$  is the rest mass of the electron, and  $\beta$  the ratio of the velocity of the particle to  $c$ , the velocity of light.



The applicability of the quantum relation to the  $\gamma$ -ray part of the spectrum was investigated in some preliminary experiments by Ellis, to whom most of our knowledge of this part of the subject is due. The  $\beta$ -ray spectra produced by the absorption of the  $\gamma$ -radiation from a single source (radium B+C) in a number of different elements of high atomic weight were measured. The spectra obtained were identical except that they were shifted more and more in the direction of lower energies, as the atomic number, and hence the energy, of the absorbing level increased. If  $E$  is the energy of the  $\beta$ -particles forming some particular line in the spectrum and  $w$  the energy of the absorption level of the absorbing element,  $E+w$  should be constant and equal to the energy of the corresponding  $\gamma$ -ray. This is found to be the case, as is shown in Table IX, which is taken from one of Ellis's early papers.

TABLE IX

Absorbing element	Ba	W	Pt	Pb	U	
$E$	2.53	2.20	2.12	2.03	1.74	$\times 10^5$ volts
$w$	.37	.69	.78	.89	1.18	„
$E+w$	2.90	2.89	2.90	2.92	2.92	„

The accuracy of the method has been greatly increased by careful technique, and measurements of the frequency of radiation are now made to an accuracy which is certainly better than one part in a thousand. It may be mentioned that conversion may also take place in the  $L$  or even the  $M$  levels of the absorbing material. The probability of these conversions is much smaller than for the  $K$  level, and the corresponding lines are usually faint.

The lines in the  $\gamma$ -ray spectrum of a given radio-active element show the same type of interrelation which we found between the lines of the characteristic X-ray spectrum of an element, that is to say, there are numerous lines which can be related by an expression of the type  $\nu_a = \nu_b - \nu_c$ . It is reasonable to suppose that the characteristic  $\gamma$ -rays are emitted during transitions between various energy levels existing in the nucleus of the radio-active atom. We shall discuss this in more detail in a later chapter (§ 158).

**125. The continuous  $\beta$ -ray spectrum. The neutrino.** Whereas both the  $\alpha$ -particles and the  $\gamma$ -rays from a radio-active substance are, as we have seen, emitted with a few perfectly definite energies characteristic of the substance, the  $\beta$ -particles show a continuous range of energies

from zero up to a maximum. The distribution of energy in the  $\beta$ -ray spectrum can be studied by replacing the photographic plate in Ellis's apparatus (Fig. 94) by a solid screen pierced with a suitable slit. Different parts of the spectrum can be focused in turn on the slit, by varying the magnetic field, and the relative number of  $\beta$ -particles passing through the slit can be determined either by collecting them in a Faraday cylinder or using a Geiger counter.

Fig. 95, in which the relative number of particles is plotted against the energy shows the general form of the relation. The upper energy

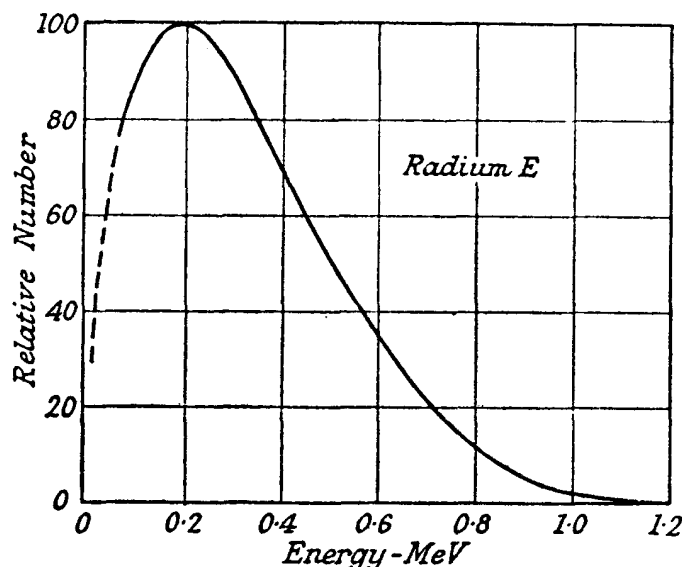


FIG. 95.  $\beta$ -ray spectrum of Radium E

limit of the spectrum is characteristic of the particular substance emitting the radiation. For radium B it is 0.65 MeV. For radium C it is as high as 3.15 MeV.

The existence of a *continuous*  $\beta$ -ray spectrum raises rather important theoretical difficulties. Consider, for example, the case of radium E, which disintegrates to form polonium, with the emission only of  $\beta$ -radiation and a quite negligible amount of  $\gamma$ -radiation. The total number of  $\beta$ -particles given out per second is known to be equal to the number of atoms disintegrating in the same time, so that presumably each  $\beta$ -particle is the result of a single atomic disintegration. But one of these  $\beta$ -particles may have energy exceeding 1 MeV and another certainly less than one-tenth of this amount. The two atoms of polonium formed by these two disintegrations should presumably differ in energy by a million volts. One would expect so large an energy deficit to produce some detectable effect, but as far as is known all the polonium atoms are exactly alike.

It seemed possible at first that all the  $\beta$ -particles might be emitted with the maximum energy and that the slower particles had lost energy in escaping from the substance by some absorption process. The most probable of these on general grounds would be the conversion into  $\gamma$ -radiation; but radium E emits no appreciable amount of  $\gamma$ -rays. To test the matter further Ellis and Wooster allowed the disintegration of radium E to proceed inside a closed calorimeter, the walls of which were sufficiently thick to absorb all the known products of disintegration. By measuring the heat produced it was found that the average energy given out for a single disintegration was 0.35 MeV. This is far smaller than the 1.2 MeV carried off by the most energetic  $\beta$ -particle,

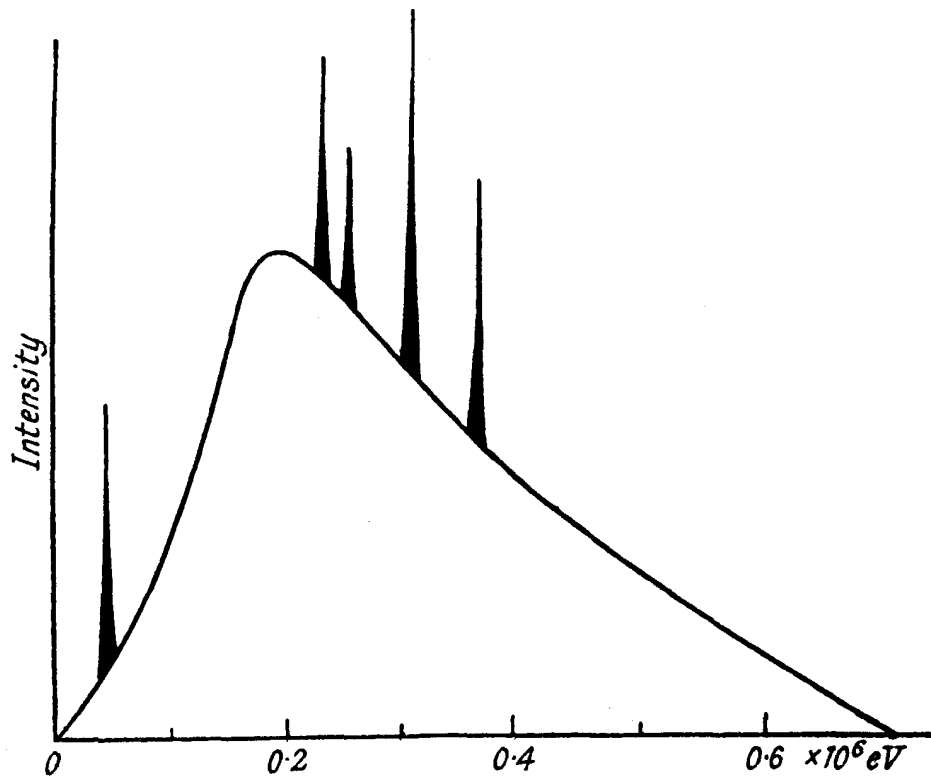


FIG. 96.  $\beta$ -ray spectrum of Ra B

and corresponds very closely to the average energy of the  $\beta$ -rays, as deduced from the spectrum of Fig. 95.

The only remaining possibility (unless indeed we are prepared to abandon the principle of the conservation of energy) seems to be the suggestion, due to Pauli, that the energy given out in each disintegration is indeed 1.2 MeV, corresponding to the upper limit of the  $\beta$ -ray spectrum, but that this energy is divided, in variable amounts, between the  $\beta$ -particle itself, and some hitherto unobserved particle, to which he proposed to give the name *neutrino*.

The neutrino must be uncharged, since it does not produce any ionization. Its mass must be very small, since no effects have yet been observed which could be attributed to the collision of the neutrino with the atoms through which it passes—effects which have made possible the experimental study of the more massive uncharged particle known as the neutron (§ 140).

It has recently been found possible to study  $\beta$ -ray disintegrations in a cloud chamber.<sup>(10)</sup> The disintegrating atom recoils as the  $\beta$ -particle is ejected, and the energy of recoil can be estimated from the number of ions formed by the recoiling atom. The energy of the  $\beta$ -particle can be determined from the length of its track. It is found that neither energy nor momentum are conserved for the individual disintegrations, unless some third particle is postulated to balance the equations.

The *ad hoc* assumption of a particle unobserved, and possibly unobservable, is not altogether satisfactory. There are, however, other nuclear phenomena which seem to demand a similar solution, and the neutrino seems to be gradually finding general acceptance.

**126.  $\beta$ -ray line spectra.** In addition to the continuous spectrum, some  $\beta$ -ray emitting substances also give out a characteristic line spectrum; that is, groups of  $\beta$ -rays of quite definite velocity. Such substances all emit  $\gamma$ - as well as  $\beta$ -radiation, and the characteristic groups of  $\beta$ -rays are due to the absorption of these  $\gamma$ -rays in one or other of the extranuclear levels of the radioactive atom itself, with the consequent ejection of an electron from that level.

If one half of a linear source of, say, radium B is covered with sufficient thickness of platinum to absorb all the natural  $\beta$ -radiation from the source, and exposed in Ellis's apparatus (Fig. 94) two sets of lines appear on the plate; the one being the natural  $\beta$ -ray spectrum of radium B, the other due to the absorption of the  $\gamma$ -rays in platinum. The two spectra are identical save for a slight shift of one set of lines relative to the other.

If  $h\nu$  is the energy of the particular  $\gamma$ -ray responsible for one of these lines, the energy  $E_1$  of the  $\beta$ -particles ejected from the platinum will be given by  $E_1 = h\nu - W_{Pt}$ , where  $W_{Pt}$  is the energy of the  $K$  level in platinum (assuming the conversion to take place in the  $K$  level). If the corresponding "natural"  $\beta$ -ray line is due to the conversion of the  $\gamma$ -ray in radium B itself, its energy should be  $E_2 = h\nu - W_B$  where  $W_B$  is the energy of the  $K$  level of radium B. There is, however, the possibility that the radium B may have already been transformed by disintegration into its daughter product radium C *before* the emission of the  $\gamma$ -radiation. In this case the energy of the natural  $\beta$ -ray will be

$E_3 = h\nu - W_C$ . In the first case the energy shift between the two lines should be  $W_B - W_{Pt}$ ; in the second  $W_C - W_{Pt}$ . Radium B is an isotope of lead, and radium C an isotope of bismuth, so that the values of  $W_{Pt}$ ,  $W_B$ , and  $W_C$  are known from X-ray measurements. The energy difference between the two lines, which are close together, can be measured with high accuracy. Ellis and Wooster<sup>(12)</sup> found that the shift was equal to  $W_C - W_{Pt}$ ; that is to say, the radium B had been definitely transformed into radium C before the emission of the  $\gamma$ -radiation. This result is of some importance in the theory of radioactivity.

It may be mentioned that a few lines are found in the natural spectra which have no counterpart in the secondary spectra. These may be regarded as due to  $\gamma$ -rays which are so strongly absorbed in the electronic structure of the radioactive atom that they never emerge from it.

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## CHAPTER XIII

### COSMIC RADIATION, POSITRONS, MESOTRONS

**127. Discovery of Cosmic Radiation.** Until about 1910 it was generally assumed that the residual ionization always to be found in the air of an electroscope, even in the absence of any specific source of ionization, was to be ascribed either to traces of radioactive material, which in minute quantities is widely distributed in the air, sea, and soil, or possibly to some spontaneous process of ionization in the air itself. In that year Hess, by sending up electroscopes attached to balloons, found that the rate of discharge of the instruments actually increased with height of ascent; a result which was confirmed a year or so later by Kohlhörster, who found that the residual ionization at a height of six miles was thirty times that at ground level. The effect could not, therefore, be due to terrestrial sources, and Hess made the suggestion that it was due to some radiation (Hohenstrahlen) coming into the atmosphere from outside. As the mass per unit area of the whole atmosphere corresponds to 10 metres of water or a thickness of nearly 1 metre of lead, it was evident that this radiation must have a penetrating power at least six times greater than that of any known radiation from radioactive substances.

Experimentation on the subject was resumed in 1921, after the First World War, in particular by Millikan and his assistants in America. By experiments made in deep mountain gorges in the Andes, where the electroscopes were screened by mountain massifs from any radiation except that coming almost vertically from above, Millikan was able to show that the magnitude of the ionization was independent of the particular portion of the heavens to which the electroscope was exposed at the time. It was the same with the sun overhead and at midnight, and also with the galactic plane visible or hidden. The radiation, therefore, does not emanate from the sun or even from the galaxy of which it forms a part. It falls uniformly on the earth from all directions in space and hence is appropriately called *cosmic radiation*.

The absorption of the radiation in the atmosphere was obtained by measuring the ionization at different heights above ground level; and its absorption in water by sinking electroscopes into the depths of snow-fed lakes. (Lakes fed by streams, and the sea, usually contain appre-

ciable traces of radioactive contamination.) The mean coefficient of absorption at ground level is about 0.5 per metre of water; about one-tenth of that of the most penetrating  $\gamma$ -rays. The radiation is not homogeneous, and after filtration through 230 metres of water, a coefficient as low as 0.02 per metre was recorded. An analysis of the absorption curve led some authors, including Millikan, to suggest that the radiation consisted of four definite groups of rays. Attempts to analyse an absorption curve into exponential components are always hazardous, and it is better to consider the radiation as falling into two distinct groups: an extremely penetrating, or "hard" component, capable of traversing 3 metres of lead without losing as much as one-half its intensity, and a second "soft" component which is completely absorbed in about 10 cm. of that material.

**128. The latitude effect.** It was natural, in the existing state of knowledge, to assume that radiation of such extreme penetrating power must be of the nature of gamma-radiation of very short wavelength. During a voyage from Holland to Java in 1927 Clay observed that the intensity of the cosmic radiation was some 10 or 12 per cent. less in the region of the magnetic equator than at high latitudes, either north or south. This was verified by a world survey organized by Compton.<sup>(1)</sup> Following a given line of longitude the cosmic ray intensity remains sensibly constant from either pole down to a latitude of about  $45^\circ$ , and then diminishes slowly until the magnetic equator is reached (Fig. 97). The total drop is about 11 per cent. at sea level,

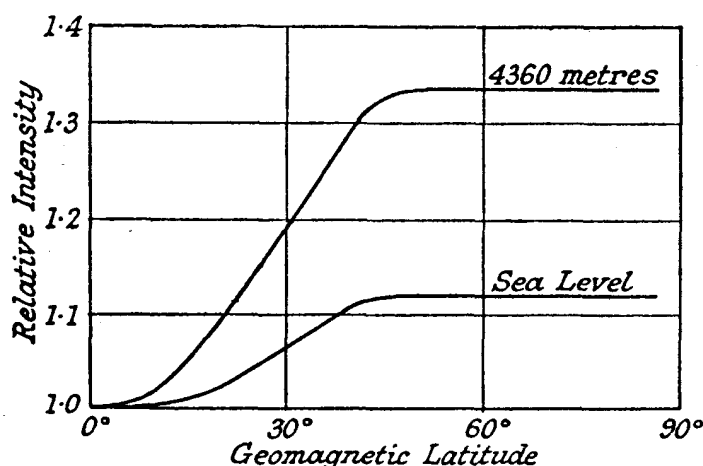


FIG. 97. Variation of cosmic ray interacting with geomagnetic latitude, at various heights

but increases to as much as 33 per cent. at a height of 4360 metres. The critical latitude at which the fall begins seems to be the same at all

altitudes according to the most recent observations. Collected observations showed that the isocosms (lines of equal cosmic ray intensity) followed almost exactly the lines of geomagnetic latitude. The most obvious explanation of this "latitude effect," as it is called, is that the paths of the cosmic rays in space are affected by the magnetic field of the earth, that is to say, they must be electrically charged particles.

Regarding the earth as a short magnet of magnetic moment  $8.1 \times 10^{25}$  e.m.u., it can be shown that the minimum energy which a particle may possess in order to be able to reach the earth in magnetic latitude  $\lambda$  is given by  $1.9 \times 10^{10} \cos^4 \lambda$ , where the energy is expressed in electronvolts. Thus particles of all energies from zero upwards can reach the magnetic poles while only those particles whose energies exceed  $1.9 \times 10^{10}$  eV can arrive in the equatorial plane. Hence, assuming that the incident particles cover a wide energy range, more will reach the atmosphere in high latitudes than at the equator. The resulting effect as measured on the earth's surface is modified by the absorbing effect of the atmosphere on the radiation. Although low energy particles could arrive at the poles, they would be unable to penetrate the protecting layer of air, which is equivalent, it must be remembered, to 10 metres of water. This absorption effect explains the increase in the latitude effect with increasing altitudes, as some of the less energetic particles which reach the earth's atmosphere at higher latitudes are able to effect only a partial penetration. Lack of penetration might also explain the levelling off of the curve of Fig. 97 at high latitudes. In this case, however, one would expect the latitude at which the sharp "knee" of the curve occurs to move to higher latitudes with increasing altitude, which is not in accordance with the observations. It is generally agreed that this "polar cap" indicates that the proportion of particles in the primary cosmic radiation having energies less than  $10^9$  eV is negligible.

The average energy of the primary particles reaching the earth's atmosphere at the magnetic equator has been estimated at  $3 \times 10^{10}$  eV and the number of such particles at about 2 per sq. cm. per minute. The total power of the rays striking the atmosphere is about a million kilowatts, and the incoming current, assuming all the rays to carry unit charges of the same sign, about 0.13 amperes.

**129. The East-West effect.** Ionization chambers, usually filled with argon under pressure in order to increase the ionization produced by the radiation, suffice for the intensity measurements so far discussed. These give no information either as to the number, direction or nature of the particles concerned. A Geiger counter (§ 100) can be



used to count the number of ionizing particles passing through its sensitive volume, the discharge which accompanies the passage of each particle being amplified, if necessary, by a suitable valve amplifier circuit. The amplifier circuit can be modified so that it only transmits a signal if two, or more, Geiger counters connected to it are simultaneously excited; that is to say (apart from fortuitous coincidences, which are reasonably rare), if the *same* particle passes through both counters. A pair of counters mounted one above the other with their axes parallel and at some distance apart, connected to such a circuit, fixes the line of travel of the particle whose presence it registers, and functions as a cosmic ray telescope.

Experiments with this apparatus have shown that, at any azimuth, there is a preponderance of particles coming from a westerly direction. The effect is a maximum at the equator, and for azimuthal angles between  $45^\circ$  and  $60^\circ$ , where it amounts to about 14 per cent. For simplicity, let us consider a positively charged particle approaching the earth in a vertical direction at the magnetic equator. It is cutting the lines of the earth's horizontal magnetic field at right angles, and an application of the well-known left-hand rule shows that it will be deflected so as to come in from a westerly direction. Conversely a negative particle would tend to approach from the east. The effect is, of course, complicated by the fact that particles are coming in from all directions. A positive particle approaching directly from the eastern horizon would, of course, reach the observer from the east, but at a smaller zenith angle. A complete analysis of the observations shows that practically all the primary particles capable of producing effects at ground level must be positively charged. They are usually supposed to be protons, though some authors have suggested that  $\alpha$ -particles may also be present.

It should be emphasised that the particles actually reaching the earth's surface are, as we shall see later, all secondaries, produced by the absorption of the primary cosmic particles in the upper atmosphere. As these secondary particles continue in the direction of the primaries from which they are produced, and as the magnetic deflection of such high-energy particles in the small thickness of the atmosphere is negligible, the east-west asymmetry gives us information about the *primary* cosmic particles.

130. Experiments with cloud chambers. The most complete information about the particles near ground level is clearly to be obtained from experiments with a Wilson cloud chamber, in which the actual tracks of the particles are made visible. If the apparatus is

operated at random, the chance of a particle passing through it during the fraction of a second for which it is in a sensitive state is uncomfortably small, though early experimenters worked in this way. The difficulty can be overcome by placing Geiger counters vertically above and below the chamber, which is usually mounted with its face vertical, so that a particle which passes through both counters must also pass through the expansion chamber. The signal generated when the two counters are fired off simultaneously is used to trigger off the expansion in the chamber. An expansion is, therefore, only made immediately after the passage of a particle. Apparatus on these lines was first designed by Blackett <sup>(2)</sup> and with various modifications and improvements has been extensively used by later experimenters.

In experiments with the radiations from radioactive substances we can easily distinguish between the tracks of, say,  $\alpha$ -particles, protons and electrons, by the density of the ionization (the specific ionization, as it is called) along the track (cf. Figs. 15 and 16). We have seen, however (§ 103), that the specific ionization due to an  $\alpha$ -particle is inversely proportional to its velocity, and a similar law applies to protons and other charged particles. The specific ionization produced by a really high-energy particle, such as those which occur in the cosmic rays, depends, in fact, mainly on its velocity and charge and only to a minor degree upon its mass; and the track of a high-velocity proton will not be markedly denser than that of an electron of similar velocity. Thus to identify a high-speed particle with certainty we need further information.

This can be supplied by applying a strong magnetic field across the cloud chamber and observing the resulting curvature of the track of the particle. Since the radius of curvature  $\rho$  is given by the relation  $H\rho = (mv)/e$  (Eqn. 29), the momentum of the particle can be determined, assuming its charge to be known. Thus the curvature of a proton track would be much smaller than that of an electron showing the same density of ionization. Since the specific ionization for different values of  $H\rho$  is known for each type of particle, the nature of the particle forming a given track can be determined by combining the two sets of observations. Incidentally, if an absorbing screen is stretched across the interior of the chamber, the curvature of the track before and after passage through the screen can be observed and hence the absorption of momentum.

In most instances where a single track crosses the cloud chamber, comparison of the magnetic curvature and the specific ionization enables the particles to be identified as high-speed electrons with

energies ranging up to  $10^{11}$  eV. Similar tracks, showing no measurable curvature, are probably those of electrons with energy too great to show any detectable deflection in the magnetic fields available. Proton tracks have occasionally been recorded; in the proportion of about 1 proton to 2000 electrons. The protons probably originate in some nuclear disintegration produced in the neighbourhood of the cloud chamber by the cosmic rays.

**131. Discovery of the positive electron.** In 1933 Anderson <sup>(3)</sup> noticed, on some of his cloud-chamber photographs, a certain number of paired tracks, appearing to originate from the same point usually in or near the casing of the chamber. The specific ionization and the radii of curvature of the twin tracks were very similar, and showed that each must be the track of a high-speed electron. The startling point about the photographs, however, was that the magnetic curvatures of the two particles were in opposite directions. Thus if one was negatively charged, the other must be positive.

It was, of course, just conceivable, that by some almost incredible coincidence, one particle was travelling towards the common point and the other away from it. In that case both particles would, of course, have the same sign. Anderson, however, was able to settle the point by the simple device of stretching a 6-mm. lead plate across the cloud chamber. Assuming that the common point of the two tracks is above the plate, Anderson found that the curvatures of both the tracks were considerably greater below the plate; that is to say, their energies were smaller below the plate than above. Each must therefore have been travelling downwards from the apparent point of origin. Therefore one must be positively charged.

The same photograph was sufficient to rule out the possibility that the positively charged particle might be a proton. Assuming the particle to be proton, the curvature of the track below the plate showed that its energy must be of the order of 300,000 eV. Such a low-energy proton would not only produce a very dense track, but would have a range in air of only about 5 mm. The observed track had a range of at least 5 cm. before it passed out of the field of observation. The only possible conclusion is that we are dealing with an entirely new type of particle, with mass comparable with that of an electron, but bearing a positive charge. The particle is known as the positive electron, or, more conveniently, as the *positron*.

**132. Origin of the positron.** We have already seen that on Einstein's theory of the equivalence of mass and energy, the mass of an electron is equivalent to energy of 0.5 MeV. Since however the electron has a

negative charge, and production of a negative charge requires the production of an equal and opposite positive charge, it would appear that the production of electrons could only take place in pairs, one being positively and the other negatively charged, and requiring in all about 1 MeV. Something of this kind had, in fact, been foreshadowed in a highly mathematical theory due to Dirac.

Thus there exists the possibility that the absorption under suitable circumstances of a photon of energy exceeding 1 MeV might result in the production of an electron pair, one of the pair being an ordinary electron, the other a positron. Apart from the cosmic rays such radiation is rare, but thorium C'' gives out a strong  $\gamma$ -radiation of energy 2.62 MeV which could be used for the experiment. Chadwick, Blackett, and Occhialini<sup>(4)</sup> concentrated the active deposit of thorium on a wire which was placed inside a lead cylinder enclosed in a Wilson chamber. A magnetic field of 600 gauss was applied across the chamber, and the resultant tracks were photographed in the usual way.

The photographs showed numerous tracks both of positive and negative curvature, about 12 per cent. of the tracks being due to positrons. It must be remembered that in addition to the  $\gamma$ -ray absorption which leads to pair production, we have also the normal Compton absorption of the  $\gamma$ -rays in the lead sheath, which gives rise to electrons only. It appears from the figures that for the thorium C'' radiation the Compton effect accounts for about 75 per cent. of the total absorption.

On the theory outlined a positron and an electron are produced simultaneously, so that we should expect to observe numbers of paired tracks starting from the same point but deviated in opposite directions by the magnetic field. One such pair is shown in Fig. 98 (Plate VI). The right-hand track is that of the positron, and in this case the two particles have been emitted with approximately the same energy, as shown by the equal curvatures. It is, however, not usually possible to observe these paired tracks, as the majority of the conversions take place well within the lead absorber, and owing to the large scattering of the particles in the lead, their tracks are generally widely separated before emergence.

The theory can be tested more satisfactorily by deducing the energies of the particles from the observed curvature of the tracks. The  $\gamma$ -radiation from the thorium deposit contains only one strong line of energy greater than 1 million volts, namely the line  $2.62 \times 10^6$  volts. The production of positrons may, therefore, safely be ascribed to this radiation. Since the creation of the pair of particles accounts for a

million volts the combined initial kinetic energies of the two particles cannot exceed  $1.6 \times 10^6$  volts. This energy seems to be distributed at random between the two particles. Assuming that in some conversions the whole of the excess energy is carried away by the positron we see that the maximum kinetic energy of the latter cannot exceed  $1.6 \times 10^6$  volts. In a careful measurement of the positron tracks the maximum energy observed was  $1.55 \times 10^6$  volts with a probable error of about 5 per cent. This agreement may be regarded as very satisfactory.

These calculations are based on the assumption that the mass of the positron is equal to that of the electron. The cloud chamber experiments only show it to be of the same order of magnitude. Positrons are now known to be emitted by many "artificial" radioactive materials (§ 150), and using such a source Spees and Zahn have recently been able to show, using a modification of Bucherer's method (§ 115), that the value of  $e/m$  for positrons is in fact identical with that for electrons. The positron is, of course, attracted by an electron, and on impact the two should suffer mutual annihilation, with the emission of the corresponding amount of energy in the form of radiation. The life of a positron, in matter of any appreciable density, should therefore be very brief.

It is easy to see that, if the annihilation occurs in free space, at least two photons of equal energy and moving in opposite directions must be emitted in order that the principle of the conservation of energy may be satisfied. The annihilation radiation should thus consist of 2 photons each of energy 0.5 MeV or wavelength 0.024 Å. It is significant that those artificial radio-active elements which emit positrons always give a strong  $\gamma$ -ray of this wavelength.<sup>(6)</sup> It may be claimed, therefore, that both the creation and annihilation of electron pairs are matters of observation.

**133. Absorption of high energy photons and electrons.** The phenomena outlined in the previous section indicate that for photons of energy greater than 1 MeV absorption may take place not only by the well-known process of photo-electric absorption, but also by the process of pair production. The probability of the latter occurrence increases with the energy of the photon, and for the most energetic particles is the more important of the two. The penetrating power of the radiation, therefore, does not increase indefinitely with decrease in wavelength, and photons of high energy may be more absorbable than those with energies of, say, 1 MeV.

The recent development of an electron accelerator known as the *betatron* has made it possible to produce X-radiation with quantum

energy up to 100 MeV and to study directly the absorption of such radiation. The results are contained in Fig. 99. It will be seen that

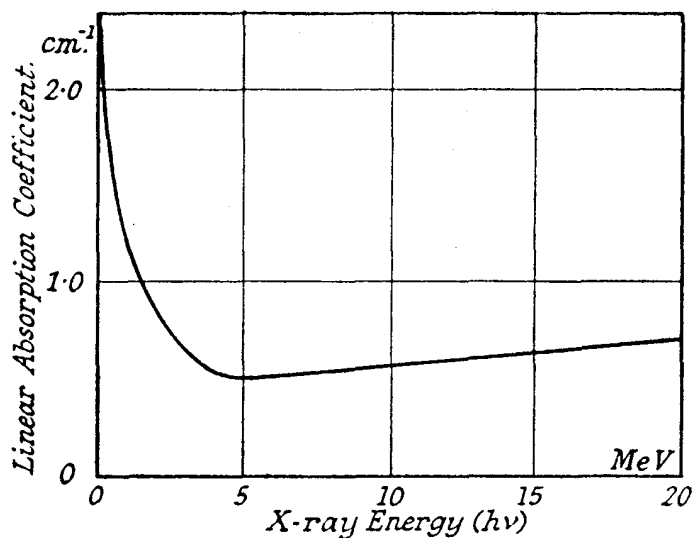


Fig. 99. Variation, with frequency of absorption coefficient in iron for high energy X-radiation

the coefficient of absorption has its minimum value in the neighbourhood of 5 MeV, and thereafter increases steadily with increasing energy.

The penetrating power of high-energy electrons takes a similar course, though for a different reason. Electrons and beta-rays of moderate energy lose their energy gradually, mainly by the process of ionization, the specific ionization decreasing with increasing velocity. Hence the range of the electron increases with its energy. We know, however, that if a beam of electrons is focused on the target of an X-ray tube, a small fraction (about 0.2 per cent. under normal working conditions) of the energy reappears as electromagnetic radiation, i.e. as X-rays.

The effect is ascribed, on the classical theory, to deceleration of the electron during a close approach to an atomic nucleus (§ 76); and the modern quantum theory differs from the classical theory mainly in regarding the energy emitted as being concentrated in a single high-energy photon. The fraction of the electron's kinetic energy so transformed increases rapidly with the actual energy of the electron. It is already quite important for the 100-MeV electrons produced by the betatron. Theory shows that at still higher energies, exceeding  $1.5 \times 10^8$  eV, this radiative loss ("Bremsstrahlung" or "braking-radiation," as it is called) is responsible for most of the absorption of the electron's energy. Thus, instead of the penetrating power of an electron increasing steadily with increasing energy, there is, in fact, an

upper limit to the penetrating power of electrons, something of the order of 10 cm. of lead. Since the whole thickness of the atmosphere corresponds to about 1 metre of lead, it is evident that the cosmic ray electrons which appear in the cloud-chamber photographs must have been formed well within the atmosphere, and, in fact, not many miles above the earth's surface.

**134. Cosmic ray showers.** Consider a  $10^{11}$  eV electron travelling downwards through the atmosphere. It loses energy rapidly by the "braking" process, giving rise to a series of high energy photons. These may well have an energy of, say,  $10^{10}$  eV each, and each of them may give rise, on approaching a nucleus, to a positron-electron pair. A million volts of the photon's energy is used up in this "materialisation," the remainder being shared between the twin particles. These again give rise by the "braking" process to more photons, which again produce fresh "pairs," so that the original electron gives rise to a numerous progeny of electrons, positrons, and photons. The process is illustrated, very diagrammatically, in Fig. 100. These particles

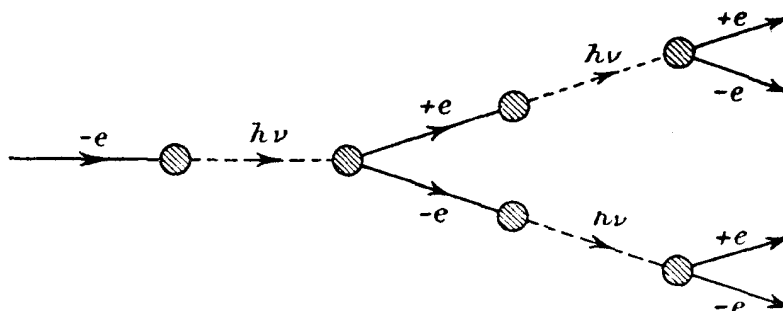


FIG. 100. Formation of a cosmic ray shower.

probably make up the greater part of the "soft" cosmic radiation, and are responsible for most of the tracks photographed in the cloud chamber. It is significant that the number of such particles increases rapidly, at first, with increasing height of ascent, decreasing again as the upper limit of the atmosphere is approached.

The process can be concentrated in space if some dense material, such as lead, is substituted for air as the absorbing medium. A single electron impinging on a lead plate enclosed in a cloud chamber may be seen to give rise to a sheaf of electrons and positrons emerging on the further side of the plate. Some of these may retain sufficient energy to produce fresh showers, either by absorption in the gas in the chamber (some of the chambers used are filled with argon at pressures up to 200 atmospheres, and hence have a very considerable absorbing power) or in lead plates enclosed in the chamber. Pair production may also

occur, owing to the photons which are also present, though they leave no "tracks."

A photograph of one of these "cosmic ray showers," as the phenomenon is called, due to Blackett, who first observed the effect, is shown in Fig. 101 (Plate VI). The shower shown originated outside the chamber, and many of the tracks are due to photo-electric effects in the walls of the chamber. As many as 300 particles, with a total energy of  $6 \times 10^{11}$  volts, have been observed in a single shower, though aggregates of energy from  $10^9$  to  $10^{10}$  eV are much more common. The very high energy manifestation is sometimes described as a "burst"; but there is no reason to suppose that it differs in any material respect from an ordinary "shower."

If a shower originates a few miles up in the atmosphere the resultant particles may produce simultaneous effects in a number of Geiger counters spread out over a fairly wide area on the earth's surface. Such "extensive" showers have been found by Auger and others to cover an area of at least 25 acres; the density of the electron hail, as we may call it, being of the order of 25 per square yard. Thus more than a million particles arrive simultaneously on the ground. Calculation shows that the particle initiating such a storm must have had an initial energy of at least  $10^{16}$  eV. It has been suggested by Heisenberg that some of the more energetic "bursts" may be due to an atomic explosion, initiated by some particularly energetic particle.

**135. Nature of the "hard" component.** In addition to the "soft" component, which, as we have seen, consists principally of electrons and positrons, we have noted that there is also a penetrating radiation, capable of producing effects through a thickness of 240 metres of water or its equivalent. The considerations detailed in § 133 show that this can consist neither of electrons nor photons. For similar reasons protons also appear to be ruled out, unless we are prepared to postulate initial energies for the particles which would be fantastic even on the cosmic-ray scale. What is required is some particle of mass intermediate between that of the electron and the proton. The deceleration of a charged particle in a nuclear field is inversely proportional to its mass, so that for a particle say 200 times the mass of the electron the "braking" effect would become negligible, and, at the same time, owing to its smaller mass the ionization loss per cm. for a given initial energy would be much less than that of the proton.

A particle of this kind had, in fact, already been postulated by mathematical physicists to explain the forces which bind together the particles forming the atomic nucleus. It is known as the *mesotron*,



sometimes shortened to *meson*. Evidence for the physical existence of this particle was sought, and discovered in the cosmic ray cloud chamber photographs.

A certain number of tracks were observed by Anderson (6), among others, which showed magnetic deflections comparable with those of the electron tracks but had a specific ionization about six times greater than that for an electron track of comparable  $H\rho$ . The value of  $H\rho$  for a particle is proportional to its momentum; the specific ionization depends mainly on its velocity. It is possible, therefore, by combining the two to obtain an estimate of the ratio of the mass of the particle forming the denser track with that of an electron. Various estimates ranging from about 100 to 300 were obtained.

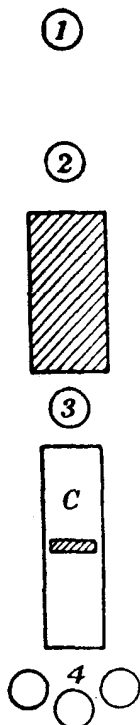


FIG. 102. Apparatus for detecting the stoppage of a mesotron in a "cloud" chamber

More convincing evidence can be procured if the particle can be observed to come to rest within the cloud chamber, so that simultaneous observations can be made of its magnetic curvature and its range. This was achieved by Street and Stevenson (7) using the ingenious arrangement shown diagrammatically in Fig. 102. Geiger counters 1 and 2 form a cosmic-ray telescope of the usual kind pointing at a block of lead 10 cm. thick to screen off the unwanted "soft" component. Counter 3 signals the passage of the particle through the lead screen. *C* is the cloud chamber, containing a lead screen to increase the probability of stopping the particle within the

apparatus, and 4 is a group of counters which will be actuated if the particle succeeds in leaving the system. The circuit is arranged so that a photograph is taken only if a particle sets off the counters 1, 2, and 3 and does not operate any of the counters in 4.

One of the photographs obtained showed the track of a particle with an  $H\rho$  of 1000 and a range of 7 cm. For a proton this value of  $H\rho$  would indicate an energy of  $4 \times 10^5$  eV, and a corresponding range of about 1 cm. On the other hand, the density of the ionization along the track was about six times that for a normal electron giving the same curvature. The particle was, therefore, demonstrably neither proton nor electron and the physical existence of the mesotron, in spite of its unusual properties, must be regarded as fully established.

**136. Properties of the mesotron.** Estimates of the rest mass of the mesotron give a mean value of about 200 times the rest mass of an electron. Different estimates range within rather wide limits, and some mathematicians are inclined to believe that these variations are real, and that the mass of the mesotron may depend on the nature of the nuclear transformation which gives rise to it. The charge may be either positive or negative, and is numerically equal to the electronic charge, since a greater charge would cause much higher specific ionization. The particle is unstable, even in free space, and disintegrates with the emission of an electron or a positron, according to the nature of its charge. In order to satisfy simultaneously both the conservation of energy and the conservation of momentum, a second uncharged particle must be emitted simultaneously with the electron. This is, presumably, the neutrino, which we invoked in § 125.

The existence of the mesotron has, in fact, cleared up the anomalies connected with  $\beta$ -ray emission from radioactive bodies. The actual emission during a so-called  $\beta$ -ray transformation is, presumably, that of a mesotron which carries away the whole of the energy released by the disintegration. The mesotron then disintegrates into an electron and a neutrino, its energy being shared in variable proportions between the two particles.

**137. Average life of a mesotron.** The mean life of the mesotron has been determined by an ingenious method. We have seen that the hard component of the cosmic rays consists of mesotrons, and the coefficient of true absorption in any material should be proportional to the mass per unit area. It is found, however, that if the absorption in, say, a mile of air is compared with the absorption in an equivalent thickness of water (about 5 ft.) the number of particles penetrating the air is less than half that penetrating the water. The missing particles must have disintegrated during the time taken to pass through a mile of air; approximately 5 microseconds, assuming that the velocity of the mesotron approaches that of light. Experiments on these lines give the mesotron a mean life of about  $2 \times 10^{-6}$  sec.

Since the velocity of the mesotron cannot exceed that of light, the average distance it could travel during its lifetime would be 600 m. It has been calculated on theoretical grounds that both the mean life and the mass of the mesotron should increase with its energy, and that a mesotron with an initial energy of  $10^{10}$  eV would have an average range of about 60 km. Since the decay of the mesotron, like other radioactive processes, presumably follows an exponential law, an appreciable fraction of the particles would thus have ranges con-

siderably greater than the average. Thus a mesotron liberated in the upper atmosphere with an initial energy of  $10^{10}$  eV would have a reasonable chance of reaching the earth's surface. It is obvious, however, that they cannot constitute the primary cosmic particles, but must be formed by collision of these particles with atomic nuclei in the upper atmosphere.

**138. Origin of cosmic radiation.** It must be remembered that in the cosmic ray field of research we are dealing with particles with energies up to a million times greater than those on whose behaviour our theories have been founded and tested. It is as if, having drawn an experimental curve 1 cm. long, we were asked to extrapolate it to a distance of 100 km. It is hardly surprising that we cannot yet provide an explanation which covers adequately the whole of the phenomena observed. The original cosmic particles are, as we have seen (§ 128), positively charged and are assumed to be high-energy protons which, by absorption in the upper atmosphere, give rise to mesotrons and high-energy electrons. The latter, by the process of pair-production, generate the soft component of the radiation found at lower levels, this being augmented by the electrons arising from the mesotron disintegration. The mesotrons themselves form the hard component. A few protons, resulting from nuclear disintegrations produced by the other particles, are also present.

As to the origin of the primary particles, much has been surmised, but nothing is known. Electric fields, large in extent rather than in intensity, existing between the stars have been suggested. Magnetic fields fluctuating at the rate actually observed in sun-spots could also produce energies of the order required if sufficiently extensive in space. The original suggestion of Millikan, that the rays are produced in the process of atom building in outer space, can be neither ruled out nor confirmed. For the present the problem remains unsolved.

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## CHAPTER XIV

### THE PHYSICS OF THE NUCLEUS

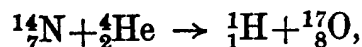
**139. Nuclear disintegration by  $\alpha$ -particles.** The fact that the masses of the atoms of all the elements could be represented approximately by a series of whole numbers (§ 53) naturally led to the assumption that the nucleus of the atom was a composite structure, made up of an integral number of elementary particles of equal mass. If so it should be possible by suitable means to disintegrate the nucleus, and thus to change one element into another. Apart from radio-activity, in which the radio-active atom spontaneously breaks down with the emission of an  $\alpha$ -particle or helium nucleus, the first evidence of nuclear disintegration was provided by Rutherford.

It was discovered by Rutherford <sup>(1)</sup> in 1919 that if a beam of swift  $\alpha$ -particles from radium C' was fired down a tube containing pure nitrogen, occasional scintillations could be observed up to distances corresponding to a range in air of 40 cm. This is far beyond the range (7 cm.) of the  $\alpha$ -particles themselves, and even beyond the range (28 cm.) of the protons projected by these particles in hydrogen. An examination of the particles by the magnetic deflection method confirmed that they were protons. Hydrogen, in the form of water, is difficult to eliminate completely in any experiment, but experiments made with oxygen and with carbon dioxide in place of the nitrogen did not yield any long-range particles, and it could safely be assumed that the particles observed in nitrogen were produced from the nitrogen atoms by the impact of the  $\alpha$ -particles.

The type of collision which produces disintegration of the nucleus is of very rare occurrence, the number of protons being of the order of 1 for  $10^4$   $\alpha$ -particles. Nevertheless Blackett, <sup>(2)</sup> using an automatic form of cloud chamber, was able to photograph seven such collisions. One of these is reproduced in Fig. 103 (Plate VII). It will be seen that only two tracks leave the fork, a faint one due to the proton ejected from the nucleus and a short dense track due to the new nucleus formed. Since there is no third track, the original  $\alpha$ -particle must have been absorbed in the nitrogen nucleus from which the proton is ejected.

The new nucleus will thus have a mass of  $14+4-1$  or 17. Since nitrogen has an atomic number and hence a nuclear charge of 7, the

new nucleus will have a nuclear charge of  $7+2-1$  or 8. It must therefore, be an isotope of oxygen of mass 17. The reaction may conveniently be expressed in the form



where the numbers above the letters indicate the atomic mass of the corresponding particle, and those below the atomic number.

Further research showed that protons could similarly be obtained from other elements of low atomic weight. In general the elements of odd atomic mass give long-range protons resembling those obtained from nitrogen. Elements of even atomic mass (with the exception of nitrogen) yield protons of much smaller range, and thus smaller kinetic energy. Elements of atomic number of the form  $4n$ , when  $n$  is an integer, as for example carbon and oxygen, are not disintegrated by  $\alpha$ -particles. Measurement of the ranges of the particles concerned in the collision indicates that the total kinetic energy of the particles after collision may be either greater or less than that of the colliding  $\alpha$ -particle.

**140. The existence of a neutron.** It was discovered by Bothe and Becker that some of the lighter elements, and in particular boron and beryllium, emit a highly penetrating radiation when bombarded by the  $\alpha$ -particles from polonium. Polonium is used in these experiments because it emits no  $\gamma$ -radiation to complicate the observations. The radiation emitted from beryllium had a coefficient of absorption of 0.22 per cm. in lead, and applying the Klein-Nishina formula (105) it was found that its energy, assuming it to be  $\gamma$ -radiation, must be about  $7 \times 10^6$  volts. This was higher than any previously known  $\gamma$ -radiation.

It was found by Curie-Joliot that if this radiation was allowed to fall on paraffin wax, or any other material containing hydrogen, protons were ejected with a maximum range of as much as 26 cm. in air, and consequently a maximum energy of the order of  $5 \times 10^6$  volts. Since there can be no question of disintegration in the case of a hydrogen nucleus, the protons could only be regarded as having been set in motion by the Compton recoil from a direct collision with the  $\gamma$ -ray quant. The maximum energy which can be so transferred from a photon of energy  $h\nu$  to a particle of mass  $M$  is given by  $\frac{2h\nu}{2 + Mc^2/h\nu}$ .

We can thus estimate  $h\nu$  for the beryllium radiation. It works out at  $55 \times 10^6$  volts; a result quite inconsistent with that deduced from its coefficient of absorption.

Chadwick<sup>(3)</sup> investigated the effect of the radiation on other elements, both in the solid and in the gaseous form. The most complete information can, perhaps, be obtained by passing the radiation through a Wilson cloud chamber containing the gas under investigation. The projected nuclei produce intense ionization along their track, so that the length of the track can be measured and the corresponding energy of projection deduced. It can also be confirmed from the absence of any second track that disintegration has not occurred. It was found that in nitrogen, for example, the projected nuclei had a range of about 3.3 mm., corresponding to an initial kinetic energy of  $1.2 \times 10^6$  volts. Using the Compton relation the corresponding energy of the beryllium radiation becomes  $90 \times 10^6$  volts. In fact, the greater the mass of the recoiling particle the greater the energy it was necessary to assume in the incident radiation.

The complete inconsistency of these observations when interpreted on the theory that the beryllium radiation is of the nature of  $\gamma$ -radiation led Chadwick to put forward the suggestion that the radiation consists of material particles of mass comparable with that of an atom. To explain the very high penetration of the particles it was necessary and sufficient to assume that they carried no charge, and it was therefore proposed that they should be called *neutrons*.

Assuming the neutron hypothesis, the atomic nuclei are set in motion by direct collision with the high-velocity neutral particle, and the problem is simply one of elastic impact between the two bodies. Let  $M$  and  $V$  be the mass and velocity of the neutron,  $A$  the mass of the atomic nucleus and  $u$  its velocity after impact. The maximum value of  $u$ , for a head-on collision, is given by

$$u = \frac{2M}{M+A} \cdot V \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

Thus, if  $u_p$  is the maximum velocity given to a proton and  $u_n$  that to a nitrogen nucleus, both of which can be deduced from the observed ranges of the particles, we have

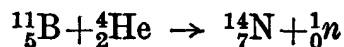
$$\frac{M+14}{M+1} = \frac{u_p}{u_n} = \frac{3.3 \times 10^9}{4.6 \times 10^8} \quad M=1.1.$$

The mass of the neutron is thus approximately equal to that of the proton. Applying the same argument to similar measurements made in argon and oxygen, it was found that the ranges observed could all be explained within the limits of experimental error on the assumption that the respective nuclei were set in motion by collision with a neutral particle of approximately unit atomic mass.

The measurements cannot be made with sufficient precision to give a very exact value for the mass of the neutron. Indirect methods, based on a study of nuclear disintegration (§ 149), show that its mass is 1.0089. It is thus slightly heavier than the proton (1.0076).

**141. Origin of the neutrons.** Neutrons, as we have seen, are produced by the  $\alpha$ -ray bombardment of the nuclei of the lighter elements, and there is evidence that all the elements up to fluorine can be made to yield neutrons under suitable conditions with the exception of helium, carbon, nitrogen, and oxygen, and of course hydrogen. The yield is greatest in the case of boron and beryllium, and a sealed tube containing a mixture of beryllium with either radium or radon provides a convenient neutron source. Neutrons are also produced when deuterium oxide ("heavy water") is bombarded by high-speed deuterons from a discharge tube, or cyclotron (§ 145). As the number of deuteron particles which can be produced in this way far exceeds the number of  $\alpha$ -particles given out even by a gram of radium, this method provides a far stronger source of neutrons than the use of radio-active material.

The neutrons themselves must result from an actual disintegration of the atom of the bombarded element. We may take the case of boron as an example. It is known that the  $\alpha$ -ray bombardment of the isotope  $^{10}\text{B}$  results in the emission of a proton. The isotope concerned with neutron emission must therefore be  $^{11}\text{B}$ . In all cases of nuclear disintegration that have been studied, disintegration is accompanied by the capture of the bombarding particle by the nucleus. The reaction must, therefore, be



where the neutron having a mass number of unity and zero charge is represented by the symbol  $^1_0\text{n}$ .

**142. Passage of neutrons through matter.** Owing to the absence of charge on the neutron, the particle is but little affected by the charges either of the electrons or of the nuclei of the atoms through which it passes, and can only transfer energy to either by what we may describe as a direct hit. On account of the very small radii of all the particles concerned this is a very infrequent event, and the neutrons are thus able to pass through very considerable thicknesses of matter before their energy is dissipated. From experiments made with a cloud chamber, Dee has estimated that a neutron in air makes an ionizing collision with an electron or a nucleus not more frequently than once in every 3 m. of its track. Its track is thus not visible in an expansion

chamber, and its claim to be included in a work on ionizing radiations is a little tenuous.

The possibility of detecting neutrons by the ionization chamber is due to the fact that, as we have already noted, occasional collisions are made with atomic nuclei which give the latter sufficient energy to produce a considerable number of ions in the gas. Each collision is accompanied by a burst of ionization, and thus can be recorded by counting apparatus of the usual kind (§ 100). We may assume that the number of collisions in a given volume of gas is proportional for neutrons of the same velocity to the number of neutrons passing through it.

From (108) it can readily be seen that the energy lost in a collision by the neutron with a nucleus decreases rapidly as the mass of the nucleus increases. Thus in substances of high atomic weight the neutrons are scattered with very little loss of energy. On the other hand, in light elements, and particularly in hydrogen, the loss is very considerable. On an average a neutron will lose one-third of its energy at each collision with a hydrogen nucleus, and thus in substances like water or paraffin wax the neutrons are rapidly slowed down until their energies are reduced to energy of thermal agitation. Some of these slow or *thermal* neutrons, as they are called, diffuse from the boundaries of the substance, and their effects can thus be studied. They remain in existence until captured by some nucleus.

**143. Nature of the neutron.** The close approximation of the mass of the neutron to that of the proton invites the suggestion that the two particles are closely related. The view held at present is that the two particles represent different states of the same fundamental particle, known as the *nucleon*, from which all atomic nuclei are constructed. It is, in fact, supposed that within the nucleus proton and neutron are constantly and rapidly interchanging their identities by emitting or absorbing the appropriate charge, the charge-carrier being supposed to be a mesotron. Thus a proton emitting a positive mesotron would become a neutron, while the neutron which absorbed the particle would be transformed into a proton. Conversely, the emission by a neutron of a negative mesotron would convert it into a proton. The theory was proposed by Yukawa in 1935 to explain the strong forces which bind the particles together in the nucleus, and which obviously cannot be coulombian, since the neutrons have no charge and the protons, being all positively charged, repel each other. The discovery of the mesotron has provided a physical basis for these mathematical conceptions.

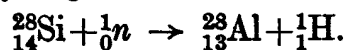


**144. Disintegration by neutrons.** For gaseous elements disintegration by neutron bombardment can be observed in a Wilson cloud chamber. The ordinary elastic collision between a nucleus and a neutron results in the formation of a single short thick track, due to the recoil of the nucleus; the track of the neutron itself being invisible. If disintegration occurs two tracks are observed, starting from the same point: a short, thick track due to the new nucleus and a longer track, of smaller density, due to the particle ejected. In the case of solid elements, disintegration can be deduced by detection of the high-speed particles ejected. In still other cases the new nucleus formed is radio-active (§ 150) and can be detected by the radiations it emits.

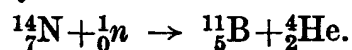
Whereas, in order to produce disintegration, the  $\alpha$ -particle must possess sufficient energy to overcome the repulsion of the bombarded nucleus, the neutron, being uncharged, is not repelled by an atomic nucleus. It is thus able to enter a nucleus of any atomic number, however high, and all elements up to and including uranium suffer neutron disintegration. Moreover, the capture is the more likely to occur the longer the neutron remains in the neighbourhood of the nucleus, and, therefore, if disintegration is one which can take place without requiring any external energy (i.e. if the reaction is exothermic) slow neutrons are more effective than fast neutrons in producing disintegration.

Neutron disintegration may result in the emission of either a proton or an  $\alpha$ -particle. A third type is also known, in which no material particle is ejected, the surplus energy being emitted as a high-energy photon. In all cases the neutron is itself captured by the nucleus.

Since the mass numbers of the proton and neutron are the same, the capture of a neutron with the emission of a proton leaves the mass number of the nucleus unaltered, but reduces its charge and hence its atomic number by unity, e.g.

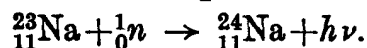


The emission of an  $\alpha$ -particle is illustrated by the following reaction, which has been studied by Feather in the Wilson chamber:



This is illustrated by the photograph in Fig. 104 (Plate VII). A neutron coming from below, and invisible in the picture, has broken the nucleus of nitrogen into a boron nucleus moving to the left, and an  $\alpha$ -particle shown by the fainter track. It is interesting to note that this reaction is the exact converse of that by which neutrons are produced. We have here an example of a reversible nuclear reaction.

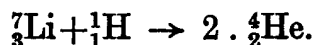
The simple capture of a neutron, with no corpuscular emission leaves the atomic number of the nucleus unchanged but increases its mass by unity, i.e. it produces a heavier isotope of the substance bombarded:



The surplus energy is emitted from the nucleus in the form of  $\gamma$ -radiation.

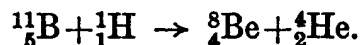
**145. Disintegration by protons and deuterons.** It was discovered by Cockcroft and Walton <sup>(4)</sup> that elements could be disintegrated by bombarding them with high-speed protons from a discharge tube. The positive particles were generated by passing a current through hydrogen in a discharge tube with a perforated cathode, and the positively charged hydrogen rays passing through the cathode were further accelerated by passing them through a long column the ends of which were at a potential difference of the order of 300,000 volts and upwards. The accelerated particles emerged through a small window in the end of the discharge tube, and were caused to impinge on a target of the element under experiment. Lithium was selected for the first experiment.

It was found that under bombardment the lithium gave off particles which could be detected by the scintillations they produced on a fluorescent screen or by an ionization chamber. They were immediately identified as  $\alpha$ -particles. By enclosing the target in a Wilson cloud chamber it was possible to photograph the tracks of the particles, and by reducing the intensity of the bombardment so that only one disintegration occurred during a single expansion, it was found (Fig. 105, Plate VII) that the particles were ejected in pairs travelling in opposite directions along the same straight line. The disintegration may thus be expressed in the form

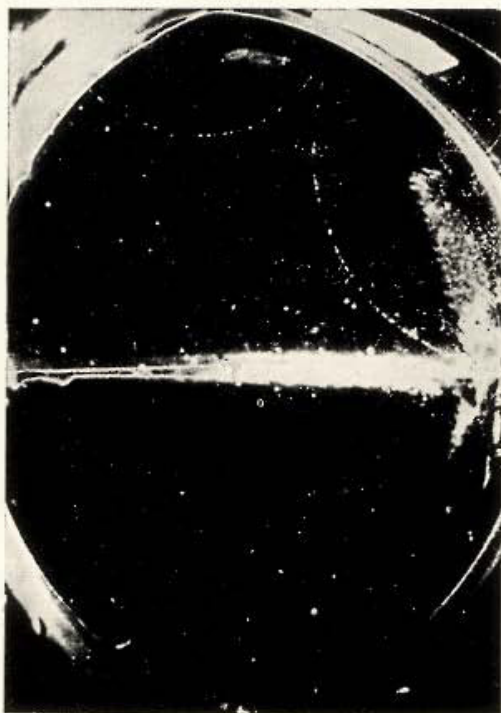


The  $\alpha$ -particles have each a range of 9 cm. in air, corresponding to an initial kinetic energy of  $8\frac{1}{2}$  million volts. The action, therefore, results in a very large evolution of energy. It should, however, be pointed out that as only about one proton in ten thousand succeeds in producing a disintegration the experiment as a whole does not leave us with a surplus of energy.

Disintegration by protons takes place by the capture of the proton and generally with the emission of an  $\alpha$ -particle, as for example in the reaction

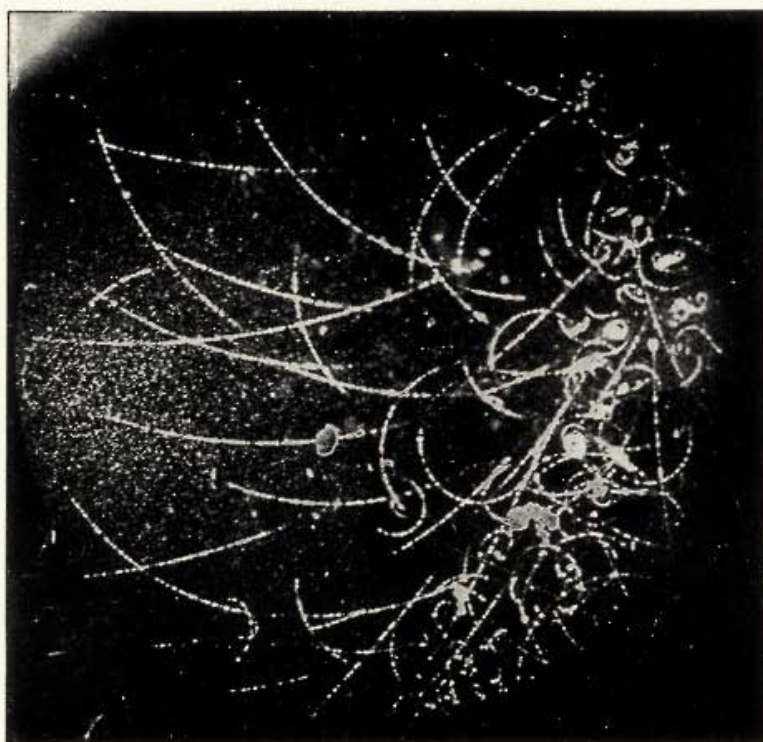


One well authenticated exception to this statement is the case of



*J. Chadwick, P. M. S. Blackett and G. Occhialini.*

FIG. 98. TRACK OF A POSITRON.



*P. M. S. Blackett.*

FIG. 101. COSMIC RAY BURST.

PLATE VII.



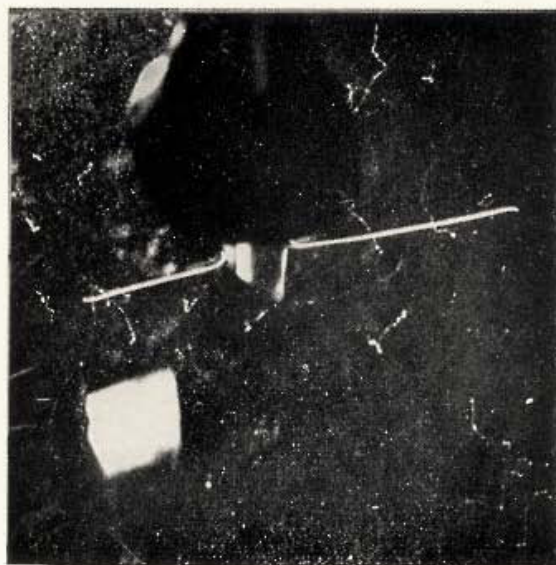
*P. M. S. Blackett.*

FIG. 103. DISINTEGRATION OF THE NITROGEN NUCLEUS BY  $\alpha$ -PARTICLES.



*Feather: Nuclear Physics, C.U.P.*

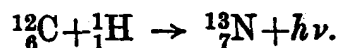
FIG. 104. DISINTEGRATION OF NITROGEN NUCLEUS BY NEUTRON.



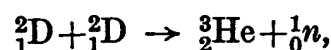
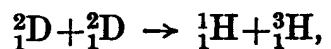
*P. I. Dee and E. T. S. Walton.*

FIG. 105. DISINTEGRATION OF LITHIUM.

carbon. Here the proton is captured, and no material particle is emitted, the new nucleus being an unstable isotope of nitrogen:



The action of deuterons is more complicated, and the particle emitted may be either a proton, a neutron or an  $\alpha$ -particle. Not infrequently the reaction between the deuteron and a given nucleus may take place in alternative ways, as in the case of the bombardment of deuterium by deuterons, where the two reactions



occur with approximately equal probability. The latter reaction is important as it provides by far the most prolific source of neutrons.

**146. Production of high-energy particles. The cyclotron.** To generate the high potentials required, Cockcroft used a system of rectifying valves and condensers, the circuit being essentially the same as the well-known "voltage doubling" circuit employed in radio engineering. An ingenious alternative method by which high-speed particles can be obtained by the use of potentials of no more than a few thousand volts is provided by the *cyclotron*, invented by Lawrence and Livingston.<sup>(5)</sup>

A charged particle moving in a plane at right angles to a uniform magnetic field describes a circular arc of radius of curvature  $\rho$  given by  $mv/eH$  (29). The time taken to describe a complete semi-circle in the field is  $\pi\rho/v$  or  $\pi m/eH$ . This is independent both of the velocity of the particle and the radius of the semi-circle. The cyclotron consists of a flat circular box (Fig. 106) divided into two parts, *A* and *B*, by a cut

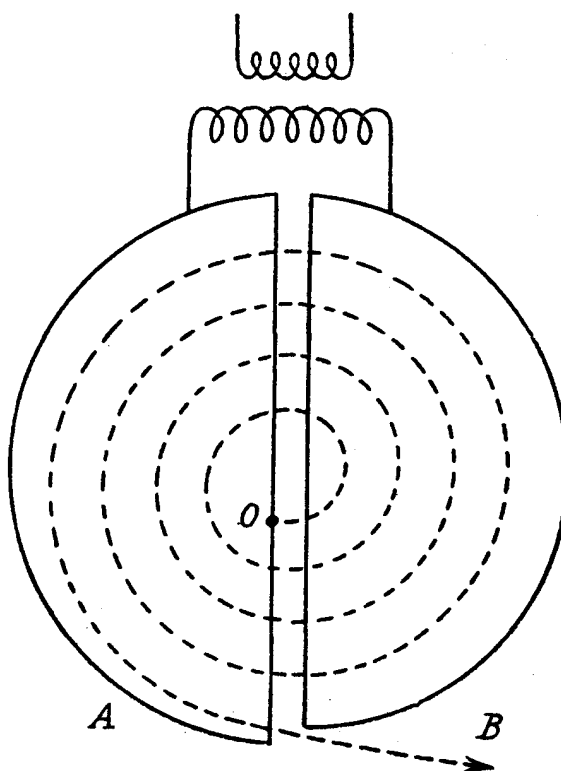


FIG. 106. Diagram showing the action of the cyclotron. The dotted lines indicate the path of a positively charged particle

along a diameter. The two halves, known as dees, are insulated and can be charged to a difference of potential of a few thousand volts by means of a valve oscillator. Positive ions are formed at  $O$ , near one edge of the gap, and, when the field is in the proper direction, will be accelerated towards the other edge, reaching it with the energy due to the P.D. between the two boxes, say 40,000 volts. On entering  $B$ , the particle is in a field-free space and moves with constant velocity. The boxes, however, are between the poles of a large electromagnet, so that the particle describes a semi-circle and reaches the edge of  $B$  in a time  $t$  equal to  $\pi m/eH$ . If the frequency of the oscillator is so adjusted that the field between  $A$  and  $B$  has reversed during this interval, the particle will be accelerated towards  $A$  and its energy will increase by another 40,000 volts. The period of the oscillator must obviously be  $2t$ . Thus the path of the particle is a spiral of gradually increasing radius, and its energy increases by 40,000 eV every time it crosses the gap.

The energy attainable depends on the number of turns in the spiral path of the particle, that is to say, on the strength of the magnetic field and the area of its pole faces. Thus very large magnets are required. The largest cyclotron working at the time of writing has poles 5 feet in diameter and can produce a proton beam of 160 microamperes, or  $10^{15}$  particles per second of energy as high as 25 MeV. A still larger cyclotron is under construction with a magnet having pole faces 184 inches across, which is expected to produce particles with energies up to 100 MeV.

The production of high-energy particles by the direct application of high potentials, as in Cockcroft and Walton's experiments, is limited by insulation difficulties, which become very serious at voltages exceeding 2 million, and probably insuperable at about 6 million volts. Such voltages can be generated by electrostatic influence machines, such as that of Van de Graaff, by Tesla transformers, or by transformers working in cascade. The two latter methods are, in fact, employed in commercial high-voltage X-ray generators. The advantage of the cyclotron is that, although the energy of the issuing particles may be of the order of 25 MeV or more, no part of the apparatus is at a potential greater than the 40,000 volts or so produced by the oscillator. The action of the cyclotron is limited by the relativity increase in mass of the particles at high velocities, which causes them to take longer to describe their orbits as their energy increases. For a 100-MeV proton, for example, with a velocity approaching one-half that of light the effect is already serious, and will be dealt with by "shaping" the magnetic poles.

**147. The betatron.** An electromagnetic device for producing high-energy electrons may be described here, although its main application so far has been in the generation of very high energy X-radiation. Consider an electron projected with momentum  $M$  at right angles to a magnetic field of instantaneous value  $H$ . It will describe a circle of radius  $r$  given by  $M=Her$ . Let  $\phi$  be the magnetic flux included in this orbit, and let it be increased. Then, by Faraday's laws of induction, the induced E.M.F.  $E$  round the electron orbit is given by  $E=-d\phi/dt$ . The electric intensity  $X$  acting round the orbit is the potential drop per cm. and is given by  $X=E/2\pi r$ . Thus the force  $P$  acting on an electron in this orbit is given by

$$P=Xe=\frac{e}{2\pi r}\frac{d\phi}{dt}$$

$$=\frac{dM}{dt}$$

by Newton's second law of motion. Thus, integrating,

$$M-M_0=\frac{e}{2\pi r}(\phi-\phi_0) \quad . \quad . \quad . \quad . \quad (109)$$

But  $M=Her$ , and thus

$$er(H-H_0)=\frac{e}{2\pi r}(\phi-\phi_0)$$

or

$$r^2=\frac{1}{2\pi}\left(\frac{\phi-\phi_0}{H-H_0}\right) \quad . \quad . \quad . \quad . \quad (110)$$

Thus  $r$  will be constant and the electron will continue in the same orbit if  $\phi$  is proportional to  $H$ . This is clearly the case if the particle is moving in the space between the poles of one and the same electro-magnet.

If  $[H]$  is the average field inside the orbit, then  $\phi=\pi r^2[H]$ . Thus the field  $H$ , in which the electron is actually travelling must be half the value of the average field inside the orbit. This can be effected by shaping suitably the pole pieces or by inserting soft-iron rods between the pole faces. Since the calculation is concerned only with the momentum of the electron, it is immaterial whether this is expressible in the classical form or whether the relativity expression is applicable. The conclusions are, therefore, not affected by the increase in mass with velocity.

In the betatron the electrons move in a hollow evacuated annulus, shown in section at  $D, D$  (Fig. 107 *a*), and known as a "doughnut," placed between the poles  $P, P$ , of an electromagnet. The pole pieces are shaped, somewhat as indicated in the diagram, and a number of soft-iron wires,  $w$ , can be inserted between the poles in order to increase the magnetic flux inside the annulus to its correct value.

The magnet is actuated by a 180-cycle (Fig. 107 *b*) alternating current. A beam of electrons, from a hot wire source, is injected into

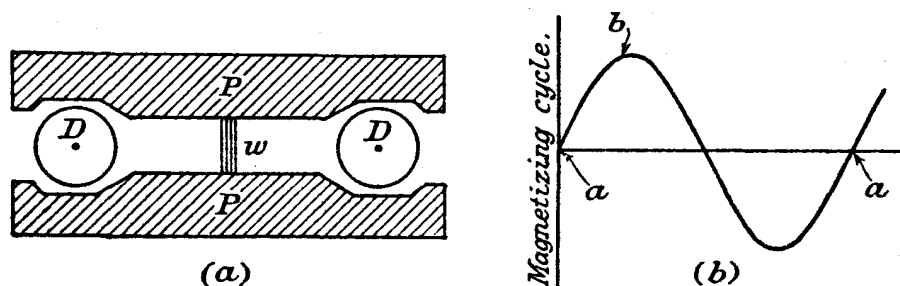


FIG. 107. Illustrating the action of the betatron: (a) Section of betatron; (b) Magnetizing cycle.

the field at the instant  $a$  when it is passing through its zero value, and is deflected out by an auxiliary magnetizing coil, at  $b$ , just before the field reaches its maximum. If required the electron beam can be made to emerge through a suitable window in the doughnut; more usually it is allowed to impinge on a target, with the production of high-energy X-radiation.

An early model, constructed by its inventor, Kerst,<sup>(6)</sup> using a magnet weighing 3.5 tons, gave electron energies of 20 MeV. A much larger machine giving an output of electrons up to 100 MeV is now commercially available.

**148. Verification of the equivalence of matter and energy.** If the exact masses of the atoms, as given by mass-spectrographic determinations (see Table XVII), are inserted in the relations given in the preceding sections it will be found that the total mass on the left-hand side of the equation is not exactly equal to that on the right. It may be slightly greater or slightly less. It is for this reason that we have written the relations with an arrow, and not with the usual equality sign. This inequality arises, of course, from the fact that the masses of the different atoms are not exactly whole numbers; in other words, from the "packing" effect discussed in § 54. Again, if the kinetic energies of the particles taking part in the reaction are estimated, by determining their range in air, it is found that the total kinetic energy



may be either greater or less after the reaction has taken place. It is clear that neither the principle of the conservation of mass nor the principle of the conservation of energy applies separately to nuclear reactions. It remains to be seen whether Einstein's suggestion (§ 117) of the equivalence of mass and energy will enable us to combine the two into a principle of conservation of mass-energy. For various experimental reasons the data are generally most accurate for disintegrations produced by artificially accelerated particles. Let us take as an example the disintegration of lithium by protons, which, as we have seen, can be written



The mass of the lithium atom is 7.0180 mass units (on the basis O=16) and that of the proton 1.0076. The mass of the lithium atom, however, includes that of its three extra-nuclear electrons, each of mass 0.00054 mass units, which take no part in the reaction. Hence the total mass on the left-hand side of the equation is  $7.0164 + 1.0076 = 8.0240$ . The mass on the right-hand side is  $2 \times 4.0029 = 8.0058$ ; there has therefore been a loss of 0.0182 mass units, or  $3.02 \times 10^{-26}$  gm. (§ 27).

On the other hand, the energy of the proton is about 0.3 MeV while the two  $\alpha$ -particles have a total kinetic energy of 17 MeV. There has thus been a liberation of energy of 16.7 MeV or  $2.67 \times 10^{-6}$  ergs. Thus the ratio of the kinetic energy gained to the mass lost is  $2.67 \times 10^{-5} / 3.02 \times 10^{-26}$ , or  $8.9 \times 10^{20}$ . The agreement between this experimental determination and the factor  $c^2$  in Einstein's relation  $E=mc^2$  is in fact surprisingly good. Many other disintegrations have been studied, with equally satisfactory results.

The full equation for the reaction can, therefore, be written

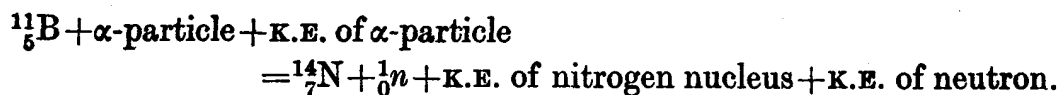
$${}^7\text{Li} + \text{proton} + \text{K.E. of proton} = 2 \times (\alpha\text{-particles}) + \text{K.E. of } \alpha\text{'s} \quad (111)$$

$$(7.0164) \quad (1.0076) \quad (0.0003) \quad (2 \times 4.0029) \quad (0.0182)$$

all the quantities concerned being, of course, expressed in the same units, mass units being usually employed.  $1 \text{ MeV} = 1.60 \times 10^{-6}$  ergs. i.e.  $1.60 \times 10^{-6} / 9 \times 10^{20}$  or  $1.78 \times 10^{-27}$  gm., using Einstein's relation. Since the mass unit is  $1.66 \times 10^{-24}$  gm., we have  $1 \text{ MeV} = 1.07 \times 10^{-3}$  mass units; or 1 mass unit = 931 MeV.

**149. The mass of the neutron.** The method can obviously be applied to determine the mass of one of the particles taking part in the nuclear reaction, and Chadwick<sup>(7)</sup> pointed out that it should be possible to obtain a value for the mass of the neutron, much more accurate than that deduced from collision experiments, by analysing

the nuclear reactions which lead to neutron emission. Taking as an example the production of neutrons from boron we may write the equation in the complete form:



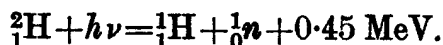
The kinetic energy of the neutron was deduced by finding the range of the fastest proton which it can eject from a sheet of paraffin wax, and the energy of recoil of the newly formed nitrogen nucleus calculated from the energy of the neutron, assuming the conservation of momentum.

Substituting the experimentally determined energy values, in mass units, in the equation, and using the best data then available for the masses of boron and nitrogen, Chadwick obtained the equation

$$11.0082 + 4.0010 + 0.0056 = 14.0042 + {}^1_0\text{n} + 0.0006 + 0.0035.$$

Allowing for the fact that the mass of the nitrogen atom includes two more extra-nuclear electrons than that of boron, this gives a value for the mass of the neutron of 1.0067.

Still greater accuracy can be obtained from the study of a reaction of a type which we have not yet considered. If deuterium is exposed to the hard  $\gamma$ -radiation from thorium C' ( $h\nu = 2.62$  MeV) the deuteron splits up into a proton and a neutron, with the evolution, in the form of kinetic energy of the particles, of 0.45 MeV. Thus we have



Substituting, after conversion into mass units, gives

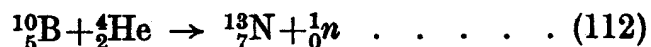
$$2.01473 + 0.00281 = 1.00813 + {}^1_0\text{n} + 0.00048,$$

or  ${}^1_0\text{n} = 1.00893$ .

**150. Artificial radio-elements.** It was discovered by the Curie-Joliot, <sup>(8)</sup> in 1934, that if a target of some light element, for example boron or aluminium, was bombarded with  $\alpha$ -particles, the target continued to emit ionizing radiation for some little time after the removal of the  $\alpha$ -ray source. The activity decayed exponentially with the time, and the substance showed the usual characteristics of a radio-active substance. A little later, Fermi showed that many of the new nuclei formed by neutron bombardment are radio-active, and was able to produce artificial radio-elements from a large number of elements throughout the atomic scale.

The chemical nature of the radio-element can often be determined by chemical experiments. Although the amount of the radio-element

is usually far too small to be detected by chemical means, its presence can always be demonstrated by the radiation it emits, and its role in a chemical reaction can thus be determined. Thus if boron nitride is bombarded with  $\alpha$ -particles (which, as we have seen, turns the boron into a radio-element) and the substance is treated with caustic soda, ammonia is evolved which carries the activity with it. Since the protons in the ammonia cannot be radio-active, the new radio-active substance formed from the boron must be a form of nitrogen. The action in this case seems to be



The radiation emitted by the artificial radio-elements usually consists either of positive or negative electrons. If the artificially formed nucleus has a mass number greater than that of the stable isotope of the element it emits a negative electron (thus providing itself with an additional positive nuclear charge) and passes into a stable form of the element of next higher atomic number. Thus radio-sodium  ${}^{24}_{11}\text{Na}$  gives out a negative electron and changes into  ${}^{24}_{12}\text{Mg}$ . On the other hand, a radio-element which has a smaller mass than the normal isotope will eject the superfluous positive charge and change into a stable isotope of the element next lower in atomic number; e.g.



As an alternative it may reduce its charge by capturing an electron from its own *K* level. In this case no material radiation is emitted, but since there is now a vacancy in the *K* level this will be filled by the capture of an electron, and the substance emits its characteristic X-ray spectrum.

Each radio-element, of which over 300 have now been listed, has its own characteristic half value period. These range from a fraction of a second to several months.

**151. Application of artificial radio-activity. Tracer elements.** Radio-active isotopes of most of the elements can now be produced in appreciable quantities. The cyclotron, for example, can produce preparations of radio-sodium having an activity equal to that of several hundred milligrams of radium in an eight-hour run. Far larger amounts can be obtained from the atomic energy piles (§ 169) now operating, and are being made available for research purposes and medical treatment.

Some of these seem to offer appreciable advantages over the natural radio-active elements in the treatment of diseased tissues. Radio-sodium, for example, has a half value period of 14·8 hours and the final

product is, of course, completely inactive. The element may thus be taken orally into the body, and the exact amount of energy it will liberate there can be calculated. The need for recovering the substance after treatment is thus eliminated. Advantage may also be taken of the body's habit of localizing certain elements in certain structures. Thus most of the iodine taken internally finds its way to the thyroid, so that the action of radio-iodine would be largely concentrated in that organ.

Extensive use has already been made of these substances as "tracer" elements in the study of the metabolism both of plants and animals. The presence of a radio-element can be detected with ease and certainty by the radiation it emits, and it can, of course, be distinguished from the mass of the stable isotope already in the body, since the latter is inactive. It has thus been shown, for example, by the use of radioactive phosphorus compounds that 62 per cent. of the phosphorus absorbed by the body finds its way into the bony structure within five days, showing a remarkably rapid rate of exchange between existing structures and the elements in the blood stream. The field for this type of research seems almost illimitable.

**152. Composition of the nucleus.** The facts outlined in the previous sections lead to the inevitable conclusion that, in spite of its very minute dimension, an atomic nucleus is a composite structure. The particles ejected under bombardment consist invariably either of protons, neutrons, or  $\alpha$ -particles, and it is clear that we must regard these as the units of which the nuclear structure is compounded.

As we have already seen, the proton and neutron are now regarded as two forms of a fundamental particle, the nucleon, from which all matter is constructed. On the other hand, the  $\alpha$ -particle, though a very stable structure, is clearly complex, since it can be produced in nuclear reactions which leave no doubt of its composite nature. For example, the addition of a proton to  ${}^7_3\text{Li}$  or of a deuteron to  ${}^6_3\text{Li}$  gives rise in each case to a pair of  $\alpha$ -particles. The  $\alpha$ -particle is thus composed of two protons and two neutrons, giving it a charge and atomic number of two and a mass number of four.

Heavier nuclei can be built up by the addition of further protons and neutrons. The atomic number  $Z$  is equal to the total charge on the nucleus, and hence to the number of protons it contains. The mass number  $A$  is clearly the total number of nucleons (protons and neutrons) in its structure. Thus a nucleus of atomic number  $Z$  and mass number  $A$  consists of  $Z$  protons and  $A - Z$  neutrons. The fact that the actual masses of the nuclei differ slightly from whole numbers, and that both

the nucleons have masses distinctly greater than unity, on the atomic mass scale, has, as we have seen, been adequately accounted for on the assumption that these differences represent the energy which has been dissipated during the formation of the nucleus from its constituent nucleons; or, conversely, the energy which would have to be supplied to disintegrate the nucleus into its constituent nucleons.

Since the atomic number, which determines the chemical nature of the element, is fixed by the number of protons it contains, it would appear that a given element might have any mass number greater than  $A$ , simply by the addition of more and more neutrons. The existence of isotopes is therefore more than adequately explained. Though some of the heavier elements consist of as many as ten or twelve isotopes, it is clear that, in general, stability considerations limit the ratio of the number of protons to the total number of nucleons in the nucleus within comparatively narrow limits. For the lighter elements the ratio is about one half; it decreases gradually as we pass up the atomic scale, and the heaviest element, uranium, has 238 nucleons in all, of which 92 are protons. This instability is clearly shown by a study of the artificial radio-elements, where, as we have already seen, if the ratio of the mass number to the atomic number is either unduly large or unduly small, the nucleus adjusts matters by the expulsion in the first case of a negative electron, in the second of a positron.

There is much evidence to suggest that the nucleons group themselves into  $\alpha$ -particles within the nucleus, just as the molecules of  $H_2O$  tend to polymerise in liquid water.

$\alpha$ -particles are ejected by the natural radio-active elements, and, as we shall see later (§ 158), seem to play an important part in the economy of those nuclei. It is also significant that elements of mass number  $4n$ , where  $n$  is an integer, are not disintegrated by  $\alpha$ -particle bombardment; and that, when disintegrated by neutrons, the result is the ejection of an  $\alpha$ -particle. It is usual to assume that a nucleus will contain as many  $\alpha$ -particles as it is possible to construct from the nucleons available.

Though both electrons and positrons are ejected from radio-active nuclei it is not believed that these particles exist as such within the nucleus. They are thought to originate from the ejection, and subsequent disintegration, of one of the mesotrons which, as we have already mentioned (§ 135), are being constantly exchanged between the nucleons.

**153. "Exchange" forces. The nucleus as a drop.** The fact that a nucleus is a very stable structure, in spite of the strong electrical

repulsion between the protons which it contains, shows that very strong attractive forces must exist between nucleons when they are brought sufficiently close together. According to the theory of Yukawa, these forces originate in the mesotron exchange between the particles, and they are known, therefore, as "exchange" forces. The existence of this attraction is further shown by the fact that in atomic disintegration the projectile, whether proton, neutron or  $\alpha$ -particle is invariably captured by the nucleus. On the other hand, the scattering of  $\alpha$ -particles has shown (§ 108) that at distances greater than  $10^{-12}$  cm. the nucleus acts as a point charge, so that the exchange forces are inappreciable at such distances. The anomalous scattering of swift  $\alpha$ -particles (§ 109), which make a still closer approach to the nucleus, clearly marks the transition point between the two types of force.

The "exchange" forces are similar in their action, though not in origin, to the forces which hold together the molecules in a liquid. A nucleon well within the nucleus will be attracted equally in all directions by the nucleons which surround it, while one on the surface will be pulled inwards by the nucleons within. The effect is, therefore, analogous to that of surface tension in liquids. Gamov therefore suggested, in 1930, that each nucleus should be regarded as a single droplet of a universal nuclear fluid.

**154. The potential barrier.** Owing to the "exchange" forces any two nuclei coming into contact would coalesce to form a single drop, just as two drops of mercury coalesce when they meet. Contact however is, in general, prevented by the fact that all nuclei are positively charged, and thus repel each other.

Consider for example an  $\alpha$ -particle with charge  $2e$  at distance  $r$  from a nucleus of charge  $Ze$ . The potential energy of the particle in the nuclear field is  $2Ze^2/r$ . At very close distances, as scattering experiments show, there is a rapid decline from the inverse square law of force, owing to the operation of the exchange forces, which at slightly nearer distances overpower the electrical repulsion and attract the particle into the nucleus. Thus if we plot the potential energy  $U$  of an  $\alpha$ -particle against its distance from the centre of a nucleus we shall arrive at a curve of the form shown in Fig. 108.

In order to penetrate into the nucleus the  $\alpha$ -particle must, according to classical theory, be projected towards it with kinetic energy equal at least to the potential energy at the highest point of the curve. We may regard the curve of Fig. 108 as a sort of potential barrier surrounding the nucleus, and warding off contact with other nuclei. The distance of the highest point of the curve from the centre which can be

estimated from scattering experiments is taken as the radius of the nucleus. The radius increases slowly with increasing atomic mass. For oxygen it is about  $3 \times 10^{-13}$  cm.; for lead,  $7 \times 10^{-13}$  cm.

The height of the potential barrier is proportional to the atomic number of the element, and to the charge on the particle. For an  $\alpha$ -particle approaching an aluminium nucleus it is about 7.5 MeV. For a proton it is half this amount. A neutron, which is uncharged, suffers no electrical repulsion, and therefore encounters no potential barrier. The potential energy curve is thus represented as a hole, into which a neutron of any energy can drop, if its path brings it into contact with the nucleus. This explains the ability of the neutron to produce disintegration even in elements of the highest atomic number.

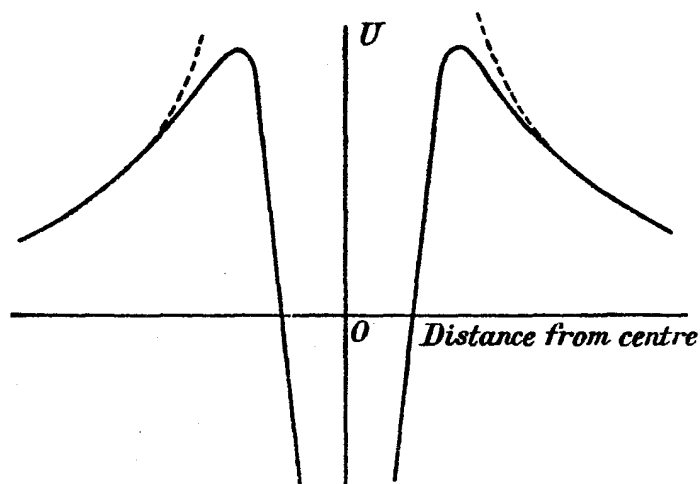


FIG. 108. Potential energy of an  $\alpha$ -particle in the field of a nucleus

If the nuclei are, as Gamov proposed, different sized drops of the same nuclear fluid, the mass  $A$  of a nucleus should be given by  $4\pi r^3 \rho / 3$ , where  $\rho$  is the density of the fluid; and the radius should therefore vary as the cube root of the atomic mass. This is found to be the case. The ratio of the atomic mass of lead to that of oxygen is  $206/16 = 12.9$ , and the cube root is 2.35. The ratio of their radii is  $7/3$ , or 2.33. Taking the radius of the oxygen nucleus as  $3 \times 10^{-13}$  cm. and its mass as  $2.66 \times 10^{-23}$  gm. (i.e.  $16 \times 1.66 \times 10^{-24}$ ) the density of the nuclear fluid works out at  $2.4 \times 10^{14}$  gm./cm.<sup>3</sup>. Its surface tension has been estimated at  $9.3 \times 10^{19}$  dynes/cm.

**155. Application of wave mechanics. Penetrability of the nuclear barrier.** Since the wavelength (§ 120) of an  $\alpha$ -particle is of the same order as the radius of a nucleus, nuclear problems should be considered

from the point of view of wave mechanics. The mathematical discussion does not lend itself to elementary treatment, and we shall assume such conclusions as we require to explain the phenomena with which we have to deal without attempting any proof. Mathematical treatises on the subject of various grades of complexity are now available in considerable variety.

On classical mechanics an  $\alpha$ -particle in order to penetrate within the nucleus must be projected towards it with kinetic energy at least equal to the highest point of the potential barrier. Conversely an  $\alpha$ -particle leaving the nucleus must come over the top of the potential barrier, and when it arrives at a distance from the nucleus must have at least the kinetic energy corresponding to its potential energy at the highest point.

Early observations by Rutherford had already thrown doubt on these conclusions. The scattering by uranium of the fast  $\alpha$ -particles from thorium C', of range 8.4 cm., is entirely normal, and we must conclude that these particles cannot rise above the lower slopes of the potential barrier of the uranium nucleus, where the inverse square law holds. On the other hand the  $\alpha$ -particles given out by uranium itself have a range of only 2.4 cm., whereas if they had left the nucleus over the top of the potential barrier their energy after emerging must at least have been greater and their range longer than the  $\alpha$ -particles from thorium C'.

On wave mechanics the wave packet representing the  $\alpha$ -particle is regarded as being reflected at the potential barrier. Just as the wave theory of the reflection of light indicates the existence of a highly damped wave motion penetrating into the reflecting surface, so wave mechanics indicates that a wave with a high coefficient of damping penetrates into the potential surface. If the barrier is thin, a portion of this wave may actually penetrate into the centre cavity representing the nucleus. The wave packet, representing an  $\alpha$ -particle is, of course, indivisible, and the intensity of the penetrating wave is taken as giving the probability that an  $\alpha$ -particle can penetrate the barrier at this point. The top of the barrier is thus definitely penetrable by  $\alpha$ -particles of suitable energy. The probability rapidly diminishes to zero as we pass further down the slope.

A more important point, however, and one which has been experimentally verified, is the complete penetrability of the barrier at certain definite levels well below the peak. To put the matter extremely crudely, we may suppose that the  $\alpha$ -particle in the nucleus will occupy such a position that it forms a stationary wave by reflection in the



opposite walls of the descending funnel. These various resonance levels, as they are called, obviously constitute an energy level system for the nuclear  $\alpha$ -particles. The energy levels normally occupied by the  $\alpha$ -particles in a stable nucleus are levels of negative energy, that is to say work would have to be done on the  $\alpha$ -particle to place it at an infinite distance from the nucleus. In this case wave mechanics agrees with classical mechanics that there is no possibility of escape for the particle. If, however, the particle occupies one of the higher energy levels, of positive energy, wave mechanics indicates that there is a strong probability, perhaps a certainty, that it will penetrate the potential barrier at this level, and will thus arrive at a distant point with kinetic energy equal to its potential energy in the resonance level.

Conversely a particle projected towards the nucleus with initial energy equal to that of one of the resonance levels of the nucleus will be able to penetrate into the nucleus along the corresponding resonance level, and may thus be captured by the nucleus, although its energy may be far less than that required to carry it over the top.

**156. Determination of nuclear resonance levels.** Very definite experimental evidence for these somewhat novel conclusions is provided by the researches of Chadwick, Pose, and others on the disintegration of the lighter elements by short-range  $\alpha$ -particles. We may take aluminium as a fairly typical example. Aluminium, as we have already seen, is disintegrated by the fast  $\alpha$ -particles from radium C', yielding protons with ranges up to 100 cm. in air. Since the energy of the radium C' particle is 7.68 million volts we may assume that it enters the nucleus either over the top or through the upper part of the potential barrier. It was discovered, however, by Pose that aluminium could also be disintegrated by the  $\alpha$ -particles from polonium the range of which is only 3.9 cm. These particles have quite insufficient energy to carry them up to the permeable part of the barrier.

A detailed analysis of the effect was made by Chadwick.<sup>(9)</sup> The polonium, deposited on a silver disc *R* (Fig. 109), was placed in a tube which could either be exhausted or filled with carbon dioxide at a varying known pressure. The end of the tube *A* was closed by aluminium leaf, the stopping power of which was equivalent to 2 mm. of air. Thus when the tube was exhausted the aluminium nuclei in the foil were being bombarded by  $\alpha$ -particles of all ranges between 3.9 and 3.7 cm. The mean range of the particles falling on the foil could be progressively reduced by admitting carbon dioxide into the tube.

This gas is chosen for the purpose because its constituents do not themselves emit long-range protons. Any protons emitted by the aluminium were collected in a small ionization chamber *C*, connected to a Wynn Williams counter. Their range could be determined in the usual way by interposing screens of known stopping power between *A* and *C*.

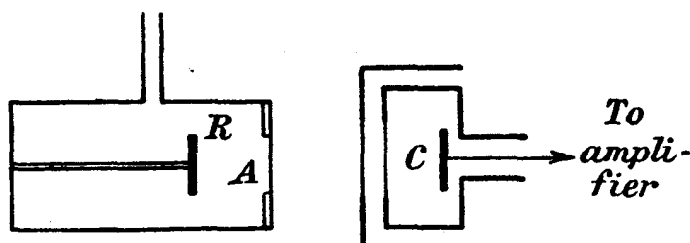


FIG. 109. Chadwick's apparatus for the study of the disintegration of aluminium by  $\alpha$ -particles

It was found that protons of maximum range 66 cm. were emitted when  $\alpha$ -particles of maximum range 3.9 cm. fell on the aluminium. If the range was reduced to 3.7 cm. by admitting carbon dioxide at low pressure, the emission of protons ceased. The aluminium was no longer suffering disintegration. If, however, the range of the particles was still further reduced to 3.45 cm. protons were again emitted, the range this time being 61 cm. Thus while particles of range 3.9 cm. and 3.45 cm. produce disintegration, and thus must be able to enter the aluminium nucleus, particles of intermediate ranges are completely ineffective.

Further investigation with still slower particles revealed two further resonance levels for  $\alpha$ -particles of maximum range 3.1 cm. and 2.7 cm. Aluminium has thus at least four resonance levels.

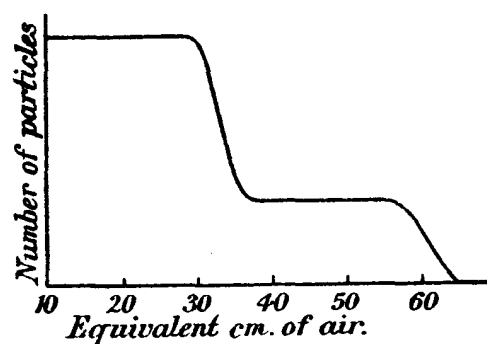


FIG. 110. Range of disintegration protons from aluminium

It was further found that the protons in each case consisted of two homogeneous groups (Fig. 110), one of much longer range than the other. It is supposed that on entering the nucleus the  $\alpha$ -particle of energy  $E_\alpha$  may fall directly to the lowest energy level of energy  $-E_n$ .

The energy thus rendered available,  $E_\alpha + E_n$ , is given to a proton which is thus ejected with kinetic energy  $E_\alpha + E_n - E_p$ , where  $E_p$  is the energy

required to remove a proton from the nucleus. On the other hand the  $\alpha$ -particle may come to rest, temporarily, in some higher energy level  $-E_m$ , in which case the proton ejected will have kinetic energy  $E_\alpha + E_m - E_p$ . The difference in energy of the two groups of protons is thus  $E_n - E_m$ . If a thick sheet of aluminium is used so that the

TABLE X

Effective $\alpha$ -particle		Range of protons ejected cm.
Range cm.	Energy volts	
3.9	$5.25 \times 10^6$	66 and 34
3.45	$4.86 \times 10^6$	61 and 30.5
3.1	$4.49 \times 10^6$	55 and 26.5
2.7	$4.0 \times 10^6$	49 and 22

$\alpha$ -particles have all velocities from 3.9 cm. down to zero, all eight groups of protons are emitted simultaneously.

The  $\alpha$ -particle, however, must ultimately fall to the lower level, giving out in the process energy also equal to  $E_n - E_m$ . We should expect this emission to take the form of  $\gamma$ -radiation. It is an interesting confirmation of the theory that  $\gamma$ -rays of precisely this energy have been detected by Webster when aluminium is bombarded by  $\alpha$ -particles from polonium. It is possible to determine in this way  $\alpha$ -ray energy levels for a number of the lighter elements. Chadwick's results for aluminium are contained in Table X, and pictorially in Fig. 111. It must be understood, however, that these are only "virtual" energy levels. An  $\alpha$ -particle in one of these levels would have positive potential energy, and would escape through the barrier, with the resultant disintegration of the nucleus. Since the nuclei are stable,

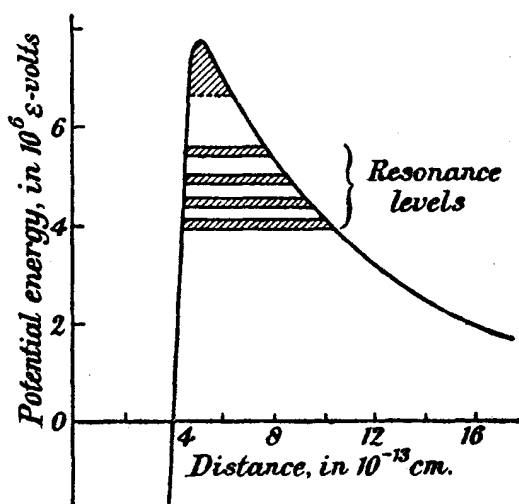


FIG. 111. Resonance levels for the aluminium nucleus

the levels actually occupied must all be below the line of zero energy. Similar experiments carried out with protons make it possible to map some of the virtual proton levels in the nucleus.

**157. The production of  $\alpha$ -radiation.** Just as the X-ray spectrum of an element gives us precise information as to the energy levels in the outer part of the atom, so the radiations emitted by a radio-active element should give us information as to the energy levels in the nucleus. We will consider first the  $\alpha$ -ray spectrum. We have no clear idea of the cause which leads to radio-active disintegration. We must suppose that some internal rearrangement takes place which leaves an  $\alpha$ -particle momentarily with positive potential energy. This particle then escapes through the potential barrier by the process of "resonance" already described. If, as is often the case, the  $\alpha$ -particles are of identical energy, this energy is the difference in energy content of the parent and daughter nuclei, and gives us no information as to the structure of either. If, however, the rays show fine structure (§ 106) or if long-range  $\alpha$ -particles are emitted we can take a further step.

Considering first the case of the fine structure, we must suppose that the swiftest  $\alpha$ -particle emitted carries away the full energy liberated by the radio-active change. The emission of an  $\alpha$ -particle with less than this energy must, therefore, leave the daughter nucleus with an excess of energy, that is to say in an "*excited*" state (§ 35). Each group of  $\alpha$ -particles thus corresponds to a definite excited state of the daughter nucleus, and the energy differences in the  $\alpha$ -ray spectrum measure the energy differences of these excited states.

The case of the long-range  $\alpha$ -particles is the converse of this. The only three substances known to emit long-range  $\alpha$ -particles (Ra C', Th C', Ac C') have a half value period of  $10^{-8}$  sec. or less. We may assume that at the moment when they are formed these nuclei are actually in an excited state, that is to say with an excess of energy. The time for which a nucleus can remain in an excited state is extremely minute. The nucleus reverts to its normal state giving out its excess energy (as we shall show later) in the form of  $\gamma$ -radiation. These particular nuclei, however, have such an extremely short life that there is a small, but finite probability that they may disintegrate with the emission of an  $\alpha$ -particle while still in an excited state. In this case the excess energy will be carried away by the  $\alpha$ -particle, which will thus have energy in excess of the normal. Energy differences between the long-range  $\alpha$ -particles thus give information as to the excitation levels of the nucleus from which they come.

**158. Origin of the  $\gamma$ -ray spectra.** An examination of the frequencies of the characteristic  $\gamma$ -ray spectrum of a radio-active element reveals the same difference relationships between the terms which are found in X-ray (§ 94) and optical emission spectra. It is therefore reasonable to assume that  $\gamma$ -rays represent the energy emitted during transitions between different energy levels in the nucleus.

In certain cases, as we have seen in the preceding section, the  $\alpha$ -ray spectra also provide us with information as to the energy levels in the nucleus. Thus, unless our theory has gone astray, there should be a correlation between the  $\alpha$ -ray spectrum, and the corresponding  $\gamma$ -ray emission. This is found to be the case.

Take for example the case of Th C which disintegrates into Th C'' with the emission of an  $\alpha$ -ray spectrum showing a fine structure containing six lines. Fig. 112 gives the energy level scheme as constructed

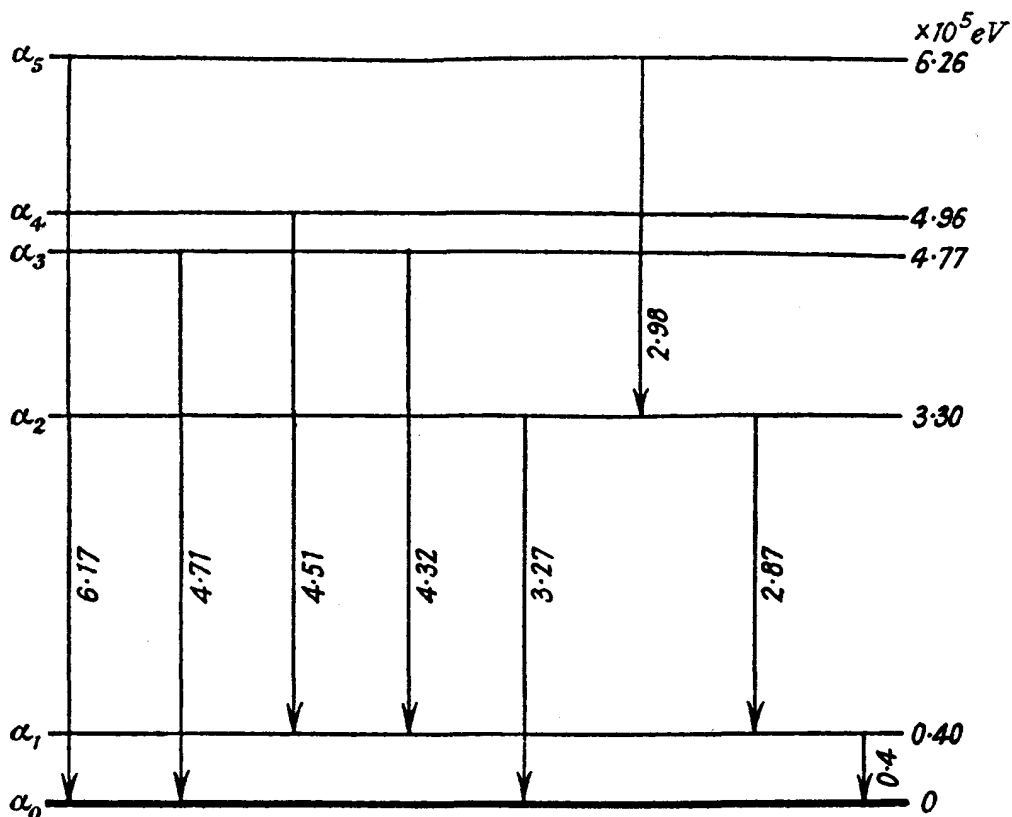


FIG. 112. Energy levels of Th C''

from the  $\alpha$ -ray data. The energy of the particle labelled  $\alpha_5$  in the diagram is less than that of the fastest particle  $\alpha_0$  by  $6.26 \times 10^5 \text{ eV}$ , and this difference is plotted upwards in the diagram because the emission of a particle of the  $\alpha_5$  group will leave the daughter nucleus, Th C'', with an excess of energy of this amount.

According to Ellis <sup>(10)</sup> the transition is accompanied by the emission of eight  $\gamma$ -ray lines which can be identified with the transitions shown by the vertical lines in the diagram. The quantum energies of these lines as determined by  $\gamma$ -ray measurements are also given in the diagram. The agreement between the  $\gamma$ -ray measurements and the values deduced from the  $\alpha$ -ray diagram is of the order of 1 per cent. Equally satisfactory agreement is obtained in the case of the long-range  $\alpha$ -particles.

**159. The wave theory of scattering.** Mathematical investigation shows that, assuming the inverse square law of force, the law of scattering on the wave-mechanical treatment of the problem is identical with that deduced by Rutherford, except in one case: the scattering of  $\alpha$ -particles in helium. Here the helium nuclei set in motion by the impact are identical in nature with the  $\alpha$ -particles, and the possibility of interference between the beam of scattered  $\alpha$ -particles and the beam of projected nuclei has to be taken into account. Thus something of the nature of interference bands are formed and at certain angles the number of scattered particles falls to zero. It can be shown that the number of particles scattered at  $45^\circ$  to the beam should be twice the number given by the classical theory.

This prediction was investigated by Chadwick,<sup>(11)</sup> who compared directly the scattering at an angle of  $45^\circ$  in helium with that in argon which is known to be normal. His results are shown in Fig. 113, where the ratio of the number of scattered particles observed to the

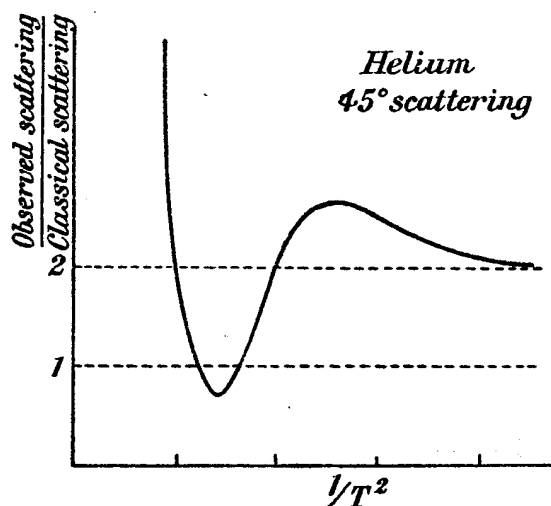


FIG. 113. Scattering of  $\alpha$ -particles by helium

number to be expected on the classical theory is plotted against the reciprocal of the energy of  $\alpha$ -particles employed. It will be observed that the ratio asymptotes to a value of twice the classical value for the more slowly moving  $\alpha$ -particles.

If we suppose that the more swiftly moving  $\alpha$ -particles actually penetrate the helium nucleus, without being captured by it, wave mechanics also provides a

possible explanation of the anomalous scattering shown by the earlier portions of the curve. Near the top of the potential barrier there is a

finite probability of penetration, that is to say a certain fraction of the incident  $\alpha$ -particles will penetrate the barrier, the remainder being reflected from its outer slope. We may suppose that the particles penetrating are themselves reflected internally in the nucleus, and thus form a second reflected beam. Interference may then take place between the two reflected beams. Calculations along these lines have been made, and lead to equations which are not too much out of harmony with the experimental results.

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## CHAPTER XV

### NUCLEAR ENERGY

**160. Magnitude of nuclear energy.** Exact measurement has shown that the mass of any nucleus is invariably less than the sum of the masses of the protons and neutrons from which it is built, and we have produced convincing experimental evidence (§ 148) that this mass defect is equivalent to the energy liberated during the formation of the nucleus from its constituents. Further, any nuclear reaction in which the total mass of the final products is less than that of the original nuclei will result in a liberation of energy, the relation between the mass lost and the energy liberated being given by  $T=c^2m$ , if the energy is measured in ergs and the mass in grams, or by  $T=931m$  if the energy is measured in MeV and the mass in atomic mass units.

Thus if it were possible to bring about the condensation of two protons and two neutrons into a single  $\alpha$ -particle, the resulting loss of mass would be  $2 \times 1.0089 + 2 \times 1.0076 - 4.0028$ , or 0.0282 mass units. This is equivalent to  $0.0282 \times 931$ , or 26 MeV. Since the mass unit is  $1.66 \times 10^{-24}$  gm. and  $1 \text{ MeV} = 4.45 \times 10^{-20}$  kW.h. the formation of 4 grams of helium from its constituents would liberate about  $7 \times 10^5$  kW.h. The disintegration of a  ${}^7_3\text{Li}$  nucleus by a proton results in the emission, as we have already seen, of 17 MeV. Thus the complete proton disintegration of 7 grams of lithium would yield  $4.5 \times 10^5$  kW.h., or about  $4 \times 10^{11}$  calories. It may be noted that the combustion of one gram of coal produces about  $8 \times 10^3$  calories.

**161. The Bohr-Wheeler theory of nuclear stabilities.** The surface energy of a drop of liquid is equal to the product of its surface area by the surface tension. Hence assuming that the forces between the nucleons in a nucleus can be regarded as constituting a surface tension,  $S$ , the surface energy is  $4\pi r^2 \cdot S$ . But the radius  $r$  is proportional to the cube root of the mass  $A$  of the drop, since the density of the fluid is constant. Hence the surface energy varies as  $A^{\frac{2}{3}}$ .

On the other hand, suppose two drops, each of mass  $A$ , coalesce to form a single drop of mass  $2A$ . The ratio of the surface energies after and before coalescence is clearly  $(2A)^{\frac{2}{3}}/2A^{\frac{2}{3}}$  or 0.79. The total surface energy has been reduced by 21 per cent. Thus energy will be evolved when two drops coalesce. The percentage is smaller if the



drops are unequal in size. Thus for a mass ratio of 10 to 1, the energy reduction is 13 per cent.; for a mass ratio of 100 to 1, which would correspond to the addition of a neutron to an atom of mass 100 the energy reduction would still be nearly 5 per cent. Thus there is always a decrease in the surface energy of the system when two drops, or two nuclei, combine.

On the other hand the nuclei are electrically charged, and, as we have already seen (§ 21), the electrical forces act in opposition to those of surface tension. The electrical potential energy of a nucleus of charge  $Z$  is  $Z^2/2r$ ; or, since  $r$  is proportional to  $A^{1/3}$ , the energy varies as  $Z^2/A^{1/3}$ . If we suppose that the mass number  $M$  is approximately proportional to the atomic number, the electrical energy of the nucleus will vary approximately as  $Z^{5/3}$ .

Now suppose a nucleus of charge  $Z$  splits into two equal nuclei, each of charge  $\frac{1}{2}Z$ . The ratio of the combined electrical energies of the

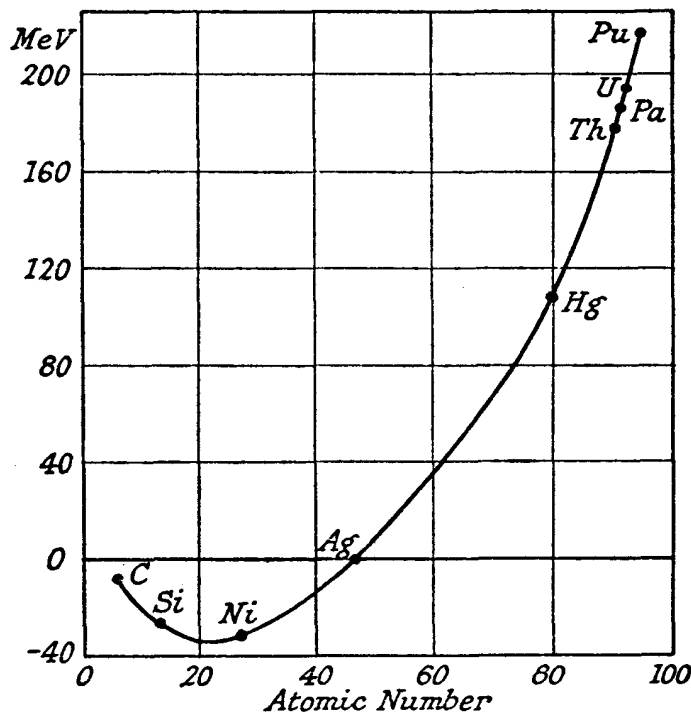


FIG. 114. Bohr-Wheeler estimate of the energy released by the fission of a nucleus into two equal parts

two fragments to the energy of the original nucleus will be  $2.(\frac{1}{2}Z)/Z^{5/3}$ , i.e.  $1/2^{2/3}$ ; or 0.63. Thus the fission will result in the emission of 37 per cent. of the electrical energy. Thus the electrification tends to promote fission; the surface tension to promote coalescence of the nuclei.

Since the electrical energy increases much more rapidly with increasing mass than the surface energy, which varies only as the two-thirds power of the mass, we should expect to reach a stage at some point in the atomic table at which the electrical energy loss due to fission would exactly balance the energy gain due to surface energy. At this point the nucleus is in exact equilibrium under the two opposing sets of forces. Work would have to be done either to increase or decrease its size. According to Bohr and Wheeler,<sup>(1)</sup> who have made an extensive survey of the subject along these lines, this point is reached almost exactly half way up the atomic table. The results of their calculations are indicated in Fig. 114, which shows the energy which should be emitted if the nucleus is split into two equal parts. For elements of lower atomic number than silver the energy is negative and the elements are stable against fission. On the other hand, two nuclei whose total masses are less than that of silver would coalesce with the emission of energy if brought into contact.

**162. Activation energy of nuclear reactions.** With the exception of the elements in the neighbourhood of silver the nuclei are, thus, only metastable. They owe their existence to the large amount of energy required to initiate the possible nuclear reactions; the activation energy, as it is called. This is on a par with many chemical reactions. Coal, for example, will not unite with oxygen unless a certain minimum energy, usually in the form of heat, is supplied, and the metastable molecule of nitro-glycerine requires some 2.2 eV to bring about its explosion.

In the case of the lighter elements which react by fusion, the activation energy is the energy required to bring the two nuclei into contact against the repulsion of their electrical charges, and is measured by the product of their charges into the sum of their radii. The figure may, however, be considerably reduced if "resonance" occurs. Thus, in the lithium proton reaction, where the height of the potential barrier is 1.2 MeV, some disintegrations have been observed with protons of energy as low as 20,000 eV.

Even in the case of the heaviest elements, the electrical forces are not sufficient to bring about the disruption of the nucleus so long as it retains its spherical form. If, however, the nucleus is set into violent vibration by the impact of some particle upon it so that it assumes something of a dumb-bell shape, the surface forces may be insufficient to withstand the electrical repulsion between the two halves of the bell, and fission will then occur, with a release of energy. The activation energy is thus the energy required to produce the critical distor-

tion. Its value has been calculated for various elements by Bohr and Wheeler<sup>(1)</sup> and, as might be expected, decreases with increasing atomic mass. The values range from 50 MeV for tin to 5 MeV for uranium. The smallest values for the activation energy are thus to be found at the two extremes of the atomic table.

**163. Thermonuclear reactions.** The energy released in the disintegration of a lithium nucleus by a proton is, as we have seen (§ 145), 17 MeV. In practice, however, if lithium is bombarded by a stream of 300,000 eV protons, only about one proton in 10,000 produces a disintegration; the remainder fritter away their energy in displacing the extra nuclear electrons. The process as a whole therefore actually results in a very considerable energy deficit.

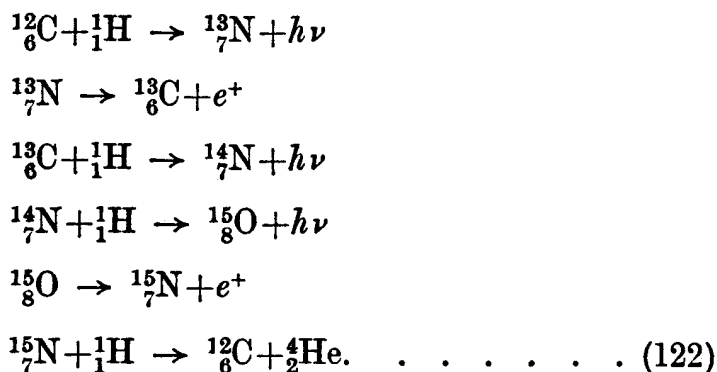
The activation energy necessary to initiate an ordinary chemical reaction is frequently supplied in the form of heat. The average kinetic energy of a gas molecule at an absolute temperature  $\theta$  is equal to  $\frac{2}{3}\beta\theta$ , where  $\beta$  is Boltzmann's constant ( $1.37 \times 10^{-16}$  erg deg.<sup>-1</sup>). Thus the molecules would, on an average, have an energy of 1 eV at a temperature of  $\frac{2}{3}(1.6 \times 10^{-12})/(1.37 \times 10^{-16})$  or 7800° K. It would appear that to initiate nuclear reactions with activation energies of 1 MeV upward temperatures of the order of  $10^{10}$  would be required.

This estimate is, however, reduced by two considerations. In the first place Maxwell's distribution law shows that in any gas there always exists a small but not insignificant proportion of molecules with actual energy several times greater than the mean. Secondly, owing to the permeability of the potential barrier occasional disintegrations occur with particles of energy much smaller than the normal activation energy. Thus disintegrations of lithium have been noted with protons of energy as low as 20,000 volts. Again, the thermal energies of the molecules are maintained so long as the temperature remains constant, and are not frittered away, as is the energy of the proton beam from a discharge tube. Any particle with the necessary energy will therefore ultimately produce a disintegration. Bethe has calculated that under the conditions existing in the interior of the sun (20 million degrees and 160 thousand million atmospheres) a mass of lithium would be half disintegrated by proton bombardment in one minute.

**164. Source of Stellar energy.** It seems highly probable, in view of these considerations, that the energy which the sun has been radiating so prodigally and so constantly throughout the greater part of geological time is actually drawn from thermonuclear reactions proceeding in its interior. It has proved possible to identify the reactions concerned

from a study of the mean reaction time of the different possible processes. All reactions in which the  $\alpha$ -particle is concerned proceed far too slowly ( $10^{13}$  years and upwards). The reaction must therefore be one in which protons are involved. The reactions between protons and the elements lighter than carbon have mean reaction times measured in days or minutes, and thus proceed far too rapidly. For elements heavier than nitrogen the mean reaction time is  $10^{12}$  years and upwards, which is far too slow to produce energy at the rate at which the sun actually radiates. The only possible reactions are those in which carbon and nitrogen are involved, and which have mean reaction times of  $2.5 \times 10^6$  and  $5 \times 10^7$  years, respectively.

According to Bethe, the process is actually a cyclic one in which both these elements are concerned. It involves the following series of reactions, all of which have been observed in the laboratory.



The net result is the condensation of four protons into one  $\alpha$ -particle, with the emission of the two superfluous positive charges as positrons. The energy released is thus some 26 MeV (§ 160). It will be noted that the original carbon atom is re-formed in the last operation. The third operation results in the formation of the common stable nitrogen nucleus. It would therefore have been possible to commence the cycle with nitrogen, instead of carbon. The energy is emitted partly in the form of  $\gamma$ -radiation, but mainly as the kinetic energy of the  $\alpha$ -particle.

**165. The fission of uranium by neutrons.** Some years ago Fermi showed that neutrons could be captured by the nuclei of the heaviest elements, including uranium, with the emission of  $\gamma$ -radiation. It was assumed that the process involved was the "radiative" capture of a neutron, the third type discussed in (§ 144), and that the resulting nucleus was a higher isotope of uranium. The emission of negative electrons, which continued for some days, showed that the nucleus was unstable, changing into an element of atomic number 93; that is higher

than any hitherto known element. Fermi considered that there was evidence for the production of several of these trans-uranic elements, as they were called.

There was, therefore, considerable excitement when in January 1939 Hahn and Strassmann <sup>(2)</sup> discovered that one of the products of the neutron bombardment of uranium was an isotope of barium, which was identifiable not only by its chemical properties, but also, since it was an unstable isotope, by its half-value period. It was clear, therefore, that a certain proportion of the neutron captures resulted not in radiation but in the actual splitting of the uranium nucleus into two not very unequal portions.

Further research showed that the fragments ejected fell into two groups: a lighter group with atomic numbers ranging from Br (35) to Mo (42), and a heavier group including the elements between Sb (51) and cerium (58). Many of the products were radio-active. A glance at the packing curve (Fig. 43) shows that fission will result in a decrease in total mass, and calculation indicated that the energy set free should amount to between 170 and 200 MeV. This was confirmed by calorimetric observations.

Uranium consists of a mixture of two isotopes:  $^{238}\text{U}$ , constituting about 99.3 per cent. of the mixture, and  $^{235}\text{U}$ . Both are fissile, but whereas U-235 (which might conveniently be called actino-uranium, since it gives rise to the actinium series of radio-active substances (§ 202)) can be split by slow neutrons of thermal energy, the higher isotope, U-238, requires high-speed neutrons for its demolition. Nuclear fission has also been observed with proto-actinium (91) and thorium (90), but only when bombarded with fast neutrons.

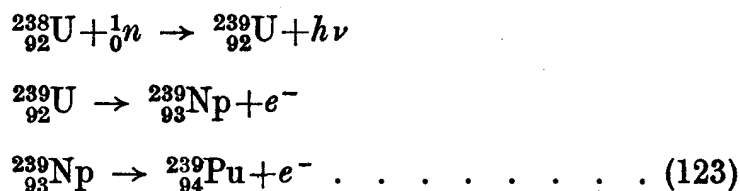
**166. Theoretical study of nuclear fission.** Calculation by Bohr and Wheeler shows that the activation energies for fission of U-235, U-238, and thorium are respectively about 5, 6, and 7 MeV. It might, at first sight, seem contradictory that a neutron with only thermal energy (about 0.04 eV at ordinary temperatures) should produce nuclear fission. It must be remembered, however, that a neutron contacting a nucleus is attracted to it by the exchange forces. Work must be done to extract the neutron from the nucleus, and an equal amount of energy is liberated when the neutron returns. On the "droplet" analogy, we may compare this to the latent heat evolved when a molecule of vapour condenses on the surface.

Assuming the value for the surface tension of the nuclear fluid of  $9.3 \times 10^{19}$  dyne/cm. it can be shown that this energy amounts to about  $5\frac{1}{2}$  MeV. This is greater than the activation energy required by

U-235 for fission. For the other elements it must be supplemented by the kinetic energy of the neutron.

The ratio of neutrons to protons in U-235 is 1.6. In the heaviest stable nuclei of the elements into which it disintegrates this ratio varies between 1.4 and 1.45. It is obvious therefore that after fission there will be a surplus of neutrons which must be disposed of, either by ejection from the system or by conversion into protons with the emission of a negative electron. Both processes occur. The fragments are radio-active, emitting  $\beta$ -radiation over a considerable period. In addition from one to three high-speed neutrons are ejected, apparently instantaneously, for each fission which takes place. Some slight additional neutron emission is observed for several seconds after the fission.

**167. The trans-uranic elements.** The nucleus of U-238 is particularly permeable to neutrons of energy between 24 and 26 eV, that is in a range intermediate between the high-speed neutrons which produce fission and the thermal neutrons. Practically every neutron in this range which strikes a U-238 nucleus is absorbed and retained by it, with the formation of a higher uranium isotope,  $^{239}_{92}\text{U}$ . This is unstable and passes, by the emission of a  $\beta$ -particle, into a new element, known as neptunium, with a mass number of 239 and an atomic number of 93. Neptunium is also radio-active, and by the emission of a further  $\beta$ -particle changes into still another new element, plutonium, of mass number 239 and atomic number 94. The reactions are as follows:



Plutonium belongs to the natural radio-active bodies, and is actually the head of the actinium series, although owing to its comparatively short half-value period (less than a hundred thousand years) it is no longer extant. It decays, with the emission of an  $\alpha$ -particle, into urano-actinium (U-235). Like U-235 it is easily fissionable by slow neutrons.

**168. Release of atomic energy. The "chain" reaction.** Consider a mass, say, of pure U-235 and let us suppose that a single fission occurs as the result of the entry of a stray neutron. As a result two fast neutrons are ejected. Although, failing immediate capture, these neutrons will rapidly be slowed down to thermal velocities, their

ability to produce fission is not thereby impaired. It is actually increased. Apart from loss by diffusion from the surface of the material, every neutron will thus be captured by a U-235 nucleus, with the production of fission and the ejection of two more neutrons.

The process is, therefore, not merely self propagating, it is also self multiplying. Thus 80 generations of neutrons would produce  $2^{80}$  or  $1.2 \times 10^{24}$  disintegrations. The number of nuclei in the original kilogram is  $2.6 \times 10^{24}$ . Disintegration would, if conditions remained unaltered, be complete before the end of the next generation, with a total energy production of some 20 million kilowatt hours. Since the time taken for fission is of the order of  $10^{-9}$  sec. the reaction might be expected to reach explosive violence.

In making these predictions we have assumed that all the neutrons produced by fission are absorbed again in a U-235 nucleus, producing fresh fissions. In practice neutrons may be lost (1) by non-fission capture in a U-235 nucleus, (2) by non-fission capture in the nuclei of impurities present in the mass, (3) by diffusion from the surface. Unless the sum of these losses is less than the production of fresh neutrons by the fission process, the reaction will die out.

The probability of (1) seems to be relatively small. Process (2), however, is very important, as many nuclei are much more effective as neutron absorbers than uranium. The presence of only one part in a million of some impurities is sufficient to stop the chain reaction. Assuming, however, that the rate of production of neutrons exceeds the rate of loss by (1) and (2), the effect of diffusion from the surface can be reduced by increasing the mass of material. The diffusion loss is a surface effect and, for a spherical mass, varies as the square of the radius; the neutron production is a mass effect and varies as the cube of the radius. Hence the diffusion loss becomes relatively less important as the radius increases. There will be a critical radius at which the two effects exactly balance. If this radius is exceeded the action gets out of control, and explosion will occur.

Although numerical data have not been released, it is understood that for both U-235 and plutonium the critical mass is of reasonably small dimensions.

**169. Controlled release of nuclear energy. The Fermi pile.** The separation of pure U-235 from its more abundant isotope is an expensive process, since chemical methods are ruled out. The first sample of U-235 was prepared, as already mentioned (§ 56), by a mass-spectrograph method. The release of nuclear energy as a commercial proposition clearly depends on our being able to induce, and control, a chain

reaction in natural uranium, that is, in the mixture of isotopes which we actually find in nature.

The problem has been solved by Fermi. The probability of capture of thermal neutrons is much smaller for U-238 than for the lighter isotope; so much smaller, in fact, that the great majority of the thermal neutrons are captured by the U-235 nuclei in spite of the great preponderance of the heavier isotope. The neutrons emitted during fission are, however, fast neutrons, and in the process of slowing down to thermal velocities their energies must pass through the range 24 to 26 eV, for which, as we have already seen, the probability of resonance capture in U-238 is very high. It is the existence of this resonance band which prevents the starting of a chain reaction in natural uranium.

Suppose, now, that we divide our uranium into lumps so small that the fast neutrons will emerge from them without making an appreciable number of collisions in the lump. These lumps are surrounded by some material which will slow down the neutrons to thermal energies, and yet has itself only a small probability for neutron capture. Since neutron energy is lost by collision light elements are the most effective, and hydrogen compounds are often used. Unfortunately the probability of capture for hydrogen is rather large, and carbon was eventually chosen. As disintegration proceeds, due to stray neutrons producing disintegrations in the uranium lumps, thermal neutrons begin to accumulate in the carbon "moderator," as it is called, some of which diffuse into the uranium lumps. Their energy is, however, now much too small for resonance capture by U-238, and most of them produce fresh fissions in the U-235 nuclei.

The probability of neutron loss is much higher than in a pure U-235 or plutonium pile. It was calculated that for an infinite mass the multiplication factor (i.e. the number of fissions produced by the neutrons from a single fission) would be about 1.07. Neutron loss through the surface must, therefore, be reduced to less than 7 per cent. of the production rate, and the critical radius may be expected to be large. The first pile constructed by Fermi in 1942 contained 12,400 lb. of uranium metal and considerably more pure graphite. The energy production was at the rate of 200 watts. The reaction was controlled by reducing the neutron population in the modulator by the insertion of rods of some element (e.g. cadmium) which has a very high absorption for thermal neutrons.

The pile was constructed not for the sake of its energy output, which was negligible, but for the plutonium generated by those neutrons which are absorbed in the U-238 nuclei. Since plutonium is not a



uranium isotope, it can be separated from the metal in the pile by purely chemical processes. It is thus much cheaper to prepare than U-235, and is equally effective for the explosive release of energy.

For fuller particulars, the reader is referred to the U.S.A. official report, prepared for the U.S.A. government by Dr. Smyth, from which most of the information in the preceding sections has been culled.

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## CHAPTER XVI

### THE STRUCTURE OF THE ATOM

**170. The extra-nuclear electrons.** We have seen that the mass of the atom resides almost entirely in a nucleus, the radius of which is somewhat less than  $10^{-12}$  cm. The nucleus carries a positive charge equal to its atomic number  $Z$ . Since the atom as a whole is electrically neutral, the nucleus must be associated with a number of electrons equal to the atomic number. The radius of the atom, as deduced from the kinetic theory of matter, is of the order of  $10^{-8}$  cm. so that we must suppose that the motions of the extra-nuclear electrons are confined in a sphere of this order of magnitude. The electron itself may be regarded as a particle with a radius of the order of  $10^{-13}$  cm. The atom is, therefore, an exceedingly open structure, which explains why it can be so easily penetrated by high speed particles of various kinds.

Earnshaw's theorem tells us that any arrangement of point charges is unstable, on classical theory, if the particles are at rest. Stability can, however, be obtained (as in the similar case of the solar system) if we suppose that the electrons are describing circular orbits around the nucleus, with appropriate velocities. The condition to be satisfied is clearly that the centrifugal force  $mv^2/r$  shall be equal to the attractive force  $Ze \cdot e/r^2$  between the nucleus and the electron. This gives an infinite number of possible orbits. On the other hand since the electrons would have an acceleration towards the nucleus they should, on classical theory, radiate energy continuously. Their velocity would thus continually diminish and the structure would ultimately collapse.

A solution of the difficulty is clearly to be found along the lines of the quantum theory. We have already seen that in those cases where we are able to follow the interchange of energy between electrons and radiation the electron does not radiate continuously, as demanded by the classical theory, but emits either a whole quantum of monochromatic radiation or no energy at all. Assuming that these conditions hold within the atom the quantum theory indicates the possibility that electrons in certain preferred orbits may not give out radiation, in which case the stability of the nuclear atom could be assured.

The exact form in which the quantum theory should be applied was

not immediately apparent. Planck's constant,  $h$ , has the dimensions of energy  $\times$  time, or momentum  $\times$  distance. The latter product is sometimes known as an "action." Bohr assumed that an orbit would be non-radiating or *stationary* if the electron in it had an amount of "action" equal to an integral multiple of  $h$ , which may thus be regarded as the natural unit of action. For an electron describing a circular orbit we have, therefore,

$$mv \cdot 2\pi r = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

where  $r$  is the radius of the orbit, and  $n$  is an integer which is known as the quantum number of the orbit. This may also be expressed in the form

$$2\pi mr^2\omega = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

where  $\omega$  is the angular velocity of the electron.

A stationary orbit must satisfy this relation in addition to the requirements of ordinary mechanics. Thus only a select few of the orbits mechanically possible are possible electronic orbits.

**171. Bohr's theory of the hydrogen atom spectrum.** The fundamental truth of Bohr's assumptions was made evident by their success when applied to the case of the hydrogen atom. Since the hydrogen atom consists of a positive nucleus and a negative electron, each of which behaves as a point charge, the calculation of the stationary orbits can be made with certainty and exactness.

Let  $e$ ,  $m$  be the charge and mass of the electron,  $Ze$  and  $M$  the charge and mass of the nucleus, and let  $r$  be the radius of the orbit in which the electron is moving with a velocity  $v$ . Equating the centrifugal force to the mutual attraction of the two charges we have as the mechanical condition of equilibrium

$$\frac{mv^2}{r} = \frac{Ze^2}{r^2}.$$

If  $\tau$  is the time of revolution of the electron in its orbit  $v = 2\pi r/\tau$ , and, substituting this value of  $v$  in the previous equation,

$$\frac{4\pi^2 mr}{\tau^2} = \frac{Ze^2}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Applying now Bohr's second postulate the "stationary" orbits are to be distinguished by the relation

$$2\pi mr^2\omega = nh,$$

or, since

$$\omega = \frac{2\pi}{\tau},$$

$$\tau = \frac{4\pi^2 m r^2}{nh} \quad \dots \dots \dots (119)$$

Substituting this value in (117) we have

$$r = \frac{n^2 h^2}{4\pi^2 m Z e^2} \quad \dots \dots \dots (120)$$

which gives the radii of the possible stationary orbits. All the quantities on the right-hand side of the equation are known. For the hydrogen atom, for which  $Z=1$ , the radius of the smallest orbit, obtained by putting  $n=1$ , is  $0.52 \times 10^{-8}$  cm. The radii are proportional to the squares of the natural numbers, so that all the orbits except the first are outside the generally accepted value for the radius of the hydrogen atom.

The kinetic energy of the electron in its orbit is  $\frac{1}{2}mv^2$  or  $Ze^2/2r$ . As the force between the electron and the nucleus is attractive the maximum potential energy will be when the electron is at an infinite distance from the nucleus. If  $W_\infty$  is this maximum value, the actual potential energy at a distance  $r$  from the nucleus is  $W_\infty - e \cdot Ze/r$ .

The total energy in the orbit is thus

$$W_r = \frac{Ze^2}{2r} + W_\infty - \frac{Ze^2}{r}$$

$$= W_\infty - \frac{Ze^2}{2r}$$

$$= W_\infty - \frac{2\pi^2 m Z^2 e^4}{n^2 h^2} \quad \dots \dots \dots (121)$$

substituting for  $r$  from (120).

Now by the ordinary quantum relation if an electron falls from an orbit of energy  $W_{r_1}$  to one of energy  $W_r$  the difference is emitted as a monochromatic radiation of frequency  $\nu$  given by

$$\nu = \frac{W_{r_1} - W_r}{h}$$

$$= \frac{2\pi^2 m Z^2 e^4}{h^3} \left\{ \frac{1}{n^2} - \frac{1}{n_1^2} \right\} \quad \dots \dots \dots (122)$$

substituting for  $W_{r_1}$  and  $W_r$  from (121). For hydrogen  $Z=1$ . Thus the lines in the spectrum of the hydrogen atom should be obtainable by substituting different integral values for  $n$  and  $n_1$  in this equation.

$n$  is the number of the orbit into which the electron falls,  $n_1$  that of the orbit from which it starts.

We have assumed, in the analysis, that the mass of the electron is negligible in comparison with  $M$ , the mass of the nucleus. The necessary correction, which is easily made, gives for the hydrogen spectrum

$$\nu = 2\pi^2 \frac{Mm}{M+m} \frac{e^4 Z^2}{h^3} \left\{ \frac{1}{n^2} - \frac{1}{n_1^2} \right\} \quad . \quad . \quad . \quad . \quad (123)$$

**172. Numerical test of Bohr's theory.** It is usual in spectroscopy to employ the wave number  $\nu'$ , where  $\nu' = 1/\lambda = \nu/c$ , in place of the frequency  $\nu$ . Balmer discovered as early as 1885 that the wave numbers of all the lines in the normal hydrogen atom spectrum could be derived by substituting integral numbers for  $n$ , in the general relation

$$\nu' = R_H \left( \frac{1}{4} - \frac{1}{n_1^2} \right) \quad . \quad . \quad . \quad . \quad . \quad (124)$$

although no explanation of the fact could be offered.  $R_H$  is a constant known as the Rydberg constant for hydrogen. Its value is  $1.09677 \times 10^5 \text{ cm.}^{-1}$ .

Putting  $n=2$  in (123) we see that the two relations are identical in form. The relation, however, is much more than formal. For hydrogen  $Z=1$ , and consequently  $R_H$  should be given by  $\frac{2\pi^2}{c} \frac{Mm}{M+m} \frac{e^4}{h^3}$ , dividing by  $c$  to convert frequencies into wave numbers. All the quantities in this expression are known to an accuracy of the order of 1 in 1000, and there are no "disposable constants" to enable us to evade the full rigour of a numerical test. The numerical value of the expression is  $1.0955 \times 10^5$ . The agreement is thus perfect within the limits of experimental error.

It will be seen that in Bohr's theory a given line of the Balmer series is emitted when an electron falls into the orbit, for which  $n=2$ , from one of the outer orbits. The  $H_\alpha$  line, for example, is emitted by an electron passing from  $n=3$  to  $n=2$ ; or in X-ray nomenclature from the  $M$  to the  $L$  level. Ultimately, however, the electron must return to the innermost level of lowest energy, so that we should expect to find a series of lines given by  $\nu' = R_H \left( \frac{1}{1} - \frac{1}{n_1^2} \right)$ , and calculation shows that the lines of this series would be in the ultra-violet. They have been identified by Lyman. A series in the infra-red for which  $n=3$  has been

discovered by Paschen. The origin of the various series is indicated in fig. 115.

It will be seen that a line is only emitted if the atom becomes ionized, or at least "excited." The electron on its return may fall from one

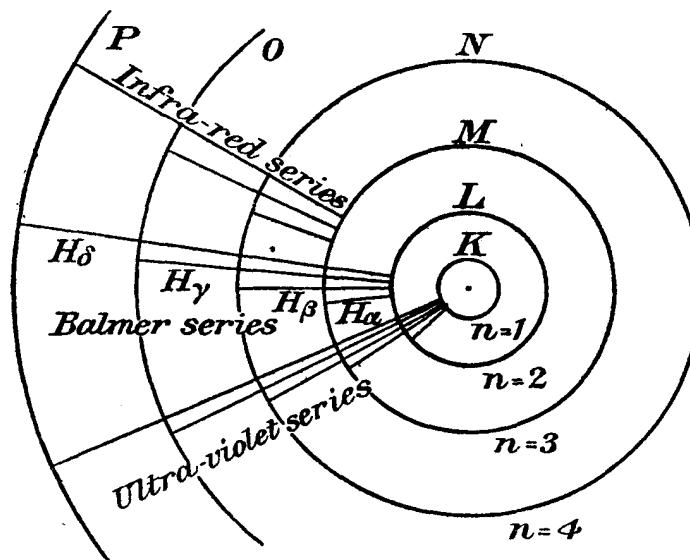


FIG. 115. Schematic diagram of the Bohr orbits for a hydrogen atom

quantum orbit to another, emitting a line at each transition, until finally it comes to rest in the most stable innermost orbit.

**173. The spectrum of ionized helium.** The hydrogen atom spectrum is the only normal spectrum for which exact calculations can be made. As soon as we add a second electron to the atom the problem becomes effectively that of three gravitating bodies, no accurate solution of which has yet been attained. If, however, a nucleus is stripped of all its extra-nuclear electrons in a very intense discharge, the return of the first electron to the nucleus will be a two-body problem, and its spectrum should be given by Equation 123. Millikan has succeeded in exciting and measuring the spectra of these stripped atoms for all the elements in the first series of the periodic table.

The most interesting case is that of helium. Both Pickering and Fowler discovered some years ago a spectrum the various lines in which could be represented by the relation

$$\nu' = 4R \left( \frac{1}{n^2} - \frac{1}{n_1^2} \right) \quad \dots \quad (125)$$

where  $R$  had very nearly, but not exactly, the value of Rydberg's constant for hydrogen. Putting  $Z=2$  in (123) we see that the Pickering-Fowler spectrum is exactly what we should expect, on Bohr's theory, from the return of a single electron to a stripped helium nucleus. The

slight difference between the two Rydberg constants is due to the slight difference in the correcting term for the motion of the nucleus.

Slight as this difference is, the accuracy of spectroscopic observations is so great that it provides one of the most accurate methods available for determining the ratio of the mass of an electron to that of a proton. Writing  $R_{He}$  for the value of Rydberg's constant in the Pickering-Fowler series and  $M_H$  and  $M_{He}$  for the masses of the hydrogen and helium nuclei we have from (123)

$$\frac{R_H}{R_{He}} = \frac{M_H}{M_H + m} \bigg/ \frac{M_{He}}{M_{He} + m} = \left(1 + \frac{M_H}{M_{He}} x\right) \bigg/ (1 + x),$$

where  $x = m/M_H$ . But

$$\begin{aligned} R_H &= 109677.69 \pm 0.06 & M_H &= 1.0076, \\ R_{He} &= 109722.14 \pm 0.04 & M_{He} &= 4.0029, \end{aligned}$$

whence

$$\frac{m}{M_H} = \frac{1}{1838}$$

with a probable error of not more than 1 in the last figure. The ratio of the mass of the electron to that of the hydrogen *atom* is thus 1/1839.

The ratio  $e/M_H$  is, as we have seen earlier, the Faraday constant in electrolysis and is equal to 9649 e.m.u. per gm. Hence

$$\frac{e}{m} = \frac{e}{M_H} \cdot \frac{M_H}{m} = 1.761 \times 10^7 \text{ e.m.u./gm.}$$

It is interesting to note that the most recent determinations of  $e/m$  by the deflection method are giving results in very good agreement with this value.

**174. The origin of spectra.** It is impossible to carry out the calculations which gave us the hydrogen spectrum, for atoms containing more than one electron. The effect produced on the motion of an electron by the other electrons in the system is too complicated to yield to mathematical analysis. It is clear, however, that the electrons will be arranged in definite quantum energy levels, although the appropriate energies cannot be exactly evaluated.

A given spectrum line will be emitted when an electron passes from one quantum level to another, and its frequency  $\nu$  will be given by  $h\nu = W_2 - W_1$ , where  $(W_2 - W_1)$  is the difference in energy of the two levels. If we write  $W_1 = h\nu_1$  and  $W_2 = h\nu_2$ , we have

$$h\nu = h\nu_2 - h\nu_1, \quad \text{or } \nu = \nu_2 - \nu_1.$$

The frequencies of the lines in a line spectrum are thus given by the differences between certain terms. This is a well-known law of spectroscopy, and had been discovered from a study of the numerical relations between the lines in line spectra, long before the Bohr theory gave it a theoretical significance.

The energies of these optical levels can be deduced from spectroscopic data, in the same way that the energies of the deeper levels can be measured by X-ray observations. In the visible spectrum the energies involved are of the order of a few volts, and the transitions are mainly those between virtual orbits and the outermost of the occupied orbits.

We can, however, also map these energy levels in an entirely different way. Since the atom can only exist in certain definite energy states, it can only absorb from a colliding electron the exact amount of energy required to raise one of its outer electrons from its normal level to one or other of the higher virtual levels. The various radiation potentials (§ 35) thus correspond to the difference in energy of an electron in the ground state and in one or other of the virtual quantum levels. It should, therefore, be possible to determine these different levels from the observed radiation potentials. Values obtained this way agree well with those deduced spectroscopically, though the latter method is, of course, much the more accurate.

Again, if the atom is excited at the minimum excitation potential, the electron is removed from the ground state to the one next above it. On returning to the ground state it can clearly only emit one spectral line. On the other hand if it is raised to one of the higher states, it can return directly to the ground state, or by a series of transitions from one to another of the intervening energy levels. The full spectrum is clearly only developed when the electron is removed entirely outside the atom, that is to say when the atom becomes ionized. If  $\nu$  is the frequency term corresponding to the electron in its normal level,  $h\nu$  should be the energy required to ionize the atom. The ionization potentials deduced in this way are in excellent agreement with those determined directly by the methods described in § 34.

The various terms in the spectral series were deduced empirically in the first place, and were denoted by a rather complicated system of letters and numerals. It is, however, possible to correlate them with the system of quantum levels which will be described in succeeding sections. The detailed application of quantum principles to actual optical spectra is a large subject which is outside the purpose of this book.



**175. X-ray spectra.** The X-ray spectra differ from optical spectra in the fact that, owing to the much larger excitation potentials an electron is removed not from the periphery of the atom, but from the deeper levels, usually either the  $K$  or the  $L$  level. This vacancy is filled in the first place by an electron from a neighbouring ring. The transitions thus take place between levels deep in the atom, which are normally occupied by electrons. In this case it is possible to make an approximate calculation of the energies of the levels, and of the frequency of the radiation emitted.

Let us suppose as a rough approximation that we may regard the electrons constituting a given energy level as equivalent to a uniform distribution of negative electricity over a spherical surface. The  $p$  electrons interior to a particular orbit of radius  $a$  will act as if their charge were at the centre of the sphere, i.e. at the nucleus, and will thus reduce the effective nuclear charge from  $Z$  to  $Z-p$ . The electrons exterior to the given level will produce no resultant field upon any electron in it since the field inside a uniformly charged spherical surface is zero. The  $q-1$  electrons in the same level as the electron we are considering will (if their charge is supposed uniformly distributed) repel the given electron with a force  $\frac{1}{2}(q-1)e^2/a^2$ . The attractive force on the electron is thus

$$(Z-p)e^2/a^2 - \frac{1}{2}(q-1)e^2/a^2 = Fe^2/a^2 \quad . \quad . \quad . \quad (126)$$

where

$$F = Z - \{p + \frac{1}{2}(q-1)\} = (Z - \sigma)$$

$F$  is the effective nuclear charge, and  $\sigma$  is known as the *screening constant*.

Thus, applying our dynamical and quantum conditions, we have

$$\frac{mv^2}{a} = F \frac{e^2}{a^2}; \quad mva = \frac{nh}{2\pi},$$

whence

$$a = \frac{n^2 h^2}{4\pi^2 F e^2 m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

To calculate the potential energy of the system we may suppose that all the orbits are magnified by a large factor  $N$ , so that the distances between the particles become infinite. Let the potential energy in this state be  $W_\infty$ . As  $N$  is progressively reduced the orbits shrink until eventually they reach their normal size. At each stage the force on the given electron is  $-Fe^2/r^2$ , where  $r$  is the actual radius at any instant. The work done on the electron in bringing it to its normal orbit of

radius  $a$  is thus  $-\int_{\infty}^a \frac{Fe^2}{r^2} dr = -\frac{Fe^2}{a}$  and its potential energy is therefore

$$U = W_{\infty} - \frac{4\pi^2 e^4 F^2 m}{n^2 h^2}.$$

The kinetic energy  $T = \frac{1}{2}mv^2 = \frac{2\pi^2 e^4 F^2 m}{n^2 h^2}$  and the total energy of the electron is thus  $U + T$ , or

$$W_{\infty} - \frac{2\pi^2 e^4 F^2 m}{n^2 h^2}.$$

The frequency of the line emitted when an electron falls from an orbit of energy  $W_i$  to one of  $W_f$  is thus  $h\nu = W_i - W_f$  or

$$\nu = \frac{2\pi^2 e^4 m}{h^3} F_f^2 \left\{ \frac{1}{n_f^2} - \frac{F_i^2}{F_f^2} \frac{1}{n_i^2} \right\} = R F_f^2 \left\{ \frac{1}{n_f^2} - \frac{F_i^2}{F_f^2} \frac{1}{n_i^2} \right\}. \quad (128)$$

writing  $R$  for  $\frac{2\pi^2 e^4 m}{h^3}$ .

The screening constant  $\sigma$ , as calculated from our rough approximation (126), has the value 0.5 for the  $K$  level, and 5.5 for the  $L$  level. Thus for elements of high or medium atomic number we can to a first approximation put  $F_i/F_f = 1$ , since  $\sigma$  is small compared with  $Z$ .

To this approximation,  $\nu = R(Z - \sigma)^2 \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$ . But a given X-ray line, say the  $K_{\alpha_1}$  line, represents the transition between the same pair of quantum levels in any atom; and thus  $n_f$  and  $n_i$  are fixed. Thus, finally, writing  $a^2 = R \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$ , we have

$$\nu^{\frac{1}{2}} = a(Z - \sigma) \quad . \quad . \quad . \quad . \quad . \quad . \quad (129)$$

which is Moseley's relation.

Our calculation shows that the relation is only approximate and that it should not be expected to apply to elements of low atomic weight. This is confirmed by experiment. The experimental values for  $\sigma$  are higher than those given by our rough calculations, being of the order of 1 for the  $K$  series, and 8 for the  $L$  series.

For the  $K_{\alpha}$ -radiation corresponding to transitions from  $n=2$  to  $n=1$ , we have approximately

$$\begin{aligned} \nu^{\frac{1}{2}} &= R^{\frac{1}{2}} \left( \frac{1}{1} - \frac{1}{2^2} \right)^{\frac{1}{2}} (Z - \sigma) \\ &= (Z - \sigma) \sqrt{\left( \frac{3}{4} R \right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (130) \end{aligned}$$

which was the form in which Moseley first gave his relation. For molybdenum ( $Z=42$ ), substituting for  $R$  from the spectroscopic value

of Rydberg's constant, we find that the  $K_{\alpha}$ -radiation should have a wavelength of  $0.70 \times 10^{-8}$  cm. The mean wavelength of the  $K_{\alpha}$  lines of molybdenum is actually  $0.71 \times 10^{-8}$  cm.

**176. Sommerfeld's theory. The azimuthal quantum number.** The circular orbits discussed by Bohr are not the only orbits which satisfy the mechanical conditions of the problem. As is well known in gravitational theory a particle moving round a central body to which it is attracted by a force varying inversely as the square of the distance may, under appropriate conditions, describe an elliptic orbit of any eccentricity. If we apply Bohr's relation (117) to elliptical motion, it can be shown that for a given quantum number  $n$  the energy of the system on Newtonian mechanics is the same whether the electron describes a circular or an elliptical orbit. The elliptical orbits do not give rise to any new energy levels, and the theory outlined in the previous sections remains unaltered.

It is possible, however, as Sommerfeld showed, to look at the matter in a somewhat different way. As a particle describes an ellipse its distance from the nucleus alternately decreases and increases. Its motion can thus be regarded as the sum of two motions, one a circular motion round the nucleus, and the other an oscillation along a radius. The former is known as the azimuthal, the latter as the radial motion. Sommerfeld suggested that each of these motions must separately satisfy the quantum relation laid down by Bohr. Thus the total quantum number  $n$  of the orbit has to be divided between these two possible motions, and the eccentricity of the ellipse is determined by the mode of division.

The mode of division was represented by giving first the total quantum number, and secondly the azimuthal quantum number of the orbit. The radial quantum number is the difference between these figures. Thus if we consider the possible orbits for which  $n=3$ , we may represent them by the symbols  $3_3$ ,  $3_2$ ,  $3_1$ ,  $3_0$ . The subscript number is known as the azimuthal quantum number. The  $3_3$  orbit has obviously a radial quantum number of 0. There is thus no radial movement and the orbit is the circular orbit already discussed. The  $3_0$  orbit conversely corresponds to motion only along the radius, and is impossible since the path would pass through the nucleus. The remaining orbits  $3_2$  and  $3_1$  represent ellipses of increasing radial movement and hence of increasing eccentricity. These orbits, together with those for  $n=2$ , are shown in Fig. 116.

The energy associated with an elliptic orbit would be identical with that of the circular orbit of the same quantum number if the mass of

the electron were independent of its velocity. This, however, owing to the relativity correction is not the case. The speed of an electron in a circular orbit is constant, but in an elliptic orbit its speed increases as it approaches its perihelion. Its mass is thus greater the nearer it approaches to the nucleus. Sommerfeld was able to show

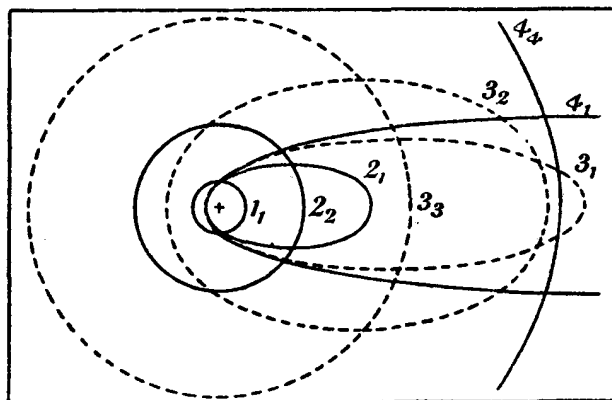


FIG. 116. Illustrating Sommerfeld's system of elliptic orbits

that when this correction is taken into account there is a very small difference in energy between the circular and elliptical orbits, the latter having the slightly greater value.

The 2 quantum level (or  $L$  level to use the X-ray nomenclature) thus consists of two levels close together, and

similarly the 3 quantum level is a close triplet of levels. This complexity in the levels gives rise to a corresponding complexity in the spectrum line emitted by the transition of an electron from one level to the other. The line  $H_\alpha$ , the first line of the Balmer series, is found when examined under very high resolution to consist of three lines very close together. It was in order to explain this fine structure of the Balmer lines (for which Bohr's original theory makes no provision) that Sommerfeld was led to put forward his theory. The three components of the  $H_\alpha$  line are formed by the transitions  $3_3 \rightarrow 2_2$ ,  $3_2 \rightarrow 2_1$ , and  $3_1 \rightarrow 2_2$ . (Experience shows that the only transitions which normally occur are those in which the azimuthal quantum number changes by unity.)

The difference in energy between an  $n_n$  and an  $n_k$  level is given by  $-RhZ^2\alpha^2/n^3k$ , where  $\alpha = 2\pi e^2/hc$  and is known as the fine structure constant.  $\alpha$  is a non-dimensional constant. Its value is  $1/137$ . The actual separation of the components of a Balmer line is so small that it is difficult to measure with accuracy, though the lines have been shown to be triplets. The separation in the ionized helium spectrum, which is 16 times greater, has been accurately measured and is in excellent agreement with the theory.

**177. The magnetic moment of an electron orbit. The Magnetron.** An electron describing a closed path is equivalent to a current flowing round that path. Since the current is measured by the quantity of

$$\mu = e v g.$$
$$\mu = \frac{1}{2} \text{ eV}.$$
$$\mu=n \cdot \frac{1}{4\pi} \frac{e}{m}\hbar \quad . \quad . \quad . \quad . \quad . \quad . \quad (131)$$

178. The spinning electron. The inner quantum number. In its earlier stages the development of the quantum theory of the atom proceeded on semi-empirical lines. We were investigating how far the application of the new and promising principles introduced by Bohr to explain the hydrogen spectrum could be adapted and extended to introduce some semblance of order into the very complicated field of atomic spectra in general. The justification for each new *ad hoc* hypothesis which had to be added to the original theory was to be found in the degree of simplification which it produced in the detailed interpretation of the spectroscopic data.

$$\nu = \frac{R}{(m-a)^2} - \frac{R}{(n-b)^2}$$

where  $m$  is a small whole number, which varies from one series to another,  $R$  is the Rydberg constant, and  $a$  and  $b$  are fractions. This is known as the Ritz combination law. Comparing this with the simpler expression for the Balmer series, it is not difficult to see that the Ritz expression represents the transition of an electron to a single lower energy level, given by the  $m$  term, from a series of higher levels represented by substituting different integral values for  $n$  in the second term. The fractions  $a$  and  $b$  are introduced by the screening effect on

the nuclear charge (§ 175) produced by the inner electrons. The problem is to account for the multiplicity of energy levels revealed by analysis of the spectra.

On general principles we should expect that the spectra of the monovalent alkali metals should be fairly closely analagous to the hydrogen spectrum. As we shall see later, the spectrum of an alkali atom is due to a single valency electron, the remaining electrons in the atom being tightly bound to the nucleus, and thus acting mainly as a partial screen to its nuclear charge. This is, in fact, the case; though, owing to the higher atomic number, the three lines which combine to form the fine structure of a Balmer line in hydrogen are now much more widely spaced, and give rise to separate spectral series in the alkali metals. Thus the series found in the alkali spectra are explicable on Sommerfeld's theory.

On closer observation, however, it is found that each of these lines is itself a close doublet. The well-known sodium-yellow line is an obvious example. Each Sommerfeld orbit, therefore, must represent not a single level but two levels differing slightly in energy. It was to account for this difference that Uhlenbeck, in 1926, suggested that the electron, which we have so far treated as a point charge, was in fact spinning about an axis, like a top.

If we write (117) in the form  $mr^2\omega = n \cdot h/2\pi$  we can regard  $h/2\pi$  as a natural unit of angular momentum, and Sommerfeld's condition may be expressed by saying that every orbit must contain an integral multiple of these natural units of angular momentum. If, however, the electron is spinning it will also have an intrinsic angular momentum which must be combined with that due to its motion round the orbit in order to obtain the total angular momentum associated with the system. The fact that the sodium line consists of two sharp lines shows that the combination can only take place in two ways. The electron spin must be parallel or anti-parallel to the orbit.

A charged sphere spinning about an axis is equivalent to an electric current, and the spinning electron will thus have a magnetic moment. If we assume this to amount to one Bohr magneton it can be shown that its spin momentum will be  $\frac{1}{2}(h/2\pi)$ , or  $\frac{1}{2}$  a quantum unit. The *spin number* is thus  $\frac{1}{2}$ . The difference in angular momentum of the two possible arrangements thus amounts to  $h/2\pi$ .

The difference in energy of the two quantum states is due to the reaction between the magnetic moment of the electron and the magnetic field (due to its own motion round its orbit) in which it is situated. In the parallel arrangement its magnetic axis is parallel to the lines of

force; in the anti-parallel arrangement it lies, so to speak, with its north pole pointing south and has thus magnetic potential energy equal to  $2MH$ , where  $M$  is its magnetic moment and  $H$  is the field. The second arrangement has, therefore, the greater energy associated with it.

To represent these facts a third quantum number must be introduced, known as the *inner quantum number* and always denoted by  $j$ . It is obtained by adding the spin number,  $\pm\frac{1}{2}$ , to the quantum number representing the angular momentum of the motion of an electron round its orbit.

It may be mentioned, in passing, that the proton and the neutron also possess spin. Their individual spins combine in the nucleus according to quantum principles, so that the total spin of a nucleus is always an integral multiple of  $\frac{1}{2}$ . The actual values range from 0 to  $9/2$ . Strangely enough, the neutron, as well as the proton, also has a magnetic moment. The reaction of the magnetic moment of the nucleus with the motion of the electrons in their orbits is responsible for the hyperfine structure exhibited by spectral lines when examined under very high dispersion.

**179. Application of wave mechanics.** The Bohr-Sommerfeld theory provides probably the best *picture* of atomic structure which we are likely to obtain. For this reason we have followed it in some detail. It must, however, be regarded as only a picture although a very useful one. Since an electron has wave properties and since the diameter of the atom is comparable with the wavelength of the electrons in it, the problem of equilibrium within the atom must clearly be treated by wave mechanics. The wave mechanical treatment illuminates many of the *ad hoc* assumptions made in the previous sections. For example, if we take Bohr's original assumption (116) and rewrite it in the form  $2\pi r = nh/mv$ , then, remembering that the wavelength of the electron is given by  $h/mv$ , we see that  $r$  for a stationary orbit is the radius of a circle whose circumference contains a whole number  $n$  of wavelengths.

The mathematical theory of quantum wave mechanics is beyond the scope of this volume. Fortunately its results agree fairly closely with those derived from the more primitive picture. The energy levels are given by the same expression as in the earlier theory, although they are no longer regarded as representing electron orbits. One modification which has emerged must be mentioned because it affects nomenclature. It is found that on the new theory Sommerfeld's azimuthal quantum number  $k$  does not, in fact, give the angular momentum of

the electron, and it has been replaced by a *subordinate* quantum number,  $l$ , where  $l=k-1$  (or more accurately  $k=\sqrt{l(l+1)}$ ), so that the angular momentum of an  $l$  state is actually  $l \cdot (h/2\pi)$ . The most eccentric of the Sommerfeld "ellipses" thus has zero angular momentum, and the possible levels in the three-quantum state are now written  $3_2$ ,  $3_1$ , and  $3_0$ , instead of  $3_3$ ,  $3_2$ , and  $3_1$  as in Fig. 116. The inner quantum number,  $j$ , is equal to  $l \pm \frac{1}{2}$ . These quantum numbers are essentially positive, so that for a quantum state in which  $l=0$ , the only possible value for  $j$  is  $+\frac{1}{2}$ .

**180. Quantization in space. The magnetic quantum number.** So far we have been considering the electron orbit as free from all external fields of force. In practice the nucleus of the atom and the motions of the other electrons in the atom produce a resultant magnetic field. There is thus a unique direction marked out in the atom, the direction of the resultant field, and the space in which the orbit lies is, to this extent, anisotropic.

A rigorous application of quantum mechanics demands that where a unique axis exists the motion relative to this unique direction shall also be quantized.

Thus an orbit for which the inner quantum number is  $j$  must lie in such a plane that the component of the total angular momentum in the direction of the resultant field can be represented by a quantum number  $m$ , where  $m$  differs from  $j$  by an integer only.  $m$  is known as the *magnetic*, or sometimes as the *equatorial* quantum number.

The maximum value of  $m$  occurs when the plane of the orbit is perpendicular to the field and the component is then equal either to  $\pm j$ , according to the direction of rotation of the electron in its orbit. The possible values of  $m$  are thus given by

$$m = -j, -j+1, \dots, j-1, j \quad \dots \quad (132)$$

The angle  $\phi$  between the axis of the orbit and the resultant field is given by  $\cos \phi = m/j$ . If  $l=0$ , so that  $j=\frac{1}{2}$ , the only possible values of  $m$  are  $\pm j$ ; that is the plane of the orbit is perpendicular to the magnetic field, but the electron may describe the orbit in either direction. We shall see in the next section that there is direct experimental evidence for this rather surprising deduction.

Since the orbit has a magnetic moment and is in a resultant magnetic field, its magnetic potential energy will depend on the angle between the axis of the orbit and the direction of the field. The total energy associated with a given orbit thus depends on its inclination, that is, on the value of  $m$ .



**181. Orientation of the atom.** The experiments of Gerlach and Stern. The conclusions arrived at as to the orientation of the electron orbits in a magnetic field have been brilliantly verified by some ingenious experiments of Gerlach and Stern. Silver was boiled in a high vacuum in an electric furnace *F* (Fig. 117) furnished with a single

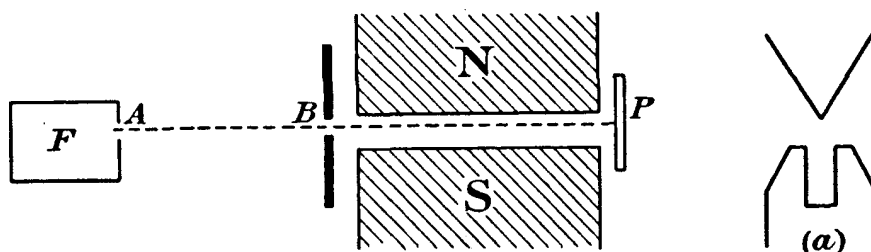


FIG. 117

narrow aperture *A*. From this aperture the atoms of silver streamed out with the velocity of thermal agitation appropriate to the temperature of the furnace. Since the space into which they passed was highly evacuated, the atoms travelled in straight lines, and were formed into a very narrow pencil by means of a second slit *B*. This pencil then passed between the poles *N*, *S* of a magnet arranged so as to give a magnetic field which varied as rapidly as possible in the direction of the field, i.e. from *N* to *S*. The pole pieces are shown in section at (*a*). After leaving this field the atoms impinged upon a cold glass plate *P* where, it was found, they left a trace which was just perceptible, and which could be intensified if necessary by chemical methods. The position at which the atoms struck the plate could thus be ascertained.

If  $M_y$  is the resolved part of the magnetic moment of the atom in the direction of the field  $H$  there will be a force acting on it equal to  $M_y \cdot \partial H / \partial y$ . This will produce a deflection  $y$  in the direction  $H$ , since a magnet always moves from weak to strong parts of the field, equal to  $\frac{1}{2} \frac{M_y}{A} \cdot \frac{\partial H}{\partial y} \cdot \frac{l^2}{v^2}$ , where  $l$  is the length of path of the atoms in the magnetic field. The velocity  $v$  can be calculated from the temperature of the furnace, and the mass  $A$  of the atom is known. By measuring  $\partial H / \partial y$  and  $l$ ,  $M_y$  can be calculated, from the deflection  $y$ .

If the electron orbits had a random orientation the resolved part of the magnetic moment would have all values from zero to  $M$ , where  $M$  is the magnetic moment of the atom. As we shall see later the electron orbits in silver and also in copper and gold form a closed system in which the magnetic moments of the electrons balance out, with one

additional electron which occupies an orbit of subordinate quantum number 0. The magnetic moment of the atom is due to this electron. For silver, copper, and gold, Stern and Gerlach found that the trace on the glass plate consisted of two definite lines, one on each side of the undeflected position, the only values found for the magnetic moment being thus  $\pm M$ . None of the atoms was undeflected in the field. The atoms must, therefore, be completely orientated during the whole of their passage through the field, with the magnetic axis pointing in either the positive or negative direction of the field. That is to say the orbit is perpendicular to the field, though the electron may revolve in either direction around it. This is in exact accord with the predictions of § 178 for an orbit for which  $l=0$ . By measuring the various quantities involved the value of the magnetic moment of the atoms of the three metals employed was found to be  $9 \times 10^{-21}$ , in excellent agreement with the calculated value of the magneton.

**182. The Zeeman effect.** If a source of light emitting a line spectrum is placed between the poles of a strong electromagnet, each line is split up into a number of components. In the simplest case the single line is split into two components equidistant from the position of the undisturbed line when viewed along the direction of the magnetic field. If viewed at right angles to this direction, there is also a line in the undisturbed position, and the single line becomes a triplet. The phenomenon is known as the Zeeman effect. The effect of a magnetic field on the electronic orbit can be determined by a well-known theorem due to Larmor. He has shown that the form of the orbit and the velocity with which it is described may be regarded as unaltered by the magnetic field if we refer all our quantities to a system of co-ordinates which rotates around the direction of the magnetic field with an angular velocity  $\omega$  given by

$$\omega = \frac{1}{2} \frac{e}{\mu} \cdot \frac{H}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

where  $H$  is the magnetic field and  $\mu$  the mass of the electron.

This velocity can be shown to be small compared with the velocity of the electron in its orbit. The orbit may thus be regarded as of constant shape and size but precessing slowly around the direction of the magnetic field. It can further be shown that, if the precession is slow, the quantum conditions will be unaffected, and an  $n$  quantum orbit will remain an  $n$  quantum orbit in spite of the new precessional motion. This is known as the principle of *adiabatic invariance*. The kinetic energy of the electron in its orbit will, however, in general, be

slightly altered. Let us consider, for the sake of simplicity, a single electron describing a one quantum orbit about the nucleus. The plane of this orbit must be (§ 180) perpendicular to the lines of the field. Now the velocity of the electron is unchanged with respect to the *moving* axes of reference. If the axes move in the direction in which the electron is rotating in its orbit its actual velocity referred to fixed axes is obviously increased. If the two rotations are opposed, it is diminished. Hence the energy of the electron in the orbit is either increased or decreased according to its direction of rotation by some amount  $\partial T$ .

It can be shown that

$$\partial T = m \frac{\omega}{2\pi} h,$$

where  $m$  is the magnetic quantum number (§ 180).

Thus for a transition in which the magnetic quantum number changes by unity, the effect of the magnetic field is to alter the frequency by an amount  $\partial\nu$ , given by

$$h \cdot \partial\nu = \frac{\omega}{2\pi} h; \quad \partial\nu = \frac{eH}{4\pi\mu c} \quad . \quad . \quad . \quad . \quad . \quad (134)$$

The undisplaced line is given by transitions in which  $m$  remains unaltered.

A similar decomposition of the spectral lines is produced by a strong electric field. This is known as the Stark effect.

**183. Pauli's exclusion principle.** Pauli has suggested that it is impossible for a given quantum state to be occupied simultaneously by more than one electron. Thus all the electrons in the atom must be assigned a different set of quantum numbers. This is known as the exclusion principle.

We have seen that four quantum numbers are necessary and sufficient to specify completely the quantum state of an electron in the atom. Those most generally employed are  $n$ ,  $l$ ,  $j$ , and  $m$ , and the state is usually written  $n, l, j, m$ . The spin number,  $s$  ( $s = \pm \frac{1}{2}$ ), can be used instead of  $j$ . Spectroscopists, for historical reasons, use the letters  $S, P, D, F \dots$  instead of the numerals 0, 1, 2, 3  $\dots$  in specifying  $l$ , and write  $2P_{\frac{3}{2}}$  for the level represented by  $2, 1, \frac{3}{2}$ . The total number of quantum states corresponding to a given shell of total quantum number  $n$  can be deduced from the relations already discussed. For a shell of total quantum number  $n$ , there are  $n$  possible values of  $l$ , ranging from 0 to  $n-1$ , and for each value of  $l$  we have two values of  $j$ .

namely  $j=l+\frac{1}{2}$ ;  $j=l-\frac{1}{2}$ . Substituting in (132) we find the following values for  $m$ :

$$\begin{array}{rcll} j=l-\frac{1}{2} : m=-l+\frac{1}{2}, & -l+\frac{3}{2}, & \dots, & l-\frac{3}{2}, \quad l-\frac{1}{2} \quad \text{total } 2l \\ j=l+\frac{1}{2} : m=-l-\frac{1}{2}, & -l+\frac{1}{2}, & \dots, & l-\frac{1}{2}, \quad l+\frac{1}{2} \quad \text{total } 2l+2 \\ & & & \text{i.e. in all } 4l+2 \end{array}$$

Thus the total number of electrons which can be contained in a set of levels of subordinate quantum number  $l$  is  $2(2l+1)$ .

For the  $K$  level, for which  $n$  is 1,  $l$  must have the value 0 and only two electrons can be present. For the  $L$  level we have  $n=2$ , while  $l$  may have the values 0 and 1. Corresponding to  $l=0$  we have again 2 electrons, and to the value  $l=1$ , 6, making 8 in all. Thus 8 electrons are required to complete the  $L$  level. Similarly the  $M$  level requires 18 electrons and the  $N$ , 32 electrons for its completion. Of these 2 are in  $4_0$  orbits, 6 in  $4_1$  orbits, 10 in  $4_2$  orbits, and 14 in  $4_3$  orbits. Assuming Pauli's exclusion principle we can thus calculate precisely the number and distribution of the electrons in any completed level.

It may be noted that the exclusion principle does not apply to particles carrying an even number of charges. Thus there is no limit to the number of  $\alpha$ -particles which can occupy the same quantum level. Thus presumably all the  $\alpha$ -particles in the nucleus are normally in the ground level, and the different energy states, discussed in § 158, possibly represent differences in energy caused by the transference of different numbers of  $\alpha$ -particles from one level to another.

**184. Electron grouping and the periodic classification of the elements.** The total number of extra-nuclear electrons in the atom is equal to the atomic number. The  $K$  level thus becomes saturated for an element of atomic number 2, that is helium, and the  $L$  level for an atomic number  $2+8$ , that is neon. Both helium and neon belong to the group of inert gases, and we can very reasonably associate their inertness with their electronic structure.

A complete energy level is a symmetrical structure which exerts very little force outside its own system. It is unable to attract or attach a further electron. It is also a stable system, and will not readily part with one of its components. This is verified by the comparatively high ionization potentials of the inert gases. The atom will thus be neither electro-negative nor electro-positive and will not form polar compounds, while its very weak residual field will debar it from forming non-polar attachments.

Let us consider what will happen if we add another proton to the neon nucleus ( $Z=10$ ), thus increasing its atomic number to 11 (sodium).

The extra positive charge requires an additional extra-nuclear electron in the atomic structure. Neon has 10 electrons ( $2+8$ ) so that both the  $K$  and the  $L$  levels are already complete. The extra electron must therefore be attached in one of the  $M$  levels, and presumably in the lowest, or  $3_0$  level. This is an elliptic orbit of high eccentricity which at its furthest point carries the electron well beyond the close tangle of the  $K$ ,  $L$  orbits which screen it, to a very large extent, from the attraction of the nucleus, so that it is very easily detached from the atom. The atom thus is strongly electro-positive, and since it can only lose one electron (the remainder being firmly bound in the  $K$  and  $L$  levels) will obviously be monovalent. We should thus expect an alkali metal to succeed each inert gas in the list of elements, as indeed we find to be the case.

Increasing the nuclear charge by another unit, i.e. passing from sodium to magnesium, causes the addition of a second electron, also in the  $3_0$  level. Conditions are still very similar except that the atom can lose two electrons, giving it a positive valency of two, but since the nucleus has now an extra charge the force retaining the extra electrons in their orbits is somewhat stronger than for sodium, and the element is not so strongly electro-positive. Thus the alkali metal is followed by a member of the alkali earth group.

Suppose, on the other hand, we reduce the nuclear charge from 10 to 9. The atom must now normally give up one of the electrons from its  $L$  level. The  $L$  level, however, will tend to become saturated by the absorption of an electron if the occasion offers, giving an atom with a resultant negative charge of unity. The element fluorine should thus be strongly electro-negative, and of unit valency. Thus each inert gas should be preceded by a member of the halogens. Our model thus accounts for the most strongly marked recurrent feature of the periodic classification.

The  $M$  level has been shown to require 18 electrons for its saturation, but the third inert gas has an atomic number 18, or  $2+8+8$ . We must suppose that the filling of the eight  $3_0$  and  $3_1$  levels produces an electron system with the symmetry, and hence much of the stability, of the  $L$  system. At any rate it is certain that the next electron is bound, not in a  $3_2$  level, but in a  $4_0$  orbit. It can be shown that the energy of binding in this orbit is greater than that in a  $3_2$  orbit, for a nuclear charge of 19, and this also applies to the next element calcium (20).

Since we are unable to calculate the energy levels for elements of high atomic number, we cannot construct our electronic scheme from first principles. We can, however, reconstruct it from spectroscopic

TABLE XI

	<i>K</i> $1_0$	<i>L</i> $2_0, 2_1$	<i>M</i> $3_0, 3_1, 3_2$	<i>N</i> $4_0, 4_1, 4_2, 4_3$	<i>O</i> $5_0, 5_1, 5_2$	<i>P</i> $6_0, 6_1$	
1. H	1						
2. He	2						
3. Li	2	1					1st short series
4. Be	2	2					
5. B	2	2, 1					
6. C	2	2, 2					
7. N	2	2, 3					
8. O	2	2, 4					
9. F	2	2, 5					
10. Ne	2	2, 6					
11. Na	2	2, 6	1				2nd short series
..							
18. A	2	2, 6	2, 6				
19. K	2	2, 6	2, 6	1			1st long series
20. Ca	2	2, 6	2, 6	2			
21. Sc	2	2, 6	2, 6, 1	2			
22. Ti	2	2, 6	2, 6, 2	2			
23. V	2	2, 6	2, 6, 3	2			
24. Cr	2	2, 6	2, 6, 5	1			
25. Mn	2	2, 6	2, 6, 5	2			
26. Fe	2	2, 6	2, 6, 6	2			
27. Co	2	2, 6	2, 6, 7	2			
28. Ni	2	2, 6	2, 6, 8	2			
29. Cu	2	2, 6	2, 6, 10	1			
30. Zn	2	2, 6	2, 6, 10	2			
31. Ga	2	2, 6	2, 6, 10	2, 1			
..							
36. Kr	2	2, 6	2, 6, 10	2, 6			2nd long series
..							
54. X	2	2, 6	2, 6, 10	2, 6, 10	2, 6		
..							Series con- taining rare earth metals
79. Au	2	2, 6	2, 6, 10	2, 6, 10, 14	2, 6, 10	1	
..							
86. Rn	2	2, 6	2, 6, 10	2, 6, 10, 14	2, 6, 10	2, 6	

data. The Zeeman effect and the Stark effect are often useful in identifying the particular quantum numbers to be assigned to a given electron. Table XI contains part of such a reconstruction due to Stoner. It is gratifying to note that an almost exactly similar table was drawn up by Main-Smith entirely on chemical grounds.

The general outlines of the periodic classification and their interpretation in terms of electron levels will be clear from the table. The first short period is caused by the regular filling up of the various  $L$  orbits, and the second short period, which closely resembles it, by the orderly filling of the  $3_0$  and  $3_1$  orbits. The completion of these levels gives a structure resembling the  $L$  level, and the corresponding element is an inert gas argon. The next electrons are added in the  $4_0$  level, giving potassium and calcium. The structure of these elements, however, differs from that of sodium in the fact that there are still 10 vacant quantum states in the  $M$  level. With scandium the process of filling these states begins, and we have a succession of elements with somewhat similar properties and mostly with two electrons in the  $4_0$  state. They are thus all electro-positive, many of them having a characteristic valency of 2, although, owing to the possibility of losing an electron from the incomplete  $3_2$  level, they also exhibit higher valencies. It will be noticed that the element of atomic number 28 is not an inert gas, as two of its electrons are found in  $4_0$  levels, instead of completing the  $M$  level.

Krypton, again, has the argon structure, with eight electrons in the  $N$  level. The development follows the lines of the first long series, and gives rise to a second long series. With xenon we have the additional complication that not only are the  $O$  levels incomplete but there are 14 vacant states in the underlying  $N$  levels. The filling of these gaps gives rise to the series which includes the rare earth metals. It will be seen that the table accounts not only for the regularities but also for the irregularities of the periodic classification of the elements.

**185. The electron theory of metals.** The electrical conductivity of metals is most readily explained by assuming that the metal contains a large number of free electrons, capable of moving about in it under the influence of an electrical field. These electrons are presumably detached from the outer levels of the metallic atoms, which are, of course, all electro-positive and thus tend to lose electrons. The positive metallic ions, so formed, are rigidly bound in a crystal lattice and take no part in the electrical transference.

In the absence of evidence to the contrary it was natural to assume

that the electrons behaved like molecules of a gas, sharing in the energy of thermal agitation of the surrounding atoms, and having an energy distribution given by Maxwell's distribution law. On these assumptions formulae were derived for the thermionic emission, for the electrical and thermal conductivities, and for the various thermoelectric effects, which agreed reasonably well with experimental observations.

If we assume that the electronic atmosphere behaves as a perfect gas, the current in a uniform conductor of cross-sectional area  $A$  is by (5)  $i = AnekX$ , where  $n$  is the number of free electrons in unit volume, and  $k$  is their mobility. By (12) we have  $k = \frac{1}{2} \frac{e}{m} \frac{\lambda}{v}$  where  $\lambda$  is the mean free path of the electron and  $v$  its root mean square velocity. Assuming the equipartition of energy we can write  $\frac{1}{2}mv^2 = \frac{3}{2}\alpha\theta$ , where  $\theta$  is the absolute temperature and  $\alpha$  is Boltzmann's constant. Hence on substituting for  $k$  and eliminating  $m$  we have

$$i = A \frac{ne^2\lambda}{2mv} X = A \frac{ne^2\lambda v}{6\alpha\theta} X \quad . \quad . \quad . \quad . \quad (135)$$

If the field is uniform we have  $X = \frac{V}{d}$ , where  $d$  is the length of the conductor and  $V$  the applied E.M.F. Thus  $i = \frac{A}{d} \frac{ne^2\lambda v}{6\alpha\theta} V$ , which leads at once to Ohm's law. Comparing with the equation  $i = \frac{A}{d} \sigma V$ , where  $\sigma$  is the specific conductivity, we see that the specific conductivity is given by  $\frac{ne^2\lambda v}{6\alpha\theta}$ , and is constant for a given temperature. The only quantities in this expression which vary for different substances are  $\lambda$  and  $n$ . It does not seem probable that the variations in  $\lambda$  will be very great. The conductivity of a substance therefore should depend mainly upon  $n$ , the number of free electrons per unit volume.

Suppose that one part of a metallic conductor is heated. The electrons in the heated part have their velocities increased to correspond with the new value of the temperature. The electrons moving away from the heated part will therefore have a greater energy than those moving towards it. There will thus be a transference of energy from the hot to the cold part of the conductor, or in other words there will be conduction of heat through the metal by the electrons. It is known that non-conductors of electricity are very poor conductors of



heat. It seems probable therefore that the conduction by the electrons is very much greater than that due to the atoms of the substance.

Assuming that the atmosphere of electrons in the metal can be treated as a gas, the thermal conductivity  $\kappa$  of the electrons should be given by

$$\kappa = \frac{1}{2} n v \alpha \lambda \quad . . . . . (136)$$

(see Jeans, *Dynamical Theory of Gases*), where  $n$ ,  $v$ ,  $\alpha$ , and  $\lambda$  have the same significance as in the equation for electrical conductivity. Neglecting any conductivity due to the fixed atoms this will be the thermal conductivity of the metal. Thus the ratio of the thermal to the electrical conductivity is given by

$$\frac{\kappa}{\sigma} = \left( \frac{1}{2} n v \alpha \lambda \right) / \left( \frac{n e^2 \lambda v}{6 \alpha \theta} \right) = 3 \left( \frac{\alpha}{e} \right)^2 \theta \quad . . . . . (137)$$

The ratio of the thermal to the electrical conductivity should at a given temperature therefore be independent of the nature of the conductor. This is the well-known law of Wiedemann and Franz. Again this ratio should be directly proportional to the absolute temperature, that is, its temperature coefficient should be  $3.67 \times 10^{-3}$  per degree C. Table XII

TABLE XII  
Ratio of thermal to electrical conductivity

Material	$\kappa/\sigma$ at 18° C.	Temperature coefficient
Copper	$6.7 \times 10^{10}$	$3.9 \times 10^{-3}$
Silver	$6.9 \times 10^{10}$	$3.7 \times 10^{-3}$
Gold	$7.1 \times 10^{10}$	$3.7 \times 10^{-3}$
Lead	$7.2 \times 10^{10}$	$4.0 \times 10^{-3}$
Tin	$7.4 \times 10^{10}$	$3.4 \times 10^{-3}$
Platinum	$7.5 \times 10^{10}$	$4.6 \times 10^{-3}$
Palladium	$7.5 \times 10^{10}$	$4.6 \times 10^{-3}$
Iron	$8.0 \times 10^{10}$	$4.3 \times 10^{-3}$

shows that, for most pure metals, these deductions from the electron theory are very approximately true.

The value of  $\kappa/\sigma$  as calculated from (137) is  $6.4 \times 10^{10}$  at 18° C. Considering the approximations made in the calculation the agreement is not unsatisfactory.

**186. Quantum theory of metals.** Difficulties, however, arose from another quarter, and of a serious kind. The atomic heat of most of the metals has the value 6, given by Dulong and Petit's law. The specific

heat is, therefore, in good agreement with the supposition that it is due solely to the change in heat energy of the atoms. Experiments by Lindemann, in which a contribution from the free electrons was specially sought for, convinced him that their contribution was certainly not more than 1 per cent. of the total. On the other hand, to explain the specific conductivity of metals on the theory outlined above it is necessary to assume that  $n$ , the number of free electrons per c.c., is of the same order as the number of atoms. They should, therefore, on classical theory, account for about one-half of the total heat energy of the metal.

The difficulty has been circumvented by Sommerfeld in the following manner. We have seen that energy of the order of a few volts is required to extract an electron from a metal. We may thus regard the surface of a metal as presenting a potential barrier of this magnitude against the escape of electrons. We may now further suppose that the motion of the electrons inside this barrier is quantized, so that there are a large number of quantum, or "phase" states to use the language of the new statistical theory, differing from each other by small but definite amounts of energy. By the exclusion principle there cannot be more than one electron in a given state at the same time. Thus all the electrons differ from each other in energy by small but definite amounts.

If the number of electrons is small in comparison with the number of levels, so that most of the levels are unoccupied, the system can take up or give out energy freely. Thus an electron in a level of small energy can, by collision with a more energetic atom, take up sufficient energy to raise it to one of the slightly higher energy levels, or conversely it can give up a definite amount of energy and fall into a somewhat lower level. The lowest level in the system is, of course, the level in which the electron has zero energy. The average energy of an electron will thus be very approximately equal to the average energy of the neighbouring atoms, i.e. about 0.023 eV at 0° C.

Let us suppose, however, that the number of electrons becomes comparable with the total number of possible states, so that most of the levels are occupied. The electrons, as we have seen, tend to have an average energy of about 0.02 volt, so that all the levels in this neighbourhood say from 0 to 0.04 volt are rapidly filled. If more electrons are present they cannot fall into levels of lower energy, since these are already filled, and must therefore rise into higher and higher levels. The more electrons there are, the higher the tide of occupied levels must rise. Thus in the case of silver at 0° C. it has been calculated that the

energies of the individual electrons range from 0 to 5.6 volts, instead of being distributed about a mean value of 0.023 volt. A system of this kind in which all the lower levels are occupied is said to be degenerate, and it can be shown that, owing to their large number and small mass the electrons in any metal form a highly degenerate system.

Such a system is almost incapable of exchanging thermal energy with the surrounding atoms. An electron in a higher level cannot give up energy to an atom, because there is no unoccupied lower energy level to which it can pass. Thus if the temperature of the conductor is lowered the total energy in the electronic system remains practically unaltered, even if the temperature sinks to absolute zero. Similarly if the temperature is raised the electrons in the lower levels cannot absorb energy from the atoms unless they can absorb sufficient at one collision to raise them to one of the unoccupied highest levels. As this requires an amount of energy some 200 times greater than the average energy of an atom at 0° C. its occurrence is extremely infrequent, and when it does occur is liable to be nullified by an electron from a higher level falling into the now vacant state.

But the specific heat of the electronic system is the rate of increase of energy with temperature. The specific heat of a degenerate system is thus very small. Exact calculation shows that in the case of metals the contribution of the electron system to the total specific heat is, at ordinary temperatures, rather less than 1 per cent. It is thus too small to be detected.

The quantization of the electron system introduces some modifications into the elementary theory described earlier. According to Sommerfeld the electrical conductivity  $\sigma$  is given by

$$\sigma = \frac{8\pi}{3} \frac{e^2 \lambda}{h^2} \left( \frac{3n}{8\pi} \right)^{\frac{2}{3}} \quad . . . . . (138)$$

and the thermal conductivity by

$$\kappa = \frac{8\pi^3}{9} \frac{\lambda \alpha^2 \theta}{h^2} \left( \frac{3n}{8\pi} \right)^{\frac{2}{3}} \quad . . . . . (139)$$

the symbols having the same meaning as before. Thus

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \left( \frac{\alpha}{e} \right)^2 \theta \quad . . . . . (140)$$

The new theory thus gives the same temperature coefficient as the classical theory. The numerical value for the ratio  $k/\sigma$  at 18° C. works out at  $7.1 \times 10^{10}$ , in excellent agreement with the experimental data.

**187. Thermionic emission.** The theory can be applied to the problem of thermionic emission, which in an earlier chapter we treated by thermodynamical methods. Calculation shows that the electron system in a metal remains degenerate up to temperatures of the order of  $15,000^{\circ}\text{C}$ . Assuming that an electron will leave the surface if its kinetic energy perpendicular to the surface is greater than some constant  $W_a$  it can be shown that the current density of emission should be given by

$$i = \frac{4\pi em\alpha^2}{h^3} \theta^2 \epsilon^{-\frac{(W_a - W_i)}{\alpha\theta}}, \quad \dots \quad (141)$$

where  $W_i$  is the energy of the highest phase state which is normally occupied in the degenerate electron system. If we suppose that a fraction  $r$  of the electrons reaching the surface with the appropriate energy are reflected back into the metal

$$i = (1-r) \frac{4\pi em\alpha^2}{h^3} \theta^2 \epsilon^{-\frac{(W_a - W_i)}{\alpha\theta}} \quad \dots \quad (142)$$

The numerical value of  $4\pi em\alpha^2/h^3$  is about 120 when expressed in amperes per square cm. It may be noticed that it is independent of the nature of the metal. The most recent determinations of the emission constant  $A$  (§ 60) for clean surfaces of specially purified tungsten and molybdenum is about 60. The reflection coefficient  $r$  is thus approximately  $\frac{1}{2}$  for all these metals. The smaller values of  $A$  shown by coated surfaces are due to a higher reflection coefficient.

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For the newer quantum wave mechanics

FLINT, *Wave Mechanics*,

MOTT, *Wave Mechanics*,

are comparatively elementary.

DIRAC, *Quantum Mechanics*,

CONDON and MORSE, *Quantum Mechanics*,

FRENKEL, *Wave Mechanics*, Vol. 1, Elementary Theory,

are more advanced texts.

## CHAPTER XVII

### RADIO-ACTIVE CHANGES

**188. Disintegration of uranium.** Uranium and its salts give off both  $\alpha$ - and  $\beta$ -rays, together with some  $\gamma$ -rays of small intensity. Using a thick layer of the substance practically the whole of the effect on a photographic plate is due to the  $\beta$ -radiation, since owing to the great absorbability of the  $\alpha$ -rays only those from a comparatively thin layer can reach the surface and at the best of times the photographic effect of an  $\alpha$ -ray is small. In 1900 Sir Wm. Crookes showed that it was possible by a single chemical operation to obtain uranium which was photographically inactive, the whole of the photographic activity being concentrated in a very small residue, free from uranium and consisting chemically of the small traces of the impurities present in the original salt. This residue, to which he gave the name of uranium X, could be obtained many hundred times as active weight for weight as uranium itself, while the major fraction which could be shown by chemical tests to be uranium was photographically inactive. The method originally used was to precipitate the uranium with ammonium carbonate and to dissolve the precipitate in excess of the reagent. On filtering, a trace of precipitate remained behind on the filter paper. This contained the uranium X. The same separation can also be carried out by other methods. So far the phenomena resemble very closely the separation of two ordinary chemical substances by the usual methods of analysis. However, if the two fractions are laid aside for two or three months it is found that the uranium has recovered the whole of its original activity while that of the uranium X has completely disappeared. The loss of activity of the uranium was therefore only temporary in character, while the increased activity of the separated uranium X was equally short lived.

It was further found that the total activity, as measured by the photographic effect, that is, by the  $\beta$ -rays, was constant at any time before or after the separation, the gain of activity of the uranium exactly corresponding to the loss of activity of the uranium X. The measurements are most conveniently made by the electrical method. The substance to be examined is spread in a thin layer over the lower plate *B* of the parallel plate ionization chamber (Fig. 118), which is

used as the high potential plate and connected to a battery of cells. The upper plate *A* is connected to the electroscope or electrometer, and

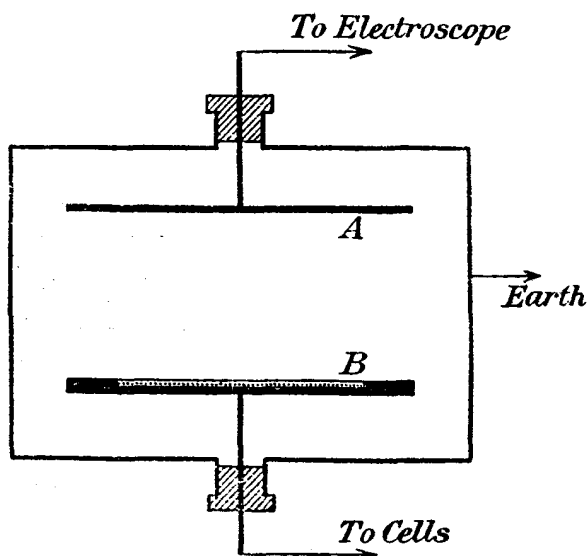


FIG. 118. Apparatus for measuring the activity of a radio-active substance

the saturation current is measured in the usual way. The saturation current is taken as proportional to the activity of the substance. In order to be comparable with the photographic effect which we have been describing and which, as has been pointed out, is due to the  $\beta$ -rays, the substance must be covered with 1/10 mm. of aluminium to cut off all the  $\alpha$ -radiation.

Using two chambers of this type we can investigate more closely the change of

activity with time in the two fractions. The results for uranium are shown in Fig. 119. The two curves are complementary, the sum of the two ordinates at any point being the same.

By plotting the *logarithms* of the currents against the time it is found that the decay curve of the uranium X follows an exponential law, that is, if  $I_t$  is the activity at a time  $t$ , then

$$\frac{I_t}{I_0} = e^{-\lambda t}, \quad \dots \dots \dots (143)$$

where  $I_0$  is the activity at the moment when the measurements began, and  $\lambda$  is a constant, which is known as the *radio-active constant* for the particular substance, in this case uranium X. Since the curves are complementary the recovery curve for the uranium can be expressed in the form

$$\frac{I_t}{I_0} = 1 - e^{-\lambda t}, \quad \dots \dots \dots (144)$$

where  $t$  is the time which has elapsed since the uranium was completely freed from uranium X, and  $\lambda$  is the same constant as before.

Differentiating the expression for the decay of uranium X we have

$$\frac{dI}{dt} = -\lambda I_0 e^{-\lambda t}.$$

But  $I_0 e^{-\lambda t}$  is the quantity of the uranium X still present =  $I$  say. Thus

$$\frac{dI}{dt} = -\lambda I \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

In other words the rate of decay at any moment is simply proportional to the quantity of radioactive matter still present, a very important result.

The uranium which has been freed from UX is inactive as measured by its  $\beta$ -ray effect. If, however, we remove the aluminium screens from the two fractions in Fig. 118 we find that the ionization is now nearly

all due to the uranium itself, the activity of the separated fraction being negligible in comparison with that of uranium itself. In other words uranium itself gives out  $\alpha$ -rays but no  $\beta$ -rays while the uranium X gives out  $\beta$ -rays but no  $\alpha$ -rays.

The rate of decay of activity of the UX has been shown by many experiments to be quite independent of external conditions. It is the same whether the two fractions are kept separate or whether they are enclosed in the same tube; the same whether the product is exposed to the air or sealed in a thick-walled lead box, so as to be freed as far as possible from all external influences. It is quite independent of temperature, being the same at the temperature of liquid air as at a red heat. It does not depend on the processes used for the separation of the two substances and is therefore presumably independent of the state of combination of the substance. The growth of activity of the uranium salt is the same if it is changed into some other salt of the metal, or whether it is solid or in solution. It is in fact a constant of the element itself.

**189. Theory of radio-active change.** It is at first sight somewhat remarkable that the processes of recovery and decay should be so closely connected even when the two fractions are separated from each other in such a way as to preclude all possibility of interaction between

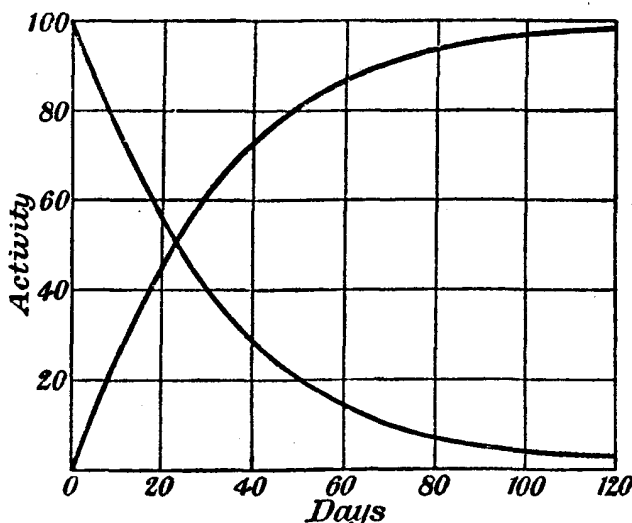


FIG. 119. Decay and recovery curves for uranium X

them. The effects however have found a complete explanation on the following assumptions:

(1) That there is a constant production of the new radio-active substance (UX) by the radio-active body (uranium).

(2) That the new substance (UX) itself disintegrates according to an exponential law, so that  $q=q_0e^{-\lambda t}$ , where  $\lambda$  is the radio-active constant for the change.

Suppose that from the given mass of uranium  $q_0$  particles of uranium X are produced per second. The activity of the particles produced in a short interval of time  $dt$  will at their moment of production be equal to  $Kq_0dt$ , where  $K$  is a constant measuring the effect produced on our electroscope by one particle of UX. The activity of these particles after a time  $\tau$  will be given by

$$Kq_0e^{-\lambda\tau}d\tau=dI \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$

Suppose we wish to know the total activity of the uranium at a time  $T$  after it has been completely freed from the product UX. Then the time which has elapsed since the formation of the particular particles we are dealing with is equal to  $T-t$ , where  $t$  represents their moment of formation. The effect due to these particles is thus  $Kq_0e^{-\lambda(T-t)}dt$  and the total activity at the time  $T$  is therefore given by

$$\begin{aligned} I_T &= \int_0^T Kq_0e^{-\lambda(T-t)}dt \\ &= \frac{Kq_0}{\lambda}(1-e^{-\lambda T}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (147) \end{aligned}$$

The maximum activity is reached when the process has been going on for a sufficient time to make  $e^{-\lambda T}$  negligible, and is given by

$$I_0 = \frac{Kq_0}{\lambda}.$$

Substituting in (147) we have

$$\frac{I_T}{I_0} = 1 - e^{-\lambda T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

which agrees with the experimental results. It will be noted that  $\lambda$  is the constant measuring the rate of decay of the product.

The equilibrium state is obviously reached when the loss of activity due to decay of the product is equal to the rate of formation of the product from the original substance. Since the product is always decaying according to the exponential law, whether it is mixed with



the parent substance or separated from it, it follows that uranium is constantly producing a new substance uranium X from itself at a definite constant rate. When equilibrium has been established it is evident that the quantities of the two substances present will bear a constant ratio to each other.

The possibility of separating uranium X from uranium in various chemical ways shows that the two substances are chemically distinct. Moreover, the uranium not only gives uranium X but in the process emits  $\alpha$ -particles. It is impossible to avoid the conclusion that the atom of uranium disintegrates spontaneously forming a new element uranium X which, although owing to its rapid decay is present in quantities too small to admit of direct chemical investigation, can be detected by the  $\beta$ -radiations which it emits. The product of the decay of uranium X has been identified as a new radio-active substance, ionium, which in its turn has been proved to be the origin of the element radium.

**190. Theory of successive transformations.** The case we have so far considered has been a very simple one for two reasons. In the first case the amount of uranium breaking up during the time of our experiments is so infinitesimally small that we can without error regard the amount of uranium present during the experiment as constant. In the second place the substance formed by the disintegration of the uranium X is so feebly radio-active that its effect upon the electroscope is inappreciable. In general these simple conditions do not hold. Radium, for example, which has been prepared for more than a few days is found to contain at least five radio-active substances whose activity must be taken into account. The same principles which were successful in explaining the decay and recovery curves of uranium and its product can also be applied to the more complex cases with equal success. For convenience these principles have been stated in the form of six propositions by Rutherford, to whom the theory is due.

(1) The activity shown by radio-active substances is due to the disintegration of the atom.

(2) A definite fraction of the total number of atoms present become unstable in a given small interval of time.

(3) In most cases this instability and disintegration is accompanied by the emission of energy in the form of  $\alpha$ -,  $\beta$ -, or  $\gamma$ -rays. If radiations are emitted it is assumed that the fraction of the atoms changed per second can be measured by the intensities of the radiations emitted.

(4) The expulsion of a single  $\alpha$ -ray of atomic weight 4 gives rise to an atom differing in atomic weight by four units from the parent atom. The expulsion of  $\beta$ -particles, which have a mass insignificant compared

with that of a hydrogen atom, produces no change in atomic weight, but increases the nuclear charge by unity.

(5) At any time after disintegration has commenced there exist together in the substance

(a) the unchanged substance,

(b) the immediate product of its disintegration of the same or smaller atomic weight,

(c) any product or products formed from the disintegration of (b) if it is itself radio-active.

(6) Each of the products of radio-active change is a new element, and has different chemical properties from those of the parent atom. This is shown by their different behaviour to various chemical reagents as in the case of uranium. It is still more clear in the case of radium, in which the first decomposition product is a gas.

The general problem may be stated as follows: Suppose we have a series of elements  $A, B, C, D, \dots$  such that  $A$  changes into  $B$ ,  $B$  into  $C$ ,  $C$  into  $D$ , and so on. Given that the number of atoms of each present at a given moment in a substance is  $a, b, c, d, \dots$  respectively to find the number of each still present in the mixture after a time  $t$  has elapsed. The general problem is capable of solution, but it will be simpler and more instructive to confine our attention to one or two illustrative cases which are met with in actual practice.

By assumption (2) we have for each of the substances

$$-dn/dt \propto n$$

where  $n$  is the number of atoms of that substance actually present at the given moment. Thus

$$\frac{dn}{dt} = -\lambda n; \quad n = n_0 e^{-\lambda t}, \quad \dots \dots \dots (149)$$

that is to say, each element by itself decays according to an exponential law.  $\lambda$  is known as the radio-active constant for the substance.

We shall see later that the disintegration follows a probability law, and that  $\lambda n$  is only the most probable value of the number disintegrating per second. When, however, as is usually the case, the total number of atoms is large, the actual value over a finite interval of time will not differ from its most probable value by an appreciable amount.

It must be noted that  $\lambda n$  is also the rate of formation of the new substance  $B$ . We have already seen that  $\lambda$  is independent of the most extreme changes in external and chemical conditions. It is also independent of the age of the substance. Experiments have been made with radium emanation, which loses half its activity in 3.85 days, over a

period exceeding three months. The rate of decay throughout was strictly exponential, the constant  $\lambda$  being the same at the end of the experiment as at the beginning. It is thus the same for emanation which is freshly formed as for emanation three months old. In fact the break up of the atom depends solely on the law of probability and the nature of the atom.

Two other ways of defining the rate of decay of the radio-active substance are often used. The *half period value* is the time taken for the activity of the substance to fall to one-half its original value. This can be obtained from (149) by putting  $n/n_0 = \frac{1}{2}$  and solving for  $t$ . Thus  $-\lambda t = \log \frac{1}{2}$ ,

$$t = \frac{\log 2}{\lambda} = \frac{0.693}{\lambda} \quad \dots \dots \dots (150)$$

The *average life* of the radio-active atom can also be calculated. The number of atoms changing during a short interval  $dt$  at a time  $t$  since the separation of the substance  $= \lambda n dt$ . Also  $n = n_0 e^{-\lambda t}$  and thus  $dn = \lambda n_0 e^{-\lambda t} dt$ . These atoms have had a life of  $t$  seconds. The average life of all the atoms of the substance is therefore

$$\begin{aligned} & \frac{1}{n_0} \int_0^\infty t \lambda n_0 e^{-\lambda t} dt \\ &= \frac{1}{\lambda} \quad \dots \dots \dots (151) \end{aligned}$$

**191. Radio-active equilibrium.** Suppose we commence with some substance such as uranium or radium the life of which is long compared with the time occupied by our experiments. The average life of uranium, for example, is about  $7 \times 10^9$  years and that of radium about 2280 years. Over any ordinary period of time therefore the quantity of the original radio-active substance which we will call  $A$  remains constant, and hence fresh radio-active matter is produced from it at an approximately constant rate, which will be equal to  $\lambda_1 a_0$ , where  $a_0$  is the number of atoms of  $A$  present, and  $\lambda_1$  its radio-active constant. If  $B$  is the immediate disintegration product of  $A$ ,  $B$  is being formed in the substance at this constant rate  $\lambda_1 a_0$ . But the rate at which  $B$  disintegrates is proportional to the amount of  $B$  actually present, and thus increases with the increase in the quantity of  $B$ . Hence a point will come when the rate of disintegration of  $B$  is equal to its rate of formation from  $A$  and the amount of  $B$  in the substance will then remain constant. The rate at which  $B$  increases is equal to the

difference between its rate of formation from  $A$  and its rate of disintegration into  $C$ . Thus

$$db = \lambda_1 a_0 dt - \lambda_2 b dt \quad . \quad . \quad . \quad . \quad . \quad (152)$$

where  $b$  is the number of atoms of  $B$  present at the given instant. Similarly for the element  $C$  we have

$$dc = \lambda_2 b dt - \lambda_3 c dt.$$

It is evident that the amounts of  $B, C \dots$  present in the mixture will become constant when

$$\lambda_1 a_0 = \lambda_2 b = \lambda_3 c = \dots \quad . \quad . \quad . \quad . \quad . \quad (153)$$

Thus when radio-active equilibrium, as it is called, has been established the various radio-products will bear to each other a constant ratio which is inversely proportional to their radio-active constants. Conversely, if on analysis two radio-active substances are always found occurring in nature in the same ratio, one of them must be a product, direct or indirect, of the other. Thus the fact that the proportion of radium to uranium in minerals is always constant is evidence that radium is a product of the disintegration of uranium.

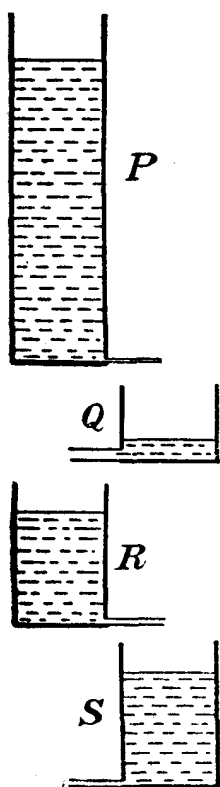


FIG. 120

A hydrostatic analogy may prove useful in understanding the process. Suppose we have a series of tanks  $P, Q, R, \dots$  emptying one into the other as shown in Fig. 120. The rate at which water flows out of any tank will be proportional to the height of water in the tank at the moment under consideration. In other words, supposing the tanks to have the same area of cross section, the flow will be proportional to the quantity of water in the tank. The rate of efflux of the water thus follows the same law as the rate of decomposition of a radio-active substance.

Supposing now that the water in the tank  $P$  is kept at a constant level. This will correspond to the case we have just been considering. Water will rise in each tank until the rate of efflux, which is proportional to the quantity of water in the tank, is equal to the rate at which water pours into it from the tank above. When equilibrium is established the rate of flow will be constant throughout, and the height of the water in the different tanks will be inversely proportional to the resistance offered by their exit tubes to the flow. If the bore is small

the height of the water will be large; if on the other hand it is large a comparatively small height of water will produce the necessary efflux. Similarly when equilibrium is established substances of quick decay, for which the radio-active constant is large, will be present in relatively small proportions while those which decay slowly and for which the radio-active constant is small, will be present in large amounts. It is thus only the comparatively long-lived radio-active substances, such for example as radium itself, which we could expect to obtain in weighable quantities.

The case which we have just considered constitutes what may be described as permanent radio-active equilibrium, the amounts of each substance present remaining constant after equilibrium has once been reached. Let us suppose now that water is no longer poured into the first tank  $P$ , but that  $P$  has a small aperture so that the rate of efflux is small. This will correspond to the case of the decomposition of a radio-active substance of small but appreciable radio-active constant. The rate of flow of water through the system will obviously be governed by that of the tank with the smallest aperture, in this case by  $P$ . It is evident, therefore, that when equilibrium has been established the quantity of water in each tank will be proportional to the quantity remaining in  $P$ , since the rate of flow through the system is governed by this factor. A little consideration will show that the relative amounts of water in the different tanks will be very nearly the same as if the level in  $P$  were maintained at the value which it has at the moment of observation by a continuous influx of water. It will actually be slightly larger. The two cases will approximate more nearly to each other the slower the rate of efflux from  $P$ .

We thus get a state which we may describe as that of *transient equilibrium*, the relative quantities of water in each tank remaining constant, but each of the quantities decreasing in absolute magnitude proportionately to the quantity in the first tank.

This case corresponds to that of a radio-active substance which gives rise by its disintegration to a series of radio-active substances, the half value period of which is short compared to its own. For example, radium emanation, which has a half value period of 3.825 days, gives rise to successive radio-active substances which have half value periods of 3.05, 26.8, and 19.7 minutes respectively. Thus, if radium emanation is placed in an enclosed space and left to decay, after equilibrium has been established, which takes place in about four or five hours, the quantity of any one of the products present is simply proportional to the quantity of the emanation remaining in the vessel. This is a case

of transient radio-active equilibrium. It can be shown that in this case the quantity of each product actually present is about 1 per cent. greater than the value which it would finally attain if the quantity of emanation present were maintained at its actual value at that instant by an external source.

The problems arising can easily be solved if required by an application of the principles enunciated by Rutherford.

Taking the case we have just considered for an example, suppose that  $A, B, C, \dots$  are a series of such elements so that  $A$  changes into  $B$ ,  $B$  into  $C$ , and so on. Then if the substance at the beginning consists only of the element  $A$  (for example only of radium emanation), and if  $a, b, c, \dots$  are the number of atoms of each element present at a time  $t$  from the start, and  $\lambda_a, \lambda_b, \lambda_c, \dots$  are the corresponding radio-active constants, we have

$$a = a_0 e^{-\lambda_a t}$$

where  $a_0$  is the number of atoms of  $A$  with which we commenced the experiment. Also by (152) we have

$$db = \lambda_a a dt - \lambda_b b dt,$$

$$dc = \lambda_b b dt - \lambda_c c dt,$$

and so on. Substituting for  $a$ , we have

$$\frac{db}{dt} = \lambda_a a_0 e^{-\lambda_a t} - \lambda_b b.$$

The solution of this equation is of the form

$$b = a_0(k_1 e^{-\lambda_a t} + k_2 e^{-\lambda_b t}) \quad \dots \quad (154)$$

Now  $b=0$  when  $t=\infty$ , since all the substances eventually disintegrate completely;  $\therefore k_1 = -k_2$ . Differentiating the equation and applying the conditions that when  $t=0$ ,  $a=a_0$  and  $b=0$ , we find

$$k_1 = \frac{\lambda_a}{\lambda_b - \lambda_a}; \text{ and thus finally}$$

$$b = \frac{a_0 \lambda_a}{\lambda_b - \lambda_a} (e^{-\lambda_a t} - e^{-\lambda_b t}) \quad \dots \quad (155)$$

The method can obviously be continued to give the corresponding value of  $c$ , and so on. If  $\lambda_b$  is large compared with  $\lambda_a$ , then when  $t$  is comparatively large,  $e^{-\lambda_b t}$  will be small compared with  $e^{-\lambda_a t}$  and the quantity of  $b$  present will thus be proportional to  $a_0 e^{-\lambda_a t}$ , that is, to the quantity of  $A$  remaining; the result we have already obtained from general considerations.

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## CHAPTER XVIII

### RADIUM AND THE RADIO-ACTIVE SERIES

**192. The nature and properties of radium.** The principles we have been considering will be made clearer, and the evidence for them more conspicuous, if we study in detail one series of radio-active changes. That of radium is the most suitable for the purpose, not merely on account of its intrinsic importance but also owing to various accidental causes which render the phenomena particularly clear.

Radium can be isolated from uranium ores, in which it exists as a product of the radio-active disintegration of the uranium. It is obtained by the processes used in the extraction of barium from the ores, to which element it bears a close chemical resemblance. It can be obtained free from barium by repeated fractional crystallization of the bromide, the radium salt being somewhat less soluble in water than the corresponding salt of barium, with which, however, it is isomorphic.

The proof of the elementary nature of radium is now complete. It is afforded in the first place by the spectrum, which is characteristic of the metal, and differs from those of all other elements. The lines in the radium spectrum have now been analysed into their component series, and are found to present the common characteristics of the metals of the alkali earths. Applying the usual laws of spectra the spectrum of radium is found to correspond to that of an alkaline earth metal of atomic weight about 225.

Radium has been obtained in sufficient quantities and of sufficient purity to enable its atomic weight to be obtained by direct chemical methods. The method employed is that of precipitating the radium chloride with silver nitrate, and weighing the silver chloride formed. In this way the atomic weight is found to be 226.0.

The value of the atomic weight of radium is strong evidence for the disintegration theory of its formation. It is known that the conversion of uranium into radium is attended with the expulsion of three  $\alpha$ -particles. Thus on the disintegration theory the atomic weight of radium should be less than that of uranium by the weight of three  $\alpha$ -particles. The atomic weight of uranium is 238. That of radium should therefore be  $238 - 3 \times 4.0$ , or 226, in satisfactory agreement with the chemical determinations.

Radium can be obtained in metallic form. It is a silver white metal, melting at about  $700^{\circ}\text{C.}$ , and beginning to volatilize at a slightly higher temperature. It is attacked by air, forming a nitride, and by water, forming the hydroxide. It forms a series of salts similar in properties and appearance to those of barium. It thus possesses in every particular the characters of a metallic element of the alkaline earth group.

**193. Radium emanation.** The first product of the disintegration of radium is the heavy inert radio-active gas known as radium emanation. It is found that a sample of pure radium emits this gas at a constant, definite rate, the emission being attended by the expulsion of an  $\alpha$ -particle. The phenomena of radio-active change are peculiarly striking in this instance, the parent substance being a metallic solid, while the disintegration product is an inert gas.

The emanation can be separated from the radium salt by dissolving the latter in water, and boiling the solution. The gas emitted, which contains hydrogen and oxygen from the decomposition of the water by the radiations emitted and carbon dioxide from the tap grease, is pumped off, and purified by suitable chemical means. The final purification is effected by condensing the emanation in a tube immersed in liquid air, and pumping off the more volatile gases present. Radium emanation is itself radio-active. It gives off  $\alpha$ -particles changing into a substance, radium A, which being solid is deposited on the walls of the vessel containing the gas. Radium emanation, though formed from radium, is itself an element. This is shown by its characteristic spectrum, which differs as much from that of radium as from the spectra of all other elements. The spectrum can easily be obtained by passing a little of the emanation into a small discharge tube.

Since the emanation is formed from radium by the expulsion of an  $\alpha$ -particle of mass 4 its atomic weight should be about 222, taking that of radium as 226. The matter was investigated directly by Ramsay and Gray, who succeeded in determining the density of the emanation using a quartz microbalance of special construction. Two bulbs of very unequal size but of equal mass are placed at the two ends of the beam of a small quartz balance enclosed in a small air-tight chamber. The weights are adjusted so that the beam is horizontal when the balance case is completely evacuated. If gas is now admitted to the space around the balance the buoyancy of the gas will reduce the effective mass of the larger bulb, and thus cause a deflection of the beam. The buoyancy of the gas can be calculated in terms of the deflection of the



beam when the sensitivity of the balance is known, and from the buoyancy the density of the gas can be at once determined.

The mean result of several experiments showed that the density of the emanation was 111.5 times that of hydrogen. This gives a molecular weight to the emanation of 223. The chemical properties of the gas resemble those of the argon group, and it is thus presumably monatomic. In this case its atomic weight is also 223. Considering that the total weight of the emanation available for the experiment was no more than 1/1000 of a milligram the agreement with the theoretical value is surprisingly good.

The emanation has been found to behave as a gas at ordinary temperatures. It obeys Boyle's law, it liquefies at  $-65^{\circ}\text{C}$ . at normal pressure, but exerts an appreciable vapour pressure down to a temperature of  $-150^{\circ}\text{C}$ . According to Ramsay and Gray its critical temperature and pressure are  $104.5^{\circ}\text{C}$ . and 63 atmospheres respectively. The density of the emanation at the temperature of liquid air, at which it is probably solid, is between 5 and 6.

Radium emanation is thus a definite chemical element belonging to the group of inert gases. In recognition of this fact it has become usual to refer to it as *radon*.

**194. Rate of decay of radon.** As radium emanation is extensively used in experimental researches it is necessary to know its rate of decay with some accuracy. The simplest method is to allow the emanation in a closed tube to attain radio-active equilibrium with its immediate products radium A, B, and C. These products are all short-lived, the most stable having a half period value of 26.8 minutes. A few hours thus suffices to set up transient equilibrium in the system. The rate of decay of any part of the activity is then, as we have already seen (§ 191), that of the most stable of the constituents, in this case the emanation. The advantage of this method is that it allows us to use the  $\beta$ - and  $\gamma$ -rays, which, as we shall see later, are actually emitted by

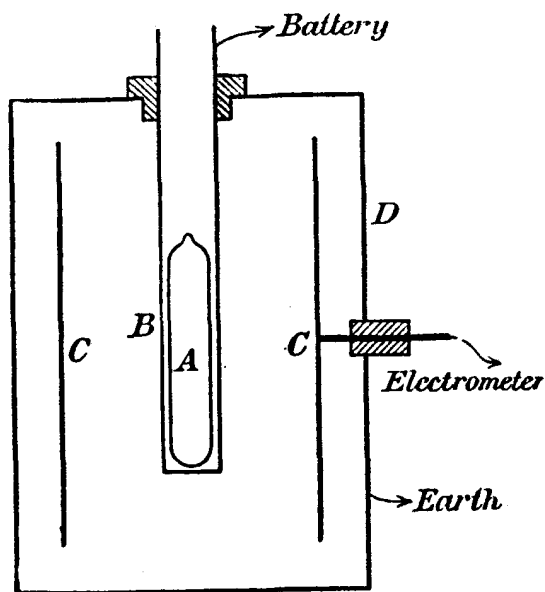


FIG. 121. Apparatus for measuring the rate of decay of radon

its disintegration products, radium B and radium C, as a measure of the activity of the emanation itself. It further allows us to evade the complications due to the growth of these products in the emanation during the first few hours after separation.

The apparatus used by Curie is shown in Fig. 121. The radon, enclosed in a sealed glass tube *A*, is placed inside the brass tube *B*, which can be connected to a battery of cells and forms one plate of the ionization chamber. The insulated plate *C* takes the form of a cylinder concentric with *B*, and is connected in the usual way to an electrometer. The whole is surrounded by a metal chamber *D* which is earthed and serves to screen *C* from electrostatic disturbances. If *D* is made perfectly air-tight, changes in the current owing to fluctuations in the temperature and pressure of the atmosphere are avoided. The  $\alpha$ -rays due to the radon itself are of course completely absorbed by the glass tube *A* and the metal tube *B*. The current is due to the  $\beta$ - and  $\gamma$ -rays from the radio-active decomposition products of the emanation. After a few hours these decay with the period of the emanation.

In this way the radio-active constant of radon was found to be  $0.1801 \text{ (day)}^{-1}$  or  $2.085 \times 10^{-6} \text{ (sec.)}^{-1}$ . Rutherford by a different method obtained a value  $0.1802 \text{ (day)}^{-1}$ . The latest determinations give a rather higher value for  $\lambda$ , namely  $2.097 \times 10^{-6} \text{ (sec.)}^{-1}$ . This corresponds to a half value period of 3.825 days.

**195. Measurement of the volume of radon in equilibrium with one gram of radium.** The quantity of radon in radio-active equilibrium with one gram of radium is by the laws of radio-active change a constant. It is termed a *curie*. Since the curie is a somewhat large standard compared with the amounts of radon generally available for experimental purposes one-thousandth part of this is taken as a subsidiary standard and is known as the *millicurie*. The value of the curie can be estimated from the radio-active constants. It has been found by the method of § 100 that one gram of pure radium (free from its products) emits  $3.7 \times 10^{10}$   $\alpha$ -particles per second. As each atom of radium disintegrates into one atom of radon with the expulsion of one  $\alpha$ -particle this is also the number of radon atoms formed per second. But the number present when equilibrium is reached is equal to  $q/\lambda$ , where  $q$  is the rate of formation of the atoms, and  $\lambda$  the radio-active constant of the radon. The total number of atoms of radon in equilibrium with one gram of radium is thus

$$3.7 \times 10^{10} / 2.097 \times 10^{-6} = 1.77 \times 10^{16}.$$

Since there are  $2.78 \times 10^{19}$  molecules in one c.c. of gas at normal temperature and pressure the volume occupied by one curie of radon under these conditions is

$$1.77 \times 10^{16} / 2.78 \times 10^{19} \text{ c.c.} = 0.66 \text{ cubic mm.}$$

**196. The active deposit from radon.** If a plate or wire of any kind is exposed to radon for a few hours and then withdrawn it is found to have become radio-active. This phenomenon was at first described as induced or excited activity. It is now known to be due to the deposition on the surface of the substance of a solid radio-active product formed by the decomposition of the radon.

If the wire is allowed to remain in the radon sufficiently long to attain equilibrium the amount of the activity is independent of the position or material of the plate, and is strictly proportional to the amount of radon present. It is also directly proportional to the area of the surface exposed.

A curious property of the excited activity is that it can be concentrated on a very small area if the latter is charged negatively. Thus if a wire is passed into a metal chamber containing a radium compound, and charged to a high potential, the activity when the charge on the wire is negative is as much as 200 times its activity when positively charged. In this way practically the whole of this "excited" activity can be concentrated on the wire, which may be made to have an activity per unit area of over 10,000 times that of the surface of radium itself.

Since the activity is concentrated on the negative electrode the carriers of the activity must have a positive charge. Since the active deposit is formed from the emanation by the emission of a positive particle carrying two electronic charges we should have expected the residual atom to have been negatively charged. We know, however, that the impact of a positive particle on matter gives rise to a number of slow-moving electrons, or  $\delta$ -rays as they have been called. It seems evident, therefore, that the escape of a positive ray from an atom must also be attended by an emission of negative electricity, probably in the form of these slow-moving electrons. Since the residual atom has a positive charge it is evident that at least three such electrons must be expelled with each positive particle.

The velocity of the carriers in an electric field can be measured by methods analogous to those used to determine the mobilities of the ions. Their velocity in a field of 1 volt per cm. is about 1.4 cm. per sec, or very nearly that of the positive ion in a gas. There is some evidence for the assumption that they carry a single negative charge.

**197. Recoil atoms.** The  $\alpha$ -particle is expelled from the radon atom with considerable velocity. The residual atom itself must thus be projected in the opposite direction with an initial velocity which is given by the consideration that the momentum of the system as a whole must be zero. Since the mass of the  $\alpha$ -particle is 4, and that of the residual atom ( $222-4$ ), the velocity of the recoil atom will be  $4/218$  or about  $1/55$  of that of the corresponding  $\alpha$ -particle. The range of the recoil atoms of radon is found to be about 0.12 mm. in air at atmospheric pressure. As we shall see later the phenomenon can be used to prepare an  $\alpha$ -ray product in a state of purity.

**198. Analysis of the active deposit of rapid change.** A body which has been exposed to radon is found after removal from the emanation

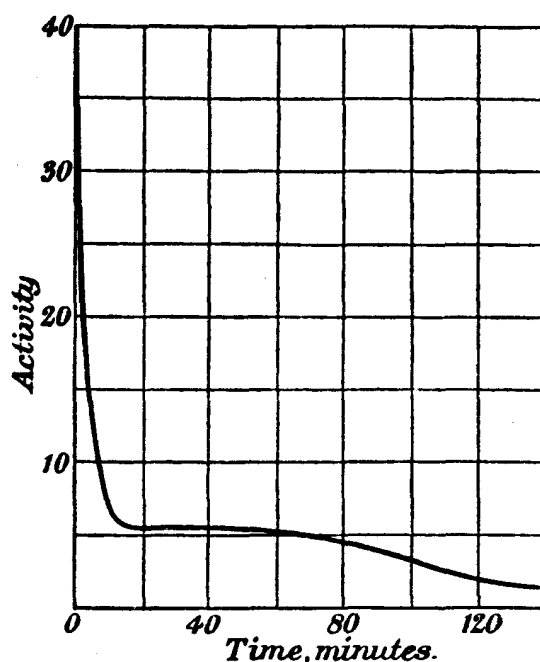


FIG. 122. Decay of excited activity;  
 $\alpha$ -ray curve

to give out all three kinds of radiations. The activity is comparatively short-lived and almost completely disappears within 24 hours. There is, however, always a small residual activity which if the body has been exposed for several days may reach as much as one-millionth of the initial activity, and the activity of this portion is found to increase steadily with time over the space of several years.

We may therefore for convenience divide the excited activity into two parts, (1) the active deposit of rapid change, (2) the active deposit of slow change.

Since the activity of the latter is in

general minute compared with that of the former, we may neglect the small residual effects due to it in our analysis of the short-lived deposit.

The active deposit can be obtained on a thin rod or wire by charging the wire to a negative potential of some 200 volts and exposing in a vessel containing radon. If the activity is to be measured by the  $\alpha$ -rays the rod may then be made the central electrode of a cylindrical ionization chamber, the saturation current being measured by an electrometer in the usual way. If  $\beta$ - or  $\gamma$ -rays are to be used to measure the activity a simple electroscope is convenient. If  $\beta$ -rays

are to be employed the opening of the electroscope is covered with only sufficient thickness of aluminium foil to absorb all the  $\alpha$ -rays. If only  $\gamma$ -rays are to be dealt with the electroscope may be placed on a sheet of lead some half centimetre in thickness, which will cut off everything except the  $\gamma$ -rays.

The shape of the decay curves depends on whether the  $\alpha$ - or the  $\beta$ -rays are used for the measurement, and also on the time of exposure of the wire to the radon. The active deposit consists therefore of a mixture of radio-active substances, all in the solid state, some emitting  $\alpha$ -rays, and some  $\beta$ - and  $\gamma$ -rays only.

The decay curve obtained after a short exposure of the wire to the radon, and measured by the  $\alpha$ -ray activity, is shown in Fig. 122. It will be seen to be divisible into three parts. The first stage shows a rapid decay falling to some 10 per cent. of its initial value in 15 minutes. This is followed by a period of some 20 minutes in which there is little variation in the activity. Then a gradual exponential decrease follows, the curve falling to half value in about 28 minutes.

Since the exposure was short the initial substance consisted almost entirely of the first decomposition product of the emanation, which we will call radium A. An analysis of the first part of the decay curve shows that

this substance emits  $\alpha$ -particles and decays to half value in about three minutes, giving rise to a product which emits  $\alpha$ -rays and decays with a half value period of 28 minutes.

Further evidence on the nature of the changes can be obtained from a study of the  $\beta$ -ray curves. The  $\beta$ -ray curves for short and long exposures are shown in the curves of Fig. 123. The curve for a short exposure of one minute shows that the  $\beta$ -ray activity is initially absent, and that therefore radium A emits only  $\alpha$ -rays. If, however, the product emitting the  $\beta$ -rays was the immediate product of the decomposition of the radium A, the curve for the rise of activity of the  $\beta$ -ray

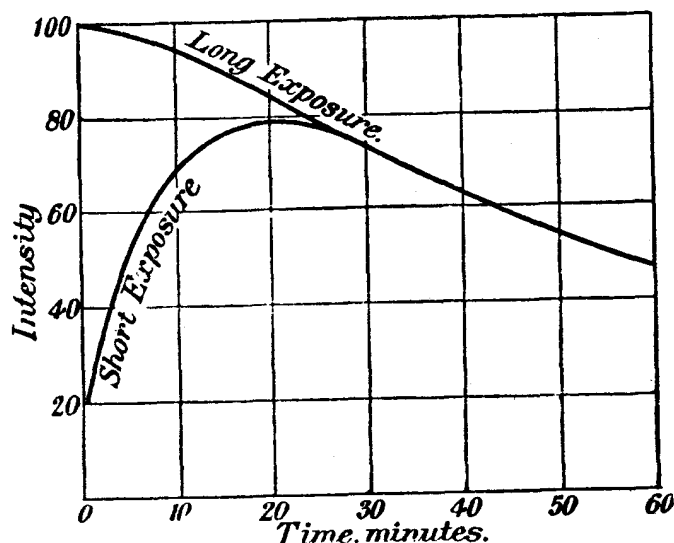


FIG. 123. Decay of excited activity ;  $\beta$ -ray curves

product should be (neglecting the decay of the latter) complementary to the decay curve of radium A, that is, it should rise to half value in about three minutes. As a matter of fact the rise to half value occupies rather more than ten minutes and a substitution of the numerical values in the formulae shows that this discrepancy cannot be explained by the small decay of the  $\beta$ -ray product in that interval of time. We are thus led to the conclusion that there is between the radium A and the  $\beta$ -ray product another radio-active substance which we will term radium B, which without itself emitting ionizing rays gives rise to the product radium C which does.

The period of the two products radium B and C can best be obtained from a study of the curves obtained for the  $\beta$ -ray activity with long exposure. In this case the three products will be in radio-active equilibrium, and the relative amount of the short-lived radium A will consequently be very small. Moreover as it has a short period it will rapidly cease to affect the shape of the decay curves. The whole of the radium A (except 1 per cent.) is, in fact, disintegrated within 20 minutes after removal from the emanation and the curve is then due solely to radium B and C.

Now it is found that this curve can be accurately expressed by an empirical formula of the form

$$I_t/I_0 = a e^{-\lambda_1 t} - (a-1) e^{-\lambda_2 t} \quad \dots \quad (156)$$

where  $a$  is a numerical constant and  $\lambda_1$  and  $\lambda_2$  have the values  $5.86 \times 10^{-4}$  and  $4.31 \times 10^{-4}$  (sec.)<sup>-1</sup> respectively. It will be seen that the second of these gives a half value period of 26.8 minutes, the other a half value period of about 19.8 minutes.

Now we have already seen that the final rate of decay will be governed by the decay of the substance of longest period, irrespective of its position in the chain of decomposition. Hence we cannot immediately infer to which of the substances B or C the two constants refer.

Fortunately, however, it is possible to separate the two products. If the wire is heated to a temperature of some 600° C. surrounded by a cooled outer cylinder it is found that the activity can be separated into two parts, one part remaining on the wire while the other part is volatilized and condenses on the cool outer cylinder. The portion remaining on the wire emits  $\alpha$ -radiation and  $\beta$ -radiation, the activity in each case falling to half value in 19.5 minutes. The other portion is initially inactive but gradually begins to emit both  $\alpha$ - and  $\beta$ -rays corresponding to those of the substance left behind on the wire. The shorter period therefore corresponds with the final product, radium C,

which emits both  $\alpha$ - and  $\beta$ -rays, while the longer period, 26.8 minutes, is that of the radium B, which without emitting radiations is the direct parent of the radium C and the immediate product of the decomposition of radium A.

It has been found that the change from radium B to radium C is not actually rayless. A quantity of slowly-moving  $\beta$ -particles comparable with the  $\delta$ -rays are given off. These, however, have so little penetrating power that they are completely absorbed in the thickness of aluminium necessary to cut off the  $\alpha$ -rays, and hence have no influence on the  $\beta$ -ray measurements. All the changes described as rayless are really of this nature.

Theoretical decay curves can be constructed from the equations in § 191, when the radio-active constants are known. They agree with those obtained experimentally to an accuracy of at least 1 per cent.

**199. Separation of the products by the method of "recoil." Disintegration of radium C.** Although the analysis of the radio-active product of short period was made in the manner explained above we have now a more powerful means of separating the various products of radio-active change. This can be done by making use of the "recoil" atoms already described. We have seen that the atom of radium A produced by the decomposition of radium emanation starts life with a velocity which will enable it to travel across several centimetres of gas at a sufficiently low pressure. The same considerations apply also to the atoms formed by the decomposition of radium A and indeed to any atoms whose formation is attended by the expulsion of an  $\alpha$ -particle. Thus if we place a wire or plate (which has been exposed to radon and thus carries a mixture of A, B, and C) in a vacuum at a short distance from another inactive plate, the atoms of radium B formed by the decomposition of radium A will in many cases be shot across the intervening space, and collect on the inactive plate. In this way radium B of considerable purity can be obtained, since the atoms of radium C, which are formed by the disintegration of the radium B on the active plate have no appreciable velocity and thus remain. The individual properties of the separate substances can thus be studied.

An application of this method to the case of radium C has led to interesting results. Most radio-active atoms disintegrate with the emission of either  $\alpha$ -rays or  $\beta$ -rays but not both. Radium C, as we have noted, gives all three types. Pure radium C can be obtained from the radio-active deposit obtained from radon by dissolving it in acid, and suspending a nickel plate in the solution. The radium C is deposited on the plate, leaving the radium B in solution. It must be

remembered that each radio-active product is an element with definite chemical properties, and differs from ordinary elements only in its instability. The various products can thus be separated by the usual methods of chemical analysis. If the nickel plate bearing the radium C is placed in vacuo near a negatively charged disk recoil atoms from the radium C collect on the disk, which thus becomes radio-active. The substance thus collected decays to half value in 1.32 minutes, with the emission of  $\beta$ - and  $\gamma$ -radiation. It is known as radium C''. Since it can be collected by recoil, it must represent the result of an  $\alpha$ -ray disintegration of radium C.

It has been shown that the  $\beta$ -ray disintegration of radium C gives rise to an extremely unstable substance, radium C', with a half value period of the order of  $1.5 \times 10^{-8}$  sec. This emits a high-speed  $\alpha$ -particle. Radium C can thus disintegrate in two distinct ways. Normally it emits a  $\beta$ -ray and gives rise to Ra C'. About 3 in 10,000 of the atoms, however, emit an  $\alpha$ -particle and change to radium C''. The final product in each case is radium D. It has been shown experimentally that the energy released in the transformation of Ra C into Ra D is the same whichever route is taken.

The series of changes we have been considering can thus be represented by the following chain:

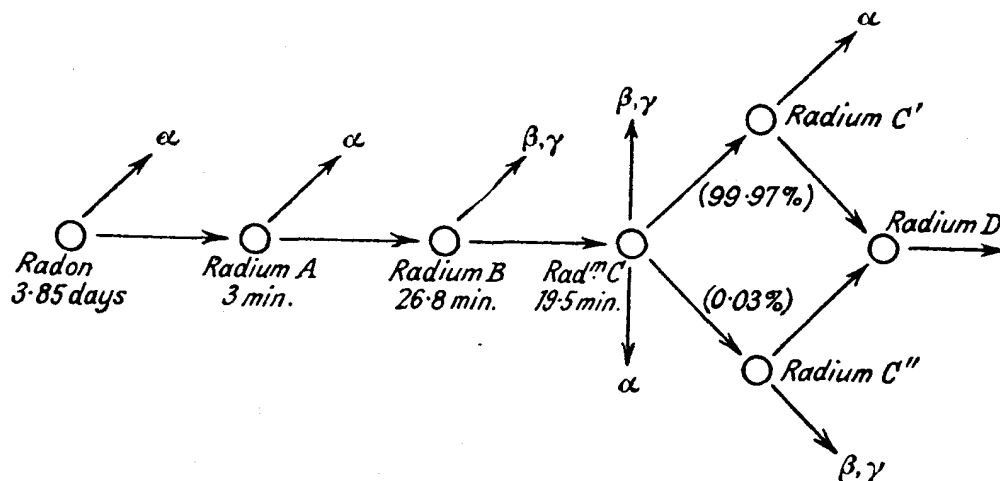


FIG. 124. The disintegration chain for radon

**200. The radio-active product of slow change.** As we have seen, radium C normally transforms into a product radium D. Radium D marks a period of comparative quiescence in the radio-active decay, its half value period being about 16.0 years. Owing to its long period it is present in minute but appreciable quantities in ores containing radium, from which it can be separated by the chemical processes used



for the separation of lead, to which metal indeed it bears so close a resemblance that no chemical test has yet been devised for distinguishing between them. Radium D gives out neither  $\alpha$ - nor  $\beta$ -rays of the ordinary type. It has, however, been shown to give off very slow  $\beta$ -rays (or  $\delta$ -rays) of the type given off by radium B.

The succeeding product, radium E, gives off  $\beta$ - and  $\gamma$ -rays and has a period of 4.85 days. The relation between radium D and its immediate product is thus the same as that for uranium and uranium X, and the decay and recovery curves follow the same laws. Thus in freshly prepared radium D the  $\beta$ -ray activity rises according to equation (147) to a maximum which owing to the slow decay of radium D remains sensibly constant for a considerable time.

Radium F, the product of the decay of radium E, emits  $\alpha$ -rays and decays to half value in 136 days. Its rate of decay and the range of its  $\alpha$ -particles identify it with the substance polonium, the first radio-active substance actually isolated from pitchblende. The product of the disintegration of polonium is apparently not radio-active, no trace of any radiations being observed after the  $\alpha$ -ray activity of the polonium has disappeared. The final product of the chain is an isotope of lead.

The period of radium D is too long to be determined by direct measurements. It can, however, be deduced by indirect methods which form an interesting illustration of the principles we have been discussing. Suppose we commence with a definite quantity of radon, say, 1 millicurie. The number of atoms of radon present is then known from the determination of the curie already discussed (§ 195). Let it be  $N$ . The number of atoms breaking up per second at the beginning of the experiment is therefore  $\lambda_1 N$ , where  $\lambda_1$  is the radio-active constant of radon. This will be the number of  $\alpha$ -particles  $n_1$ , emitted per second, a quantity which we can determine by the method already described (§ 100).

The radon is then allowed to decay for a month or so, at the end of which time it is practically all in the form of radium D. The number of atoms of radium D present is thus the same as the number of atoms of emanation with which we started, that is  $N$ , and the number breaking up per second is therefore  $\lambda_2 N$ , where  $\lambda_2$  is the radio-active constant of radium D. Now radium D itself emits no  $\alpha$ -particles. As, however, the period of radium D is long compared with that of its products E and F, the latter will be in radio-active equilibrium with it and hence the number of radium F particles breaking up per second will be the same as the number of radium D atoms disintegrating in the same time, that is  $\lambda_2 N$ . But this is the number  $n_2$  of  $\alpha$ -particles

emitted per second by the mixture of radium D, E, and F in our tube, since D and E give no  $\alpha$ -radiation. Hence

$$\frac{\lambda_2 N}{\lambda_1 N} = \frac{n_2}{n_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

from which we can determine  $\lambda_2$  since all the other quantities are known. The value of  $\lambda_2$  is found to be  $0.043 \text{ (year)}^{-1}$ , corresponding to a half value period of 16 years.

**201. Relation between the radio-active constant and the energy of the  $\alpha$ -particle.** Gamow has shown that on wave mechanics the energy  $E$  of emission of an  $\alpha$ -particle should be related to the radio-active constant  $\lambda$  of the element emitting it by an equation of the form

$$\log \lambda = a + b \cdot E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (158)$$

where the constant  $a$  depends on the particular radio-active family to which the element belongs. A relation somewhat of this type was proposed on empirical grounds many years ago by Geiger and Nuttall. The relation is useful for determining approximately the radio-active constants in cases where direct determination is for some reason impossible. The constant for U II, for example, has been deduced in this way from the measured velocity of its  $\alpha$ -rays.

**202. The radio-activity of thorium and actinium.** The element thorium is radio-active and gives rise to a chain of radio-active products in the same way as uranium gives rise to the radium chain. The disintegration process can be studied in detail by the methods already discussed, and involves no new principles. The results are sufficiently summarized in Table XIV. For further details the books mentioned at the end of the chapter may be consulted.

Another element, actinium, isolated originally from pitchblende, also gives a radio-active chain of products. The presence of actinium in uranium ores indicated that this element was not the head of the series, but that it was derived in some way from the disintegration of uranium. A detailed analysis has shown that the disintegration of uranium is considerably more complex than indicated in the previous chapter, and that traces of some half dozen other radio-active substances are eventually to be found in what we have called uranium X. The relationship of these substances remained in doubt until the discovery and isolation of the radio-active isotope of uranium of mass 235, described in § 165. The sequence of events is shown in Fig. 125. U-238 gives rise to the radium series, and U-235 to the actinium series and may be called actino-uranium. The two uraniums, while chemically identical, thus belong to different radio-active series. On the

other hand the elements known as  $UX_1$ ,  $UX_2$ , and  $U_F$ , are not forms of uranium:  $UX_1$  and  $U_F$  are isotopes of thorium, and  $UX_2$  an element of which there is no stable isotope. As we have already seen, U-235 is formed by the  $\alpha$ -ray decay of the trans-uranic element, plutonium (§ 167), which is thus (though now deceased, so to speak) the true head of the actinium chain.

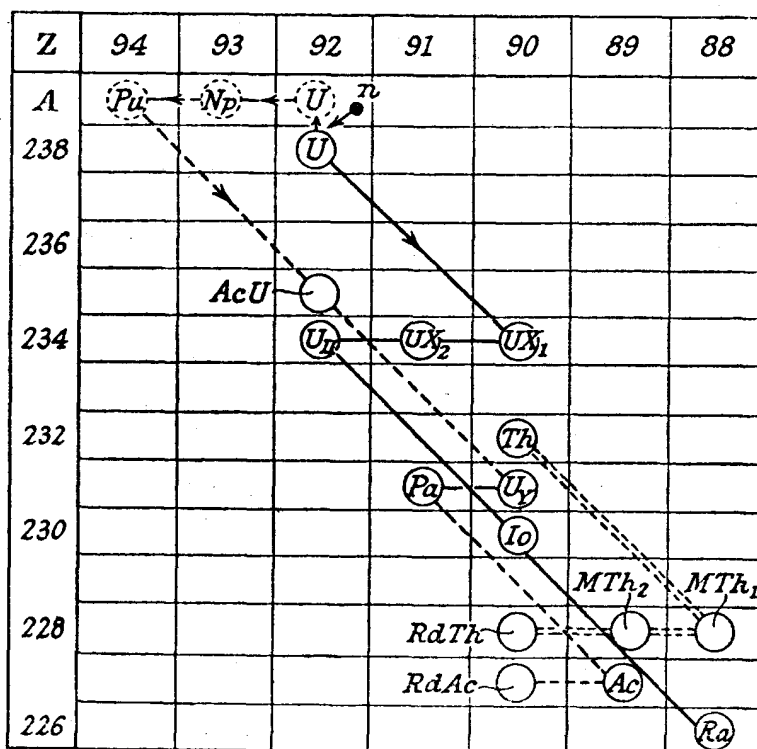


FIG. 125. The Genesis of the Uranium, Actinium and Thorium series. (U239 is formed from U238 only by the absorption of a neutron. U238 normally disintegrates to  $UX_1$  with the emission of an  $\alpha$ -particle.)

**203. Position of the radio-active elements in the Periodic table.** The main facts outlined in the previous sections were established as early as 1902, some ten years before Rutherford propounded the nuclear theory of the atom or Aston began his work on the determination of atomic masses. There was, therefore, at first some legitimate doubt as to the reality of the new elements postulated by the disintegration theory, and, when this was dispersed by the facts which we have already emphasized in preceding sections, some doubt as to how these 35 or so new elements were to be fitted into the nine spaces in the Periodic table which intervene between uranium and the final product, lead.

It was known that the ejection of an  $\alpha$ -particle (which Rutherford had already identified with a charged helium atom) would reduce the

TABLE XIII

## Uranium-Radium series

SUBSTANCE AND ATOMIC NUMBER	At. Wt.	Radio-active constant $\lambda$ in (sec.) <sup>-1</sup>	Half value period	Radia- tions emitted	Mean range of $\alpha$ -particles in air in cm.
92 Uranium ..	238	$4.9 \times 10^{-18}$	$4.5 \times 10^9$ years	$\alpha$	2.70
↓					
90 Uranium X <sub>1</sub> ..	234	$3.275 \times 10^{-7}$	24.5 days	$\beta$ (slow)	—
↓					
91 Uranium X <sub>2</sub> ..	234	$1.0 \times 10^{-2}$	1.15 min.	$\beta$	—
↓					
92 Uranium II ..	234	$2 \times 10^{-14}$	$2.7 \times 10^5$ years	$\alpha$	3.28
↓					
90 Ionium .. ..	230	$2.9 \times 10^{-13}$	$8.3 \times 10^4$ years	$\alpha$	3.194
↓					
88 Radium .. ..	226	$1.39 \times 10^{-11}$	1580 years	$\alpha$	3.91
↓					
86 Radon .. ..	222	$2.097 \times 10^{-6}$	3.825 days	$\alpha$	4.014
↓					
84 Radium A ..	218	$3.78 \times 10^{-3}$	3.05 min.	$\alpha$	4.620
↓					
82 Radium B ..	214	$4.31 \times 10^{-4}$	26.8 min.	$\beta$ (slow)	—
↓					
83 Radium C* ..	214	$5.86 \times 10^{-4}$	19.7 min.	$\alpha, \beta$	—
↓ ↘					
81 Radium C'' ..	210	$8.7 \times 10^{-3}$	1.32 min.	$\beta$	—
↓					
84 Radium C' ..	214	$4.5 \times 10^7$	$1.5 \times 10^{-8}$ sec.	$\alpha$	6.870
↓					
82 Radium D ← ..	210	$1.37 \times 10^{-9}$	16.0 years	$\beta$ (slow)	—
↓					
83 Radium E ..	210	$1.66 \times 10^{-6}$	4.85 days	$\beta$ (slow)	—
↓					
84 Radium F .. (Polonium)	210	$5.88 \times 10^{-8}$	136.5 days	$\alpha$	3.805
↓					
82 Lead .. .. (Uranium Lead)	206	—	—	—	—

\* The production of radium C' from radium C is attended by the expulsion of a  $\beta$ -particle only. The expulsion of an  $\alpha$ -particle from radium C produces radium C''.

TABLE XIV  
Thorium and Actinium Series

SUBSTANCE AND ATOMIC NUMBER	At. Wt.	Radio-active constant $\lambda$ in (sec.) <sup>-1</sup>	Half value period	Radia- tions emitted	Mean range of $\alpha$ -particles in air in cm.
90 Thorium ..	232	$1.3 \times 10^{-18}$	$1.34 \times 10^{10}$ years	$\alpha$	2.75
88 Mesothorium 1..	228	$3.26 \times 10^{-9}$	6.7 years	$\beta$ (slow)	—
89 Mesothorium 2..	228	$3.14 \times 10^{-5}$	6.13 hours	$\beta$	—
90 Radiothorium	228	$1.16 \times 10^{-8}$	1.90 years	$\alpha$	3.81
88 Thorium X ..	224	$2.20 \times 10^{-6}$	3.64 days	$\alpha$	4.13
86 Thoron ..	220	$1.27 \times 10^{-3}$	54.5 sec.	$\alpha$	4.967
84 Thorium A ..	216	4.95	0.14 sec.	$\alpha$	5.601
82 Thorium B ..	212	$1.82 \times 10^{-3}$	10.6 hours	$\beta$ (slow)	—
83 *Thorium C ..	212	$1.90 \times 10^{-4}$	60.8 minutes	$\alpha, \beta$	4.729
84     ↓ Thorium C'	208	$7 \times 10^{10}$	$10^{-11}$ sec.	$\alpha$	8.533
81     ↓ Thorium C''	212	3.61	3.20 sec.	$\beta$	—
82     ↓ Lead	208				
94 Plutonium ..	239	(?)	(?) $< 10^8$ years	$\alpha$	?
92 Actino-Uranium	235		$7.1 \times 10^8$ years	$\alpha$	3.2
90 Uranium Y ..	231	$5.4 \times 10^{-6}$	1.5 days	$\beta$ (slow)	—
91 Proto-actinium	231	$1.9 \times 10^{-12}$	12,000 years	$\alpha$	3.48
89 Actinium ..	227	$1.08 \times 10^{-9}$	20 years	$\beta$	—
90 Radioactinium..	227	$4.24 \times 10^{-7}$	18.9 days	$\alpha, \beta$	4.43
88 Actinium X ..	223	$7.14 \times 10^{-7}$	11.2 days	$\alpha$	4.14
86 Actinon..	219	$1.77 \times 10^{-1}$	3.92 sec.	$\alpha$	5.203
84 Actinium A ..	215	474	0.0015 sec.	$\alpha$	6.420
82 Actinium B ..	211	$3.2 \times 10^{-4}$	36.0 min.	$\beta$ (slow)	—
83 *Actinium C ..	211	$5.35 \times 10^{-3}$	2.15 min.	$\alpha, \beta$	5.392
81     ↓ Actinium C'	207	—	—	$\beta$	—
84 Actinium C'     ↓	211	$2.26 \times 10^{-3}$	4.71 min.	$\alpha$	6.518
82     ↓ Lead	207				

\* The C→C' transformation is attended by the expulsion of an  $\alpha$ -particle only that from C→C'' by the expulsion of a  $\alpha$ -particle.

atomic weight by four units, and that the ejection of an electron would leave the mass practically unaltered. Soddy, from a study of the chemical behaviour of the various radio-active products, suggested that the ejection of an  $\alpha$ -particle moved the element two places to the left in the table (or in modern parlance, reduced its atomic number by 2) while the ejection of a  $\beta$ -particle moved it one space to the right (i.e. increased its atomic number by unity).

The application of Soddy's rules to the radium series is shown in Table XV. To avoid confusion it has been necessary to omit the

TABLE XV  
Classification of Radio-active Elements of Radium series.

At. Wt.	0	I	II	III	IV	V	VI	VII
206					Radio Lead			
210				RaC <sub>2</sub>	RaD	RaE	RaF	
214					RaB	RaC	RaC <sub>1</sub>	
218							RaA	
222	RaEm							
226			Ra					
230					Io			
234					UX <sub>1</sub>	UX <sub>2</sub>	U <sub>II</sub>	
238							U <sub>I</sub>	

actinium and thorium series from the table. Even so, we see that two other elements, Ra B and Ra D, occupy the same space in the table as lead, to which we must add Ac B and Th B from the other series. These elements differ in their atomic weights and in their radio-active behaviour, but have identical chemical properties. Similarly in Fig. 125 we find that six elements in all, with atomic masses ranging from 227 to 234 occupy the space assigned to thorium in the Periodic classification. For such groups of elements Soddy proposed the name *isotopes*. The theory of isotopes thus originated from the study of natural radio-activity. The theoretical basis for Soddy's rules will, of course, be evident from § 152.

204. The end products of radio-active decay. It will be seen that the end product of each radio-active chain is an element of atomic number

82, that is, it is isotopic with lead. In the case of the uranium-radium series it is formed from uranium of atomic weight 238 with the emission of eight  $\alpha$ -particles, and its mass should thus be 206. Similarly the atomic mass of the end product of the thorium chain should be 208 and that of the actinium series, 207.

Uranium ores invariably contain small quantities of lead, and the ratio of the lead constant to that of uranium is approximately the same for all. This provides an indirect proof that the lead contained in the ore is a disintegration product of uranium, since it is highly improbable that the ores, which are widely distributed, should have all become contaminated to the same extent with lead from an extraneous source.

Lead has now been extracted from both uranium ores and thorium ores, and the atomic weights of the samples have been determined by purely chemical means. That from the uranium ores gives a value of 206.1; that from the thorium ores of about 208. The close agreement of these values with the predictions of the disintegration theory provides a very satisfactory confirmation of the theory.

The presence of Pb (207) in the lead extracted from uranium ores was verified by Aston, by mass spectrograph. The fact that the lead Pb (208) line was very faint in the specimen showed that the ore had been practically free from contamination, since 208 is the strongest line in ordinary leads.

**205. Production of helium from uranium. Geological age.** Since each  $\alpha$ -particle is an atom of helium, helium must be in process of formation in all minerals containing uranium. The number of  $\alpha$ -particles given out by one gram of uranium in equilibrium with all its products has been found to be  $9.7 \times 10^4$  per sec. The number of atoms of helium formed per annum by 1 gram of uranium is thus

$$(9.7 \times 10^4) \times 60 \times 60 \times 24 \times 365$$

or, since a gas at N.T.P. contains  $2.7 \times 10^{19}$  molecules per c.c., about  $11 \times 10^{-5}$  cubic mm. The direct measurement of the rate at which helium is produced in uranium ores is in fair agreement with this estimate.

Thus if we can assume that the helium produced is all occluded by the mineral, the ratio of the helium to the uranium present affords us a method of estimating the age of the mineral. The results are found to be qualitatively consistent with the ages suggested by geological considerations, that is to say, the rocks of older formations give a higher ratio of helium to uranium than those of more recent formation. Rayleigh, to whom most of our information is due, estimates the age of the

oligocene rocks as eight million years, and the ages of the eocene, carboniferous, and archæan periods as respectively 31, 150, and 700 million years. Since there is always the possibility of a certain loss of helium from the rocks by diffusion into the air, these are obviously minimum estimates.

The ratio of lead to uranium may clearly also be used as a criterion of the age of the minerals. This will give a maximum estimate since loss of lead from the ore is improbable and on the other hand the presence of traces of ordinary lead in the original ore is always a possibility, although Aston's experiments (§ 203) seem to indicate that its amount is very small. The ages deduced from the lead contents are thus probably not far from the truth. In this way the age of the carboniferous rocks has been calculated as about 340 million years, and of the archæan rocks as about 1300 million years.

**206. The heating effect of radio-active bodies.** Since the various radiations from radio-active substances are emitted with very high velocities it is evident that a radio-active substance is giving out energy at a very appreciable rate. If it is surrounded by sufficient material to absorb all the radiations this energy will manifest itself eventually in the form of heat. Thus radium is constantly producing heat and will maintain itself at an appreciably higher temperature than its surroundings. Owing to their relatively large mass the bulk of the energy is carried by the  $\alpha$ -rays.

The heating effect of the  $\alpha$ -rays can be calculated from their known velocities. Let  $n$  be the number of  $\alpha$ -particles of velocity  $v$  emitted per second by each of the  $\alpha$ -ray products, present;  $n$  will be the same for each when equilibrium is established. The total energy of the  $\alpha$ -rays is thus  $\frac{1}{2}mn\Sigma v^2$ , where  $m$  is the mass of an  $\alpha$ -particle. To this we must add the energy of recoil of the various recoil atoms which as we have seen is  $m/M$  of that of the corresponding  $\alpha$ -particle, where  $M$  is the mass of the recoil atom. Thus the total energy associated with the  $\alpha$ -radiation is

$$\frac{1}{2}mn\Sigma\left(1+\frac{m}{M}\right)v^2.$$

For a gram of radium in equilibrium with its short period products this is equal to  $1.45 \times 10^6$  ergs per second, or about 125 gram calories per hour. If only the  $\alpha$ -rays are absorbed in the apparatus this should be the rate of emission of heat by 1 gram of radium in equilibrium with its short-lived products. If the radium has been prepared for several years so that an appreciable amount of polonium is present the heating



effect of this must be added. If the apparatus, as is usually the case, absorbs an appreciable percentage of the  $\beta$ - and  $\gamma$ -rays the heating effect will be somewhat greater; if all the rays are absorbed it should be about 140 calories per hour, of which 6.3 calories are due to the  $\beta$ -rays and 9.4 calories to the  $\gamma$ -rays.

The total heating effect of a curie of radon of volume 0.66 c.mm., when in equilibrium with its immediate products, is 113.6 cal. per hour. The total heat emitted by the radon during its complete disintegration into radium D is thus

$$\int_0^{\infty} H_0 e^{-\lambda t} dt = H_0 / \lambda,$$

where  $\lambda$ , the radio-active constant of radon, is  $2.085 \times 10^{-6}$  per sec., or 0.0075 per hour. This is equal to  $1.52 \times 10^4$  gram calories per curie, or  $2.2 \times 10^9$  gram calories per gm.

The formation of 1 gram of water is attended with the liberation of about  $3.8 \times 10^3$  calories of heat. It will be seen therefore that the heat liberated in any chemical reaction is infinitesimal compared with that produced by radio-active changes.

**207. Radio-activity of ordinary matter.** The air in an ionization chamber always shows a slight conductivity even in the absence of any recognized radio-active sources. A very considerable amount of labour has been spent in attempting to determine whether any of this ionization were due to a genuine radio-activity on the part of the ordinary materials forming the case of the chamber. The experiments are most conveniently carried out with a simple electroscope of the type shown in Fig. 1.

The matter is complicated by the fact that radium, in minute traces, is widely disseminated. Many soils contain an appreciable quantity, and most river water is feebly radio-active owing to minute traces of radium salts. As radium emanation is a gas, traces of it are usually to be found in the air. Thus the air in the electroscope is subjected to  $\gamma$ -radiation from all these sources, in addition to the cosmic radiation.

By sinking electroscopes into the depths of the mountain lakes of California and the Andes, which are particularly free from radio-active impurities, Millikan was able to reduce the residual ionization to as little as 1 ion per c.c. per sec. As a single  $\alpha$ -particle will produce, on an average, about 30,000 ions per cm., the passage of a single  $\alpha$ -particle through each cubic centimetre of the gas once every eight hours or so would be sufficient to account for the residual effects. The quantity of a known radio-active material necessary to produce such a trifling

emission is so minute that its absence could never be certified. We must conclude that there is no evidence for any radio-activity among elements in general.

The survey of the subject has, however, led to the discovery of natural radioactivity in four other elements. Potassium salts emit  $\beta$ - and  $\gamma$ -radiation of a type very similar to that emitted by uranium X. The activity of a surface of potassium salt is about 1 per cent. of that of a salt of uranium. Only one isotope is concerned,  $^{40}\text{K}$ , which is present in ordinary potassium to the extent of about 1 part in 8000. It has a half value period of about  $2 \times 10^9$  years, and presumably changes into  $^{40}\text{Ca}$ . Rubidium and lutecium also have  $\beta$ -ray emitting isotopes. Samarium emits  $\alpha$ -particles of range 1.15 cm. These substances do not appear to initiate a chain of reactions, and it must be assumed that the disintegration product in each case is stable.

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# ATOMIC CONSTANTS

TABLE XVI

Some Atomic Data

Electronic charge, $e$	$4.803 \times 10^{-10}$ e.s.u.
$e/m$ for electron	$1.601 \times 10^{-20}$ e.m.u.
Mass of electron	$1.759 \times 10^7$ e.m.u. per gm.
	$9.115 \times 10^{-28}$ gm.
$h$ , Planck's constant	0.00054 mass unit
Mass unit, = mass of an atom of unit atomic weight [O=16.00]	$6.61 \times 10^{-27}$ erg sec.
$M_p/m$ , ratio of mass of proton to mass of electron	$1.660 \times 10^{-24}$ gm.
Mass of proton	1838
Mass of $\alpha$ -particle	1.0076 mass units
Avogadro's number	4.0029 mass units
Boltzmann's constant $\alpha$ or gas constant per molecule	$6.023 \times 10^{23}$ per mol.
Faraday constant	$1.371 \times 10^{-16}$ erg per degree
1 electron-volt	9649 e.m.u. per gm. equiv.
Bohr magneton $\mu$	$1.601 \times 10^{-12}$ erg
	$0.917 \times 10^{-20}$ erg per gauss

TABLE XVII

The Masses of some Light Atoms  
(The masses quoted include the mass of the extra-nuclear electrons)

Isotope	Mass	Isotope	Mass
$^1_0n$	1.0090	$^9_4\text{Be}$	9.0149
$^1_1\text{H}$	1.0081	$^{10}_5\text{B}$	10.0160
$^2_1\text{H}$	2.0147	$^{11}_5\text{B}$	11.0128
$^4_2\text{He}$	4.0040	$^{12}_6\text{C}$	12.0040
$^6_3\text{Li}$	6.0170	$^{13}_6\text{C}$	13.0076
$^7_3\text{Li}$	7.0182	$^{14}_7\text{N}$	14.0075
$^9_4\text{Be}$	8.0078	$^{16}_8\text{O}$	16.0000

These masses are expressed in mass units, where one mass unit equals  $\frac{1}{16}$  of the mass of the predominant oxygen isotope  $^{16}\text{O}$  (§ 55).

## MISCELLANEOUS EXAMPLES

1. Assuming that the vertical potential gradient in the air near the ground is 100 volts per metre, and the vertical electric current density through the air is  $3 \times 10^{-16}$  amp. per sq. cm., how many ions of average mobility 1.5 cm./sec. per volt/cm. are there in each c.c. of air ?

[Ans. 1260 per c.c.]

2. The coefficient of diffusion of positive ions in oxygen is 0.025, and the mobility of the ions is 1.36 cm./sec. per volt/cm. Calculate the total charge carried by unit volume of oxygen at N.T.P. if all the molecules were ionized.

[Ans.  $1.65 \times 10^{10}$  e.s.u./c.c.]

3. A volume of 500 c.c. of saturated air at 15° C. is expanded adiabatically so that its volume is increased by 30 per cent. What is the final temperature after expansion, and what mass of water will be deposited ?

[Ans. -13.7° C. ; 0.0054 gm.]

4. A drop of oil of density 0.920 gm./c.c. falls freely under gravity in air of viscosity  $0.824 \times 10^{-4}$  c.g.s. with a uniform velocity of 0.0858 cm./sec. When a vertical electric field of 3180 volt/cm. is applied the drop moves at various speeds which differ from each other by small multiples of 0.00539 cm./sec. Calculate a value for the electronic charge.

[Ans.  $4.88 \times 10^{-10}$  e.s.u.]

5. What is meant by an electron-volt ? Calculate in electron-volts the energy (a) of a cathode particle moving with a velocity of  $10^9$  cm. per sec., (b) of an  $\alpha$ -particle with a velocity of  $10^9$  cm. per sec., (c) of a quantum of sodium light of wavelength  $5.89 \times 10^{-5}$  cm.

[Ans. (a) 283, (b)  $2.08 \times 10^6$ , (c) 2.1]

6. According to Langmuir the energy necessary to dissociate into atoms one gram molecule of hydrogen molecules is 84,000 calories. Express this result in electron-volts per molecule.

[Ans. 3.65]

7. The first radiation potential of mercury is 4.9 volts. Mercury when bombarded by electrons at this potential emits a single spectral line of wavelength  $2.536 \times 10^{-5}$  cm. Discuss these observations.

8. A linear source of homogeneous  $\beta$ -particles is placed parallel to the lines of force in a uniform magnetic field. The  $\beta$ -particles are limited to a narrow pencil by passing through a narrow slit placed parallel to the source. Show that the pencil of  $\beta$ -particles will be brought to a linear focus and find its position.

9. In one of Thomson's early experiments on cathode rays, the cathode stream passed through coterminal electric and magnetic fields for a distance of 5 cm., entering the fields in a direction at right angles to the

lines of force. The beam was undeflected when the fields were respectively 100 volt/cm. and 3.6 gauss. The magnetic field alone produced a deflection of  $3^{\circ} 40'$ . Calculate values for  $e/m$ , and the velocity of the particles.  
[Ans.  $0.99 \times 10^7$  e.m.u./gm.;  $2.78 \times 10^9$  cm./sec.]

10. Positive rays are generated in a discharge tube working on 30,000 volts. What is the minimum deflection which the particles can experience in passing between a pair of plates 5 cm. in length and 1 cm. apart, charged to a p.d. of 200 volts, assuming that the charge on the particle remains constant throughout its course? What will be the deflection if the particle has a charge of  $8e$  in the discharge tube and loses  $7e$  between leaving the discharge tube and reaching the plates?  
[Ans.  $0.955^{\circ}$ ;  $0.119^{\circ}$ ]

11. Taking the atomic mass of hydrogen as 1.00778 and that of helium as 4.00216, calculate the packing fractions for these two atoms. Calculate also (a) in calories, (b) in volts the energy liberated if 4 atoms of hydrogen were condensed to form an atom of helium. If the energy were liberated as monochromatic radiation calculate the wavelength of the radiation.

[Ans. (a)  $1.03 \times 10^{-12}$  cal.; (b)  $2.70 \times 10^7$  e-volts; (c)  $4.57 \times 10^{-12}$  cm.]

12. A pair of parallel zinc plates in air are connected to the 210-volt 50-cycle mains, and ions are formed at the surface of the lower plate by illuminating it with ultraviolet light. What is the greatest distance of separation at which the upper plate can receive a negative charge, assuming the mobility of the ions to be 1.5 cm./sec. per volt/cm.? [Ans. 1.32 cm.]

13. How many quanta of radiation of wavelength  $5 \times 10^{-5}$  cm. must fall per sec. on a blackened plate to produce a force of 1 dyne? If the mass of the plate is 1 gm. and its specific heat 0.1 at what rate would its temperature rise? [Ans.  $7.6 \times 10^{21}$  quanta per sec.;  $7.2 \times 10^3$  C. per sec.]

14. What is the minimum wavelength in the spectrum of an X-ray tube working at 100,000 volts?  
[Ans.  $0.12 \times 10^{-8}$  cm.]

15. Derive an expression for the Compton change of wavelength on scattering. According to Sharp the Compton change amounts to  $0.04825 \times 10^{-8}$  cm. at an angle of  $169^{\circ} 45'$  with the direction of the primary beam. Assuming that  $h/e$  is  $1.372 \times 10^{-17}$  estimate the value of  $e/m$  for the electron.  
[Ans.  $1.772 \times 10^7$  e.m.u./gm.]

16. The  $K_{\alpha}$  radiation from palladium is reflected in the first order from sylvine at glancing angles of  $5^{\circ} 23'$  from the  $\{100\}$  planes,  $7^{\circ} 37'$  from the  $\{110\}$  planes, and  $9^{\circ} 23'$  from the  $\{111\}$  planes. To what order of cubic symmetry would you assign this crystal on this evidence?

[Ans. simple cubic]

17. The wavelength of Mo  $K_{\alpha 1}$  radiation as measured by a reflection grating is  $0.7083 \times 10^{-8}$  cm. The glancing angle in the third order from the  $\{100\}$  planes of rock salt is  $22^{\circ} 10'$ . What value does this give for the lattice constant of rock salt? Assuming that the density of rock salt is 2.170 gm./c.c. and the atomic weights of sodium and chlorine are 23.00

and 35.46, what value do these measurements give for the electronic charge ?  
 [Ans.  $2.816 \times 10^{-8}$  cm. ;  $4.79 \times 10^{-10}$  e.s.u.]

18. If  $Q$   $\alpha$ -particles fall normally on a scattering foil of thickness  $t$  show that the number  $y$  of  $\alpha$ -particles falling on unit area of a fluorescent screen placed at a distance  $r$  from the foil and in a direction making an angle  $\phi$  with the incident particles is given by

$$y = \frac{Qntb^2 \operatorname{cosec}^4(\phi/2)}{16r^2},$$

where  $n$  and  $b$  have the meaning assigned in § 108.

19. Assuming that the mass of an  $\alpha$ -particle is  $6.6 \times 10^{-24}$  gm. and its velocity  $1.92 \times 10^9$  cm./sec., calculate the intensity of illumination of a fluorescent screen on which  $\alpha$ -particles fall at the rate of 1000 per sec. per sq. cm. The efficiency of the  $\alpha$ -particles as a light source may be assumed to be 1 watt per candle.  
 [Ans.  $1.2 \times 10^{-9}$  candles]

20. Calculate, on Bohr's theory, the velocity of an electron in the innermost orbit of the hydrogen atom.  
 [Ans.  $2.18 \times 10^8$  cm./sec.]

21. Calculate, on the wave mechanic theory, the wavelength associated with (a) an electron moving with the speed of the electron in the  $K$ -ring of the hydrogen atom, (b) the particle from radium C', (c) a molecule of hydrogen gas at  $0^\circ$  C. (its velocity =  $1.84 \times 10^4$  cm. per sec. at  $0^\circ$  C.).

[Ans. (a)  $3.3 \times 10^{-8}$  cm., (b)  $5.17 \times 10^{-13}$  cm., (c)  $2.15 \times 10^{-8}$  cm.]

22. Assuming that the nucleus of "heavy" hydrogen has a mass twice that of ordinary hydrogen, calculate the difference in wave numbers of the  $H$  line of ordinary hydrogen and the  $H$  line of "heavy" hydrogen. Assume the values for the Rydberg constant of hydrogen and the ratio of the masses of proton and electron given on page 261.  
 [Ans.  $4.2$  cm. $^{-1}$ ]

23. When boron is bombarded with protons generated at 100,000 volts the boron nucleus disintegrates into three  $\alpha$ -particles. Calculate, in electron-volts, the energy liberated in each disintegration (the atomic masses of hydrogen, helium, and boron are 1.0078, 4.00216, and 11.0110 respectively).  
 [Ans.  $11.7 \times 10^6$ ]

24. Calculate the energy set free by the mutual annihilation of an electron and a proton. If this energy is radiated as a single quantum, calculate its wavelength.  
 [Ans.  $1.495 \times 10^{-8}$  erg ;  $1.31 \times 10^{-13}$  cm.]

25. A radio-active element  $P$  has a half value period of 20 years, and the succeeding products  $Q$ , and  $R$ , half value periods of 19.5 and 11.4 days respectively. A quantity of  $P$  is left in a closed vessel for several months. Calculate the relative numbers of atoms of the three substances which will then be present.  
 [Ans. 1 : 0.0027 : 0.0016]

26. Assuming that the atomic mass of radon is 222 and its radio-active constant is  $2.1 \times 10^{-6}$  per sec., calculate (a) the number of  $\alpha$ -particles

emitted per sec. by 1 gm. of radon when free from its disintegration products, (b) the volume of helium formed during the complete transformation of 1 gm. of radon into radium D. [Ans.  $5.7 \times 10^{15}$  ; 304 c.c.]

27. 100 millicuries of radon are contained in a capillary glass tube 5 cm. long, with internal and external diameters of 2 mm. and 6 mm. respectively. Assuming that the thermal conductivity of glass is 0.001, and that the particles are stopped at the inner surface, what will be the difference in temperature between the inner and outer walls of the tube ? [Ans.  $0.106^\circ \text{C.}$ ]

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